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# Free Vibration Analysis of Simply Supported Square Plates Resting on Non-homogeneous Elastic Foundations

by

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A discrete method is developed for analyzing the free vibration problem of square plates resting on non-homogeneous elastic foundations. The fundamental differential equations are established for the bending problem of the plate on elastic foundations. The Green function, which is obtained by transforming these differential equations into integral equations and using numerical integration, is used to get the characteristic equation of the free vibration. The effect of the modulus of the foundation on the frequency parameters is discussed. By comparing the present numerical results with those previously published, the efficiency and accuracy of the present method are investigated.

## 1. Introduction

The free vibration problems of the plates on the elastic foundations have been studied for many years. Several numerical methods, such as the Rayleigh-Ritz method [1,2] a mixed finite element method [3,4], the method of power series expansion [5], the finite strip method [6] and the finite element method [7], were used to solve this kind of problem.

In this paper, a discrete method is proposed for analyzing the free vibration of square plates resting on non-homogeneous elastic foundations. No prior assumption of shape of deflection, such as shape functions used in the finite element method, is employed in this method. Therefore there is no need to consider the continuity of the element. The spring system is used to simulate the foundations. The fundamental differential equations of a plate on non-homogeneous foundations are established and satisfied exactly throughout the whole plate. By transforming these equations into integral equations and using numerical integration, the solutions are obtained at the discrete points. The Green function, which is the solution for deflection, is used to obtain the characteristic equation of the free vibration. Numerical results are obtained for the plates on homogeneous foundations, local uniformly distributed supports and non-homogeneous foundations. The effect of the foundation modulus on the frequency parameter is discussed. The efficiency and accuracy of the present method for the free vibration of square plates on elastic foundation are investigated.

## 2. Fundamental Differential Equations

Figure 1 shows a square plate of length $a$, thickness $h$ and density $\rho$ resting on non-homogeneous foundations of foundation modulus $k_f$($f$=1 or 2). The foundation modulus in the central square part is $k_2$, and that for the other part is $k_1$. An $xyz$ coordinate system is used in the present study with its $x-y$ plane contained in middle plane of the square plate, the $z$-axis perpendicular to the middle plane of the plate and the origin at one of the corners of the plate.

In this paper, the elastic foundation is modelled as a spring system and the intensity of the reaction of the foundation is assumed to be proportional to the deflection $w$ of the plate. By considering the reaction of the foundation as a kind of lateral load, the fundamental

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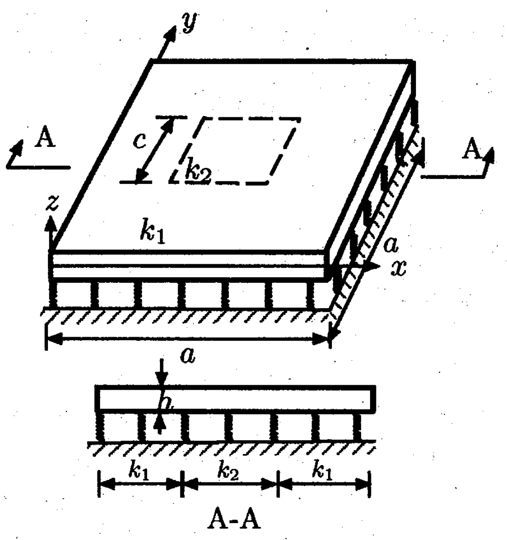


Figure 1 : A square plate resting on non-homogenous elastic foundations.

differential equations of the plate having a concentrated load $\overline{P}$ at a point $(x_q, y_r)$ and resting on a Winkler foundation of the foundation modulus $k_f$ are as follows:

