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Effects of Bottom Configurations on Wave-Current Field

by

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An attempt is taken to apprehend the coexistence phenomenon of wave, current and submerged structures numerically. The governing equations of this model are derived by vertical integration of continuity equation and the equation of motion for gradually varying impermeable sea bottom and discretized with a semi-implicit technique, such that, at every time step, the resulted tridiagonal matrix can be solved by the method of double sweep. As an application, this model is applied to study the characteristics of the interacted waves and currents over submerged mounds of parabolic and trapezoidal cross sections. Results show that the change of wave characteristics for both cases are significant over the mounds.

1. INTRODUCTION

A better understanding of the local flow conditions and their resulting impact over the nearshore zone is obviously necessary in order to achieve sufficient knowledge on protecting the territory and local inhabitants. In reality most of the nearshore dynamic processes such as sediment transport, beach erosion and the resulting beach topographic change, etc. are directily influenced by the wave, current and their combined effect. Thus the wave-current coexistence phenomenon has drawn keen interest among coastal engineers and researchers. In relatively shallow waters the bottom topography also contributes to the phenomenon through altering both the wave and current. Tayfun *et al.*[1] showed that current and bottom topography can have tremendous influence on the spectral characteristics of the random waves. Kirby[2] and Liu[3] studied the effects of the current on waves by applying the modified mild slope equation. On the other hand, the studies of wave induce current has been confided on the concept of radiation stress introduced by Longuet-Higgins and Stewart[4),5]. Very recently the use of Boussinesq equations of different forms (*see* McCowan[6], Kabiling *et al.*[7], Sato *et al.*[8], Sφrensen *et al.*[9], and Watanabe *et al.*[10]) have received significant attention for the computation of wave characteristics, wave-current components and beach evolution, but still now these types of models are not yet applicable for the computation of wave-current field in considerably deep water. A direct method is thus necessary to discern the combined effects of waves, currents and topography variations. In this connection the present approach is to propose a more generally applicable numerical method which describes the coexistence of waves, currents and where the traditional iterative concept has been discarded. As an application, this model is utilized to study the interactions of waves and currents with submerged mounds of two different cross sections.

2. GOVERNING EQUATIONS

Let us assume a vertically two dimensional problem where the fluid is moving over a gradually varying impermeable sea bottom. The governing equations of the phenomenon can be described by the following continuity and equations of motion.

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$$u_x+w_z=0 \tag{1}$$

$$u_t+uu_x+wu_z+\left(\frac{p_d}{\rho}\right)_x=0 \tag{2}$$

$$w_t+uw_x+ww_z+\left(\frac{p_d}{\rho}\right)_z=0 \tag{3}$$

where u and w are the horizontal and vertical velocity components, respectively ; p_d the dynamic pressure and ρ is the fluid density. Subscripts x, z and t denote the derivative of the term with respect to space in the horizontal and vertical direction and time, respectively.

According to the exact solution for linear waves propagating over a uniform current in constant water depth, the dynamic pressure for a wave-current coexistence field can generally be postulated in the following form

$$p_d=\rho g\eta\frac{\cosh k(d+z)}{\cosh k(d+\eta)} \tag{4}$$

where η is the elevation of the free surface from the still water level, d the still water depth, k the local wave number and g is the gravitational acceleration. The wave number k can be evaluated for the known wave period, depth and current velocity from the following dispersion relation

$$(\sigma-kU)^2=gk\tanh k(d+\eta) \tag{5}$$

Using the pressure assumption Eq.(4), integration of Eqs.(1) and (2) can be performed from bottom to the free surface along with invoking the traditional kinematic boundary conditions for the free surface and bottom to give the following governing equations

$$\eta_t+Q_x=0 \tag{6}$$

$$Q_t+\left(\frac{McQ^2}{d+\eta}\right)_x+(\eta g\kappa_1+C^2)\eta_x+\eta g\kappa_2 d_x=0 \tag{7}$$

where $Q=\int_{-d}^{\eta}udz$ is the mass flux in the x direction and C is the local intrinsic wave celerity, M_C is the momentum correction factor and κ_1 and κ_2 are the relative water depth dependent parameters.