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\overline{P}\delta(x-x_q)\delta(y-y_r)-k_f w=0,$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=0,$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0,$$

$$\frac{\partial \theta_x}{\partial x}+\nu\frac{\partial \theta_y}{\partial y}=\frac{M_x}{D},$$

$$\frac{\partial \theta_y}{\partial y}+\nu\frac{\partial \theta_x}{\partial x}=\frac{M_y}{D},$$

$$\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}=\frac{2}{(1-\nu)}\frac{M_{xy}}{D},$$

$$\frac{\partial w}{\partial x}+\theta_x=\frac{Q_x}{Gt_s},$$

$$\frac{\partial w}{\partial y}+\theta_y=\frac{Q_y}{Gt_s}, \qquad (1)$$

where $Q_x$, $Q_y$ are the shearing forces, $M_{xy}$ the twisting moment, $M_x$, $M_y$ the bending moments, $\theta_x$, $\theta_y$ the slopes, $w$ the deflection, $D=Eh^3/(12(1-\nu^2))$ the bending rigidity, $E$, $G$ modulus, shear modulus of elasticity, respectively, $\nu$ Poisson's ratio, $h$ the thickness of plate, $t_s=h/1.2$, $\delta(x-x_q)$, $\delta(x-x_r)$ Dirac's delta functions.

By introducing the non-dimensional expressions,

$$[X_1, X_2]=\frac{a^2}{D_0(1-\nu^2)}[Q_x, Q_y],$$

$$[X_3, X_4, X_5]=\frac{a}{D_0(1-\nu^2)}[M_{xy}, M_y, M_x],$$

$$[X_6, X_7, X_8]=[\theta_y, \theta_x, w/a],$$

the equation (1) is rewritten as the following non-dimensional forms:

$$\mu\frac{\partial X_2}{\partial \eta}+\frac{\partial X_1}{\partial \zeta}+P\delta(\eta-\eta_q)\delta(\zeta-\zeta_r)-\overline{k}_f X_8=0,$$

$$\mu\frac{\partial X_3}{\partial \eta}+\frac{\partial X_4}{\partial \zeta}-\mu X_1=0,$$

$$\mu\frac{\partial X_5}{\partial \eta}+\frac{\partial X_3}{\partial \zeta}-\mu X_2=0,$$

$$\mu\frac{\partial X_7}{\partial \eta}+\nu\frac{\partial X_6}{\partial \zeta}-\overline{D}X_5=0,$$

$$\nu\mu\frac{\partial X_7}{\partial \eta}+\frac{\partial X_6}{\partial \zeta}-\overline{D}X_4=0,$$

$$\mu\frac{\partial X_6}{\partial \eta}+\frac{\partial X_7}{\partial \zeta}-\frac{2}{1-\nu}\overline{D}X_3=0,$$

$$\frac{\partial X_8}{\partial \eta}+X_7-\overline{H}X_2=0,$$

$$\frac{\partial X_8}{\partial \zeta}+\mu X_6-\mu\overline{H}X_1=0, \qquad (2)$$

where $\mu=1.0$, $\overline{D}=\mu(1-\nu^2)(h_0/h)^3$, $\overline{H}=((1+\nu)/5)(h_0/a)^2h_0/h$, $P=\overline{P}a/(D_0(1-\nu^2))$, $D_0=Eh_0^3/(12(1-\nu^2))$ is the standard bending rigidity, $h_0$ is the standard thickness of the plate, $k=5/6$ is the shear correction factor, $\delta(\eta-\eta_q)$ and $\delta(\zeta-\zeta_r)$ are Dirac's delta functions, $\overline{k}_f=\mu K_f/(1-\nu^2)$, $K_f$ is the dimensionless modulus of the foundation, it is defined as follows:

$$K_f=k_f a^4/D_0 \quad (f=1,2),$$

The equation (2) can also be expressed as the following simple form.

$$\sum_{s=1}^{8}\{F_{1ts}\frac{\partial X_s}{\partial \zeta}+F_{2ts}\frac{\partial X_s}{\partial \eta}+F_{3ts}X_s\}$$
$$+P\delta(\eta-\eta_q)\delta(\zeta-\zeta_r)\delta_{1t}=0 \quad (t=1\sim 8), \qquad (3)$$

where $\delta_{1t}$ is Kronecker's delta, $F_{111}=F_{124}=F_{133}=F_{156}=F_{167}=F_{188}=1$, $F_{146}=\nu$, $F_{212}=F_{223}=F_{235}=F_{247}=F_{266}=\mu$, $F_{257}=\mu\nu$, $F_{278}=1$, $F_{318}=-\overline{k}_f$, $F_{321}=F_{332}=-\mu$, $F_{345}=F_{354}=-\mu(1-\nu^2)\overline{D}$, $F_{363}=-2\mu(1+\nu)\overline{D}$, $F_{372}=-((1+\nu)/5)(h_0/a)^2\overline{DT}$, $F_{377}=1$, $F_{381}=-\mu((1+\nu)/5)(h_0/a)^2\overline{DT}$, $F_{386}=\mu$, other $F_{kts}=0$.

## 3. Discrete Green Function

As given in Ref[8], by dividing a square plate vertically into $m$ equal-length parts and horizontally into $n$ equal-length parts as shown in Figure 2, the plate can be considered as a group of discrete points which are the intersections of the $(m+1)$-vertical and $(n+1)$-horizontal dividing lines. To describe the present method conveniently, the rectangular area, $0\leq\eta\leq\eta_i$, $0\leq\zeta\leq\zeta_j$, corresponding to the arbitrary intersection $(i, j)$ as shown in Figure 1 is denoted as the area $[i, j]$, the intersection $(i, j)$ denoted by ◯ is called the main point of the area $[i, j]$, the intersections denoted by ○ are called the inner dependent points of the area, and the intersections denoted by ● are called the boundary dependent points of the area.

By integrating the equation (3) over the area $[i, j]$, the following integral equation is obtained:

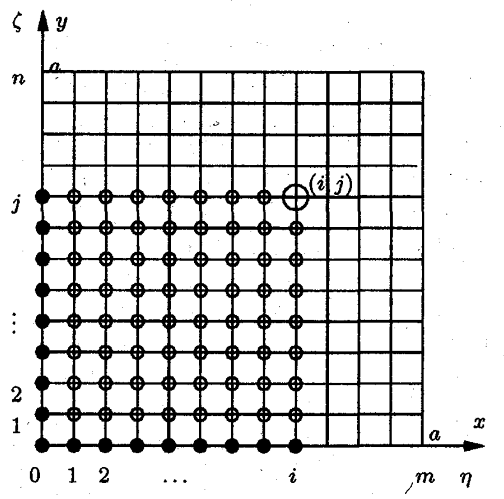


Figure 2 : Discrete points on a rectangular plate.

$$\sum_{s=1}^{8}\Big\{F_{1ts}\int_0^{\eta_i}[X_s(\eta,\ \zeta_j)-X_s(\eta,0)]d\eta + F_{2ts}\int_0^{\zeta_j}[X_s(\eta_i,\ \zeta)-X_s(0,\ \zeta)]d\zeta + F_{3ts}\int_0^{\eta_i}\int_0^{\zeta_j}X_s(\eta,\zeta)d\eta\, d\zeta\Big\} + Pu(\eta-\eta_q)u(\zeta-\zeta_r)\delta_{1t}=0 \qquad (4)$$

where $u(\eta-\eta_q)$ and $u(\zeta-\zeta_r)$ are the unit step functions.

Next, by applying the numerical integration method, the simultaneous equation for the unknown quantities $X_{sij}=X_s(\eta_i,\ \zeta_j)$ at the main point $(i, j)$ of the area $[i, j]$ is obtained as follows:

$$\sum_{s=1}^{8}\Big\{F_{1ts}\sum_{k=0}^{i}\beta_{ik}(X_{skj}-X_{sk0}) + F_{2ts}\sum_{l=0}^{j}\beta_{jl}(X_{sil}-X_{s0l}) + F_{3ts}\sum_{k=0}^{i}\sum_{l=0}^{j}\beta_{ik}\,\beta_{jl}X_{skl}\Big\} + Pu_{iq}u_{jr}\delta_{1t}=0, \qquad (5)$$

where $\beta_{ik}=\alpha_{ik}/m$, $\beta_{jl}=\alpha_{jl}/n$, $\alpha_{ik}=1-(\delta_{0k}+\delta_{ik})/2$, $\alpha_{jl}=1-(\delta_{0l}+\delta_{jl})/2$, $t=1\sim8$, $i=1\sim m$, $j=1\sim n$, $u_{iq}=u(\eta_i-\eta_q)$, $u_{jr}=u(\zeta_j-\zeta_r)$.