3 . NUMERICAL METHOD

Equations (6) and (7) have been discretized in a semi-implicit manner. After discretization (for complete derivation *see,* Zaman *et al.*[11]), and Togashi *et al.*[12]), the above equations constitute a second order semi-implicit finite difference scheme

$$\eta_{j+1/2}^{n+1}=\eta_{j+1/2}^{n}-\frac{\Delta t}{\Delta x}Q_{j+1}^{n+1/2}+\frac{\Delta t}{\Delta x}Q_{j}^{n+1/2} \tag{8}$$

$$A_j^{n+1}Q_{j-1}^{n+3/2}+B_j^{n+1}Q_j^{n+3/2}+C_j^{n+1}Q_{j+1}^{n+3/2}=D_j^{n+1} \tag{9}$$

where A_j^{n+1}, B_j^{n+1}, C_j^{n+1} and D_j^{n+1} are the constants and Δx and Δt are the spatial and temporal increments, respectively; the interger super- and subscripts express the values at the relevant temporal and spatial grids. The computational points of the discharge Q are defined at the integer grid points and fractional time steps and those of η are defined at the frictional grid points and integer time steps. The computation of η is performed directly by the Eq.(8) when Q is known. Eq.(9) which is used for the computation of Q, forms a tridiagonal matrix for every time step, can effectively be solved by the double sweep algorithm.

4 . RESULTS AND DISCUSSIONS

The established model is utilized to study the wave-current coexistence field over a parabolic and a trapezoidal mound shown in Figures 1 (a) and 1 (b), respectively. Both mounds are haveing the similar height and length but differs only by their structural definition.

Figs. 2 and 3 and, 4 and 5 describe the distrbu-

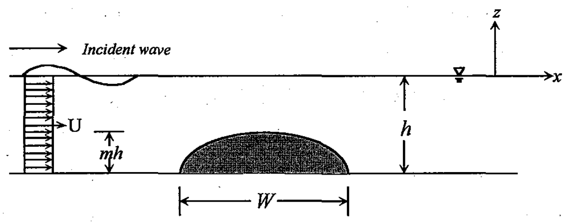

Fig. 1 (a) Schematic view of the computational domain (Parabolic mound)

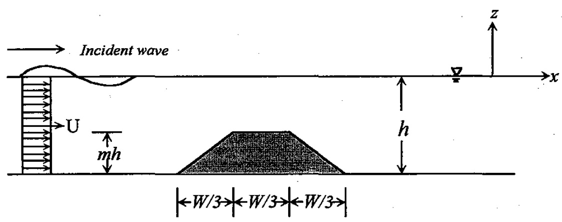

Fig. 1 (b) Schematic view of the computational domain (Trapezoidal mound) tion of wave heights and the drop of the mean water levels with various forced currents over parabolic and trapezoidal mound, respectively. In terms of non dimensional parameters, U/C_i varies from 0.14 to 0.22 while the relative water depth h/L_i is 0.3, the wave steepness H/L_i is 0.03, the relative mound height m is 0.5 and the relative mound length mh/W is 0.2. Where C_i, L_i and H_i stand for the celerity, length and height of the incident waves, respectively. From Figs. 2 and 3 it may be observed that the wave heights