By retaining the quantities at main point $(i, j)$ on the left hand side of the equation and putting other quantities on the right hand side, and using the matrix transition, the solution $X_{pij}$ of the above equation (5) is obtained as follows:

$$X_{pij}=\sum_{t=1}^{8}\Big\{\sum_{k=0}^{i}\beta_{ik}A_{pt}[X_{tk0}-X_{tkj}(1-\delta_{ik})] + \sum_{l=0}^{j}\beta_{jl}B_{pt}[X_{t0l}-X_{til}(1-\delta_{jl})] + \sum_{k=0}^{i}\sum_{l=0}^{j}\beta_{ik}\,\beta_{jl}C_{ptkl}X_{tkl}(1-\delta_{ik}\delta_{jl})]\Big\} - A_{p1}Pu_{iq}u_{jr}, \qquad (6)$$

where $p=1\sim8$, $A_{pt}$, $B_{pt}$ and $C_{ptkl}$ are given in Ref[8].

In the equation (6), the quantity $X_{pij}$ is not only related to the quantities $X_{tk0}$ and $X_{t0l}$ at the boundary dependent points but also the quantities $X_{tkj}$, $X_{til}$ and $X_{tkl}$ at the inner dependent points. The maximal number of the unknown quantities is $6(m-1)(n-1)+3(m+n+1)$. In order to reduce the unknown quantities, the area $[i, j]$ is spread according to the regular order as [1,1], [1,2], ⋯, [1,$n$], [2,1], [2,2], ⋯, [2,$n$], ⋯, [$m$,1], [$m$,2], ⋯, [$m$, $n$]. With the spread of the area according to the above mentioned order, the quantities $X_{tkj}$, $X_{til}$ and $X_{tkl}$ at the inner dependent points can be eliminated by substituting the obtained results into the corresponding terms of the right hand side of equation (6). By repeating this process, the quantity $X_{pij}$ at the main point is only related to the quantities $X_{rk0}$ $(r=1,3,4,6,7,8)$ and $X_{s0l}$ $(s=2,3,5,6,7,8)$ at the boundary dependent points. The maximal number of the unknown quantities is reduced to $3(m+n+1)$. It can be noted the number of the unknown quantities of the present method is fewer than that of the finite element method for the same divisional number $m(\geq 3)$ and $n(\geq 3)$. Based on the above consideration, the equation (6) is rewritten as follows.

$$X_{pij}=\sum_{d=1}^{6}\left\{\sum_{f=0}^{i} a_{pijfd}X_{rf0}+\sum_{g=0}^{i} b_{pijgd}X_{s0g}\right\}+\bar{q}_{pij}P,$$

where $a_{pijfd}$, $b_{pijgd}$ and $\bar{q}_{pij}$ are given in Appendix A.

The equation (7) gives the discrete solution of the fundamental differential equation (3) of the bending problem of a plate resting on an elastic foundation and having a concentrated load, and the discrete Green function is chosen as $X_{8ij}a^2/[PD_0(1-\nu^2)]$, that is $w(x_0,y_0,x,y)/\bar{P}$.

## 4. Characteristic equation

By applying the Green function $w(x_0,y_0,x,y)/\bar{P}$ which is the displacement at a point $(x_0,y_0)$ of a plate with a concentrated load $\bar{P}$ at a point $(x, y)$, the displacement amplitude $\hat{w}(x_0,y_0)$ at a point $(x_0,y_0)$ of the square plate during the free vibration is given as follows:

$$\hat{w}(x_0,y_0)=\int_0^b\int_0^a \rho h\omega^2\hat{w}(x,y)[w(x_0,y_0,x,y)/\bar{P}]dxdy, \quad (8)$$

where $\rho$ is the mass density of the plate material.

By using the numerical integration method and the following non-dimensional expressions,

$$\lambda^4=\frac{\rho_0 h_0\omega^2a^4}{D_0(1-\nu^2)},\quad k=1/(\mu\lambda^4),$$

$$H(\eta,\zeta)=\frac{\rho(x,y)}{\rho_0}\frac{h(x,y)}{h_0},\quad W(\eta,\zeta)=\frac{\hat{w}(x,y)}{a},$$

$$G(\eta_0,\zeta_0,\eta,\zeta)=\frac{w(x_0,y_0,x,y)}{a}\frac{D_0(1-\nu^2)}{\bar{P}a},$$

where $\rho_0$ is the standard mass density, the characteristic equation is obtained from the equation (8) as

$$\begin{vmatrix} K_{00} & K_{01} & K_{02} & \cdots & K_{0m}\\ K_{10} & K_{11} & K_{12} & \cdots & K_{1m}\\ K_{20} & K_{21} & K_{22} & \cdots & K_{2m}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ K_{m0} & K_{m1} & K_{m2} & \cdots & K_{mm}\end{vmatrix}=0 \quad (9)$$

where

$$K_{ij}=\beta_{mj}\begin{bmatrix} \beta_{n0}H_{j0}G_{i0j0}-k\delta_{ij} & \cdots & \beta_{nn}H_{jn}G_{i0jn}\\ \beta_{n0}H_{j0}G_{i1j0} & \cdots & \beta_{nn}H_{jn}G_{i1jn}\\ \beta_{n0}H_{j0}G_{i2j0} & \cdots & \beta_{nn}H_{jn}G_{i2jn}\\ \vdots & \vdots & \vdots\\ \beta_{n0}H_{j0}G_{inj0} & \cdots & \beta_{nn}H_{jn}G_{injn}-k\delta_{ij}\end{bmatrix}$$

## 5. Numerical results

To investigate the validity of the proposed method, the frequency parameters are given for the plate shown in Figure 1. The standard thickness $h_0$ is chosen as $h$ and $h_0/a=1/1000$ is used. All the convergent values of the frequency parameters are obtained for simply supported plates by using Richardson's extrapolation formula for two cases of divisional numbers $m(=n)$. Some of the results are compared with those reported previously.

### 5.1. A square plate on homogeneous foundations

Table 1 shows the numerical values for the lowest 4 natural frequency parameter $\lambda$ of square plates on homogeneous foundation with $K=0,10,100,1000,10000$. The convergent results of frequency parameter

Table 1: Natural frequency parameter $\lambda$ for a SSSS square plate on homogeneous foundations

| | | Mode sequence number | | | |
|---|---|---|---|---|---|
| $K$ | References | 1st | 2nd | 3rd | 4th |
| 0 | Present | | | | |
| | 12×12 | 4.575 | 7.336 | 7.336 | 9.311 |
| | 16×16 | 4.564 | 7.272 | 7.272 | 9.216 |
| | Ex. | 4.549 | 7.190 | 7.190 | 9.094 |
| | Ref.[5] | 4.549 | – | – | – |
| | Exact[9] | 4.549 | 7.192 | 7.192 | 9.098 |
| 10 | Present | | | | |
| | 12×12 | 4.603 | 7.343 | 7.343 | 9.315 |
| | 16×16 | 4.592 | 7.279 | 7.279 | 9.220 |
| | Ex. | 4.578 | 7.198 | 7.198 | 9.098 |
| | Ref.[5] | 4.578 | – | – | – |
| $10^2$ | Present | | | | |
| | 12×12 | 4.838 | 7.405 | 7.405 | 9.345 |
| | 16×16 | 4.829 | 7.343 | 7.343 | 9.251 |
| | Ex.12 | 4.816 | 7.263 | 7.263 | 9.131 |
| | Ref.[5] | 4.816 | – | – | – |
| $10^3$ | Present | | | | |
| | 12×12 | 6.261 | 7.950 | 7.950 | 9.635 |
| | 16×16 | 6.257 | 7.900 | 7.900 | 9.549 |
| | Ex. | 6.251 | 7.836 | 7.836 | 9.439 |
| | Ref.[5] | 6.251 | – | – | – |
| $10^4$ | Present | | | | |
| | 12×12 | 10.339 | 10.855 | 10.855 | 11.664 |
| | 16×16 | 10.338 | 10.836 | 10.836 | 11.616 |
| | Ex. | 10.337 | 10.811 | 10.811 | 11.554 |
| | Ref.[5] | 10.337 | – | – | – |

Ex.: The values obtained by using Richardson's extrapolation formula.

are obtained by using Richardson's extrapolation formula for two cases of divisional numbers $m(=n)$ of 12 and 16. The results obtained by Matsunaga [5] and the exact values of the plate with $K=0$[9] are also shown in the table. It can be seen that the numerical results of the present method have satisfactory accuracy. From this table, it can be also seen that the effect of the constant $K$ on the fundamental frequency parameter is much more significant than that on higher frequency parameters, the frequency parameters increase with increase of the constant $K$, and they increase quickly when $K$ is larger than 100.