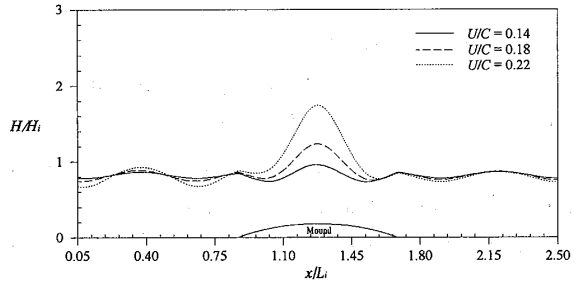

Fig. 2 Wave heights over parabolic mound

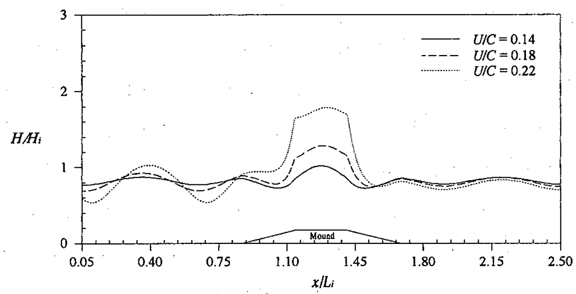

Fig. 3 Wave heights over trapezoidal mound

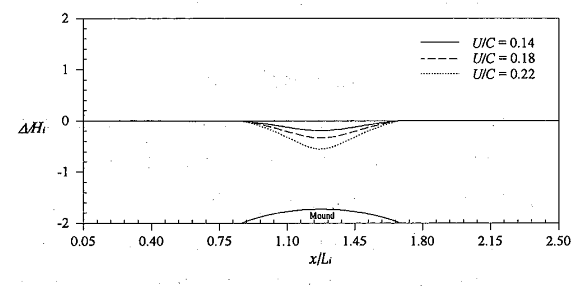

Fig. 4 Mean water level over parabolic mound

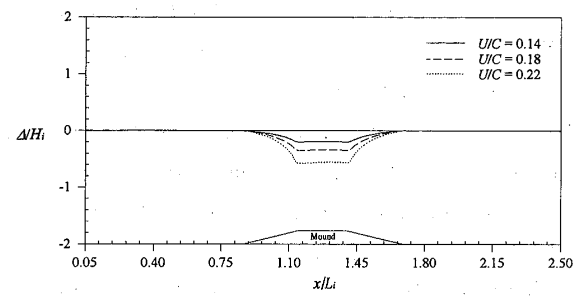

Fig. 5 Mean water level over trapezoidal mound

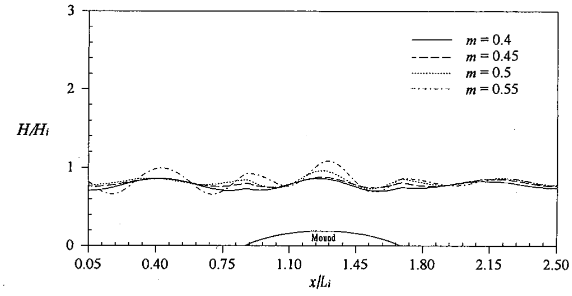

Fig. 6 Wave heights over parabolic mound

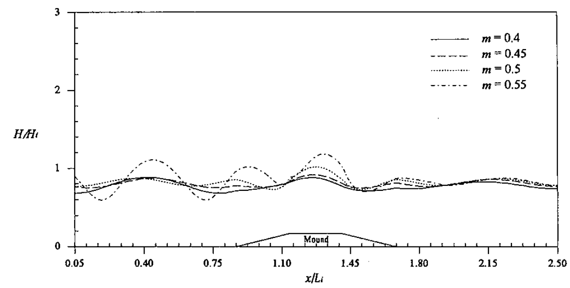

Fig. 7 Wave heights over trapezoidal mound

over the both mounds increase but the trapezoidal mound causes relatively large reflection particularly for stronger forced current. Figs. 4 and 5 describe that the drop of the mean water level over the both mounds increase with the intensified forced currents.

Figs. 6 and 7 describe the wave heights for parabolic and trapezoidal mound while Figs. 8 and 9 show the respective mean water level for varying relative mound heights where, m varies from 0.4 to 0.55 while U/C_i is 0.14, the relative water depth h/L_i is 0.3, the wave steepness H/L_i is 0.03 and the relative mound length mh/W is 0.2. Figures exhibit that the increase of the wave height and the reflection by the trapezoidal mound is greater than the parabolic one. The maximum drop of the mean water level doesn't have any significant variation in spite of the mound's shape but the water depth over it.