Table 2: Natural frequency parameter $\lambda$ for a SSSS square plate with the central part on local uniform supports ($c/a=0.6$)

| $K_1$ | $K_2$ | References | Mode sequence number 1st | 2nd | 3rd |
|---|---|---|---|---|---|
| 0 | 320 | Present | | | |
| | | 10×10 | 5.156 | 7.552 | 7.552 |
| | | 15×15 | 5.157 | 7.434 | 7.434 |
| | | Ex. | 5.158 | 7.341 | 7.341 |
| | | Ref.[2] | 5.168 | – | – |
| | | Ref.[7] | 5.123 | – | – |
| 0 | 800 | Present | | | |
| | | 10×10 | 5.766 | 7.736 | 7.736 |
| | | 15×15 | 5.782 | 7.627 | 7.627 |
| | | Ex. | 5.794 | 7.540 | 7.540 |
| | | Ref.[2] | 5.813 | – | – |
| | | Ref.[7] | 5.774 | – | – |
| 0 | 1600 | Present | | | |
| | | 10×10 | 6.491 | 8.000 | 8.000 |
| | | 15×15 | 6.514 | 7.907 | 7.907 |
| | | Ex. | 6.533 | 7.833 | 7.833 |
| | | Ref.[2] | 6.563 | – | – |
| | | Ref.[7] | 6.517 | – | – |
| 320 | 0 | Present | | | |
| | | 15×15 | 4.757 | 7.346 | 7.346 |
| | | 20×20 | 4.747 | 7.296 | 7.296 |
| | | Ex. | 4.733 | 7.232 | 7.232 |
| | | Ref.[2] | 4.715 | – | – |
| | | Ref.[7] | 4.656 | – | – |
| 800 | 0 | Present | | | |
| | | 15×15 | 4.998 | 7.447 | 7.447 |
| | | 20×20 | 4.971 | 7.425 | 7.425 |
| | | Ex. | 4.936 | 7.397 | 7.397 |
| | | Ref.[2] | 4.937 | – | – |
| | | Ref.[7] | 4.871 | – | – |
| 1600 | 0 | Present | | | |
| | | 15×15 | 5.316 | 7.618 | 7.618 |
| | | 20×20 | 5.281 | 7.595 | 7.595 |
| | | Ex. | 5.235 | 7.566 | 7.566 |
| | | Ref.[2] | 5.250 | – | – |
| | | Ref.[7] | 5.161 | – | – |

## 5.2 A square plate on non-homogenous foundations

Table 2 shows the numerical values for the lowest 3 natural frequency parameter $\lambda$ of the plate shown in Figure 1 with $K_1=0$ or $K_2=0$, which is the case of the local uniformly distributed support. The side ratio of the local square part and the plate $c/a=0.6$ and the thickness ratio $h_1/h_2=1.0$ are adopted. The convergent results of frequency parameter are obtained by using Richardson's extrapolation formula for two cases of divisional numbers $m(=n)$ pointed in Table 2. The present results are compared with those obtained by Laura and Gutiérrez[2] and Ju, Lee and Lee [7]. They are in good agreement.

Table 3 shows the numerical values for the lowest 3 natural frequency parameter $\lambda$ of the plate on non-homogeneous foundations with $h_1/h_2=1.0$, $c/a=0.6$ and four kinds of combination of $K_1$ and $K_2$. The convergent results of frequency parameter are obtained by using Richardson's extrapolation formula for two cases of divisional numbers $m(=n)$ pointed in Table 3. The present results are also in good agreement

Table 3: Natural frequency parameter $\lambda$ for a SSSS square plate with the central part on non-homogeneous foundations ($c/a=0.6$)

| $K_1$ | $K_2$ | References | Mode sequence number 1st | 2nd | 3rd |
|---|---|---|---|---|---|
| 320 | 800 | Present | | | |
| | | 10×10 | 5.878 | 7.798 | 7.798 |
| | | 15×15 | 5.883 | 7.695 | 7.695 |
| | | Ex. | 5.887 | 7.613 | 7.613 |
| | | Ref.[2] | 5.895 | – | – |
| | | Ref.[3] | 5.862 | – | – |
| 320 | 1600 | Present | | | |
| | | 10×10 | 6.571 | 8.063 | 8.063 |
| | | 15×15 | 6.588 | 7.974 | 7.974 |
| | | Ex. | 6.602 | 7.902 | 7.902 |
| | | Ref.[2] | 6.620 | – | – |
| | | Ref.[7] | 6.584 | – | – |
| 800 | 320 | Present | | | |
| | | 15×15 | 5.476 | 7.602 | 7.602 |
| | | 20×20 | 5.456 | 7.596 | 7.596 |
| | | Ex. | 5.430 | 7.589 | 7.589 |
| | | Ref.[2] | 5.446 | – | – |
| | | Ref.[7] | 5.402 | – | – |
| 1600 | 320 | Present | | | |
| | | 15×15 | 5.729 | 7.767 | 7.767 |
| | | 20×20 | 5.710 | 7.731 | 7.731 |
| | | Ex. | 5.685 | 7.684 | 7.684 |
| | | Ref.[2] | 5.685 | – | – |
| | | Ref.[7] | 5.627 | – | – |

with those obtained by Laura and Gutiérrez[2] and Ju, Lee and Lee[7]. From Tables 1～3, it can be seen the present method can be used to solve the problem of plates on homogeneous foundations, local uniformly distributed supports and non-homogeneous foundations.

## 6. Conclusions

A discrete method is extended for analyzing the free vibration problem of square plates with stepped thickness on the elastic foundations. No prior assumption of shape of deflection, such as shape functions used in the Finite Element Method, is employed in this method. Therefore there is no need to consider the continuity of the element. The spring system is used to simulate the foundations. The characteristic equation of the free vibration is gotten by using the Green function. The effects of the elastic constant of the foundations and the stepped thickness on the frequencies are considered. The results by the present method have been compared with those previously reported. It shows that the present results have a good convergence and satisfactory accuracy.

## ACKNOWLEDGEMENTS

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## Appendix A

$a_{1i0i1}=a_{3i0i2}=a_{4i0i3}=1,\quad a_{6i0i4}=a_{7i0i5}=a_{8i0i6}=1$

$b_{20jj1}=b_{30jj2}=b_{50jj3}=1,\quad b_{60jj4}=b_{70jj5}=b_{80jj6}=1,\quad b_{30002}=0$

$$a_{pijfd}=\sum_{t=1}^{8}\Big\{\sum_{k=0}^{i}\beta_{ik}A_{pt}[a_{tk0fd}-a_{tkjfd}(1-\delta_{ki})] +\sum_{l=0}^{j}\beta_{jl}B_{pt}[a_{t0lfd}-a_{tilfd}(1-\delta_{lj})] +\sum_{k=0}^{i}\sum_{l=0}^{j}\beta_{ik}\beta_{jl}C_{ptkl}a_{tklfd}(1-\delta_{ki}\delta_{lj})]\Big\}$$

$$b_{pijfd}=\sum_{t=1}^{8}\Big\{\sum_{k=0}^{i}\beta_{ik}A_{pt}[b_{tk0gd}-b_{tkjgd}(1-\delta_{ki})] +\sum_{l=0}^{j}\beta_{jl}B_{pt}[b_{t0lgd}-b_{tilgd}(1-\delta_{lj})] +\sum_{k=0}^{i}\sum_{l=0}^{j}\beta_{ik}\beta_{jl}C_{ptkl}b_{tklgd}(1-\delta_{ki}\delta_{lj})]\Big\}$$

$$\bar{q}_{pij}=\sum_{t=1}^{8}\Big\{\sum_{k=0}^{i}\beta_{ik}A_{pt}[\bar{q}_{tk0}-\bar{q}_{tkj}(1-\delta_{ki})] +\sum_{l=0}^{j}\beta_{jl}B_{pt}[\bar{q}_{t0l}-\bar{q}_{til}(1-\delta_{lj})] +\sum_{k=0}^{i}\sum_{l=0}^{j}\beta_{ik}\beta_{jl}C_{ptkl}\Big\}-A_{p1}u_{iq}u_{jr}$$