Figs. 10 and 11 represent the distribution of the

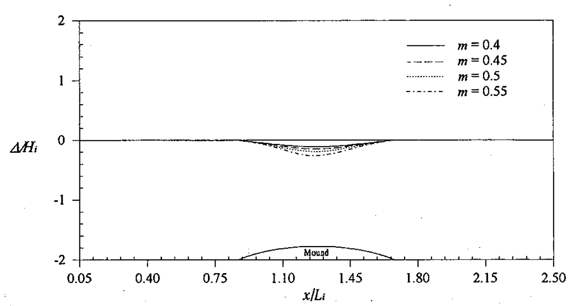

Fig. 8 Mean water level over parabolic mound

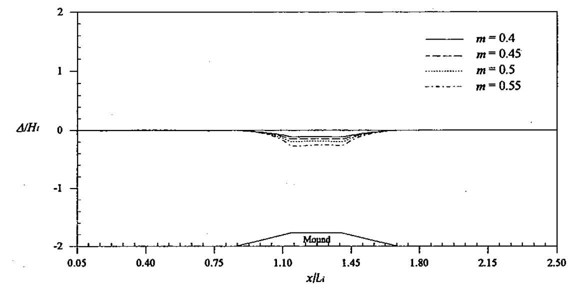

Fig. 9 Mean water level over trapezoidal mound

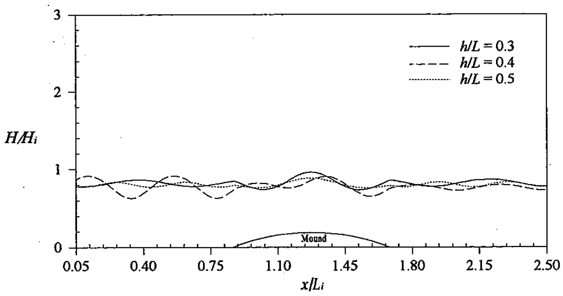

Fig.10 Wave heights over parabolic mound

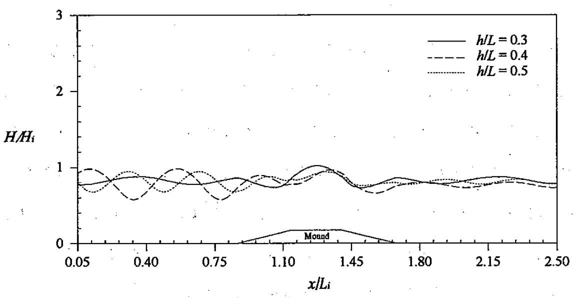

Fig.11 Wave heights over trapezoidal mound

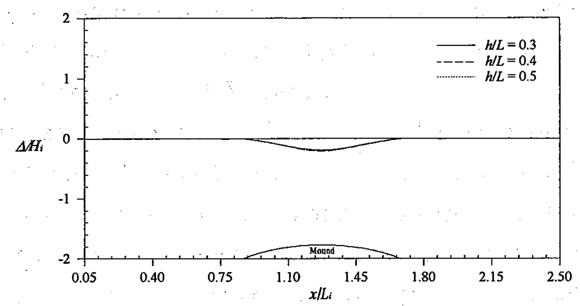

Fig.12 Mean water level over parabolic mound

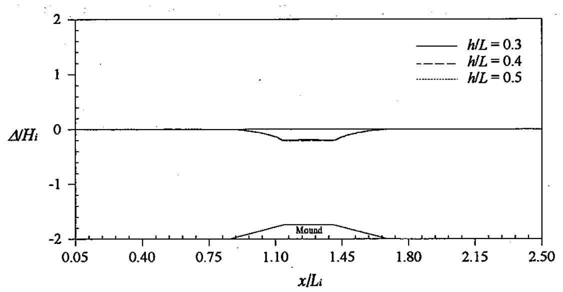

Fig.13 Mean water level over trapezoidal mound

wave heights over the mounds and, Figs. 12 and 13 notify the mean water levels for relative water depths. In this Case h/L_i varies from 0.3 to 0.5, while U/C_i being kept at 0.14, the wave steepness H/L_i at 0.03, the relative mound height m at 0.5 and the relative mound length mh/W at 0.2. Again the enlargement of the wave height over the mound as well as the reflection is relatively large for the case of the trapezoidal cross section. The maximum drop of the mean water level is the some for both mounds.

5. CONCLUSIONS

Wave-current coexistence phenomenor over a parabolic and a trapezoidal submerged mound have been investigated with a semi-implitir-numerical model. For varying forced current it is found that the enlargement of wave height and the drop of the mean water lavel over the trapezoidal mound and the reflection is in average about 5 %, 2 % and 10%, respectively than that of the parabolic mound. It is observed for varying relative mound heights that the growth of the wave height over the trapesoidal mound and the reflection is about 11% and 10% greater than the parabolic one. For the case of varying relative depths with fixed water depth, reflection by the parabolic mound is significant neither for a longer nor a shorter but for an intermediate wave length. On the other hand, reflection by a trapezoidal mound is distinct for both intermediate and longer wave lenghts.

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