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Theoretical Prediction of Shear Strength Evolution in Steel Fibre Reinforced Concrete Beams without Stirrups

By

Timothy NYOMBOI¹, Hiroshi MATSUDA², Ryu HIRAYAMA³, and Hiroshi NISHIDA⁴,

Recent research has shown that steel fibres significantly increase the shear capacity and ductility in reinforced concrete (RC) beams. Utilization of this structural capacity in RC beams has however, been limited by lack of design guidelines. Conventional methods applied in normal design are not applicable in this case. Furthermore there exists no unified expression for the complete characterization of shear strength and ductility in beams. Fundamentally, steel fibres contribution should be considered based on stress transfer mechanism, augmented by concrete and dowel action of the main reinforcements in a unified manner. This paper proposes a unified analytical model in which the complete behavior of steel fibre reinforced concrete beams is characterized. Verification of the model was found to be in agreement with the experimental results tested by the authors. Non linear behavior as well as increase in strength observed in the fibrous beams was predicted well.

Key words: Steel fibres concrete, Theoretical model, Shear Strength, Electronic speckle interferiometry (ESPI)

1. Introduction

Current application of steel fibres in concrete structures is found in areas where improved crack control, fatigue resistance, earthquakes resistance, impact loads and slope stabilizations (using fibre short-Crete) is necessary. Many researchers [1-5, 7] have also established that use of steel fibres in concrete, lead to increased shear capacity and ductility in reinforced concrete. The knowledge of the behavior and ability to predict the same is therefore paramount to the development of guidelines for design applications and utilization of the aforementioned benefit in structural systems such as beams. Researchers have acknowledged that shear phenomenon is a complex and difficult property to predict [3, 4, and 6]. A lot has been done on the computation of the ultimate shear capacity mainly with the use of simplified models and experimental data [2, 3, 5, and 7]. However, there is no much information on a rational method for the prediction of the actual contribution by the steel fibres concrete composite and the dowel action of the main reinforcements. In this paper a simplified strain ratio based analytical model for the prediction of complete evolution of shear strength in steel fibre reinforced concrete (s.f.r.c) beams failing in shear is proposed. The shear resistance due to steel fibre reinforced concrete and the dowel action of the main reinforcements have all been considered based on equilibrium of forces and the stress transfer mechanisms.

2. Shear Strength analytical model derivations

In the derivation of the analytical model, the expressions for the various forces acting to resist the shear stress were first determined as outlined in the subsequent sections. Finally equilibrium conditions between the internal forces and the applied external load were evaluated to arrive at a unified predictive relation. The following assumptions were considered.

(i) Plane sections remain plane
(ii) Failure is predominantly by shear
(iii) Shear crack occurs at an angle of 45 degrees
(iv) Concrete is brittle while steel fibres are elastic,
(v) Fibre ultimately pull out from one side
(vi) Re bars dowel action contribute to shear strength

The geometry and loading conditions used in the derivation are as indicated in Fig.1

Due to symmetry, only half of the geometry in Fig.1 (portion JKLM) has been considered and its sheared profile analyzed. Based on Gere and Timoshenko’s shear deformations in a beam [8], the cracked sheared profile of portion JKLM has been assumed to correspond to the profile shown in Fig.2.0(a) while the stress, strain and crack opening diagrams along the crack path have been considered to be as shown in Fig.3.(a), (b), (c)(d).

From the geometry in Fig.1 and the stress profile (Fig.3a), the expression for the concrete compressive and tensile forces along the idealised crack path can be determined. The compressive force component is obtained from the following relation:

\[ F_{cc} = \frac{F_{c}}{\sin \alpha} = \sigma_{ct} \left( c - \frac{w}{\psi} \right) \]  \hspace{1cm} (1)

Where \( b \) and \( w \) are the beam and crack widths respectively, while \( \psi \) is the angle of crack rotation. As idealized in Fig.3a, it is assumed that the concrete possesses some minimal tensile strength. The resistive tensile force from the concrete is expressed as:

\[ F_{ct} = \sigma_{ct} \left( \frac{w}{\psi} \right) \]  \hspace{1cm} (2)

Where \( \sigma_{ct} \) is the tensile strength of plain concrete.

(2) Shear forces in compressed and cracked region

Determination of the concrete and crack-slip shearing forces (\( F_w \) and \( F_{cc} \)) in the compressed region and cracked region respectively, are considered in a unified manner under equilibrium analysis of all the forces (see section 2.2).

(3) Fibre tensile forces \( F_{t(1)} \) and \( F_{t(2)} \)

In order to determine the expression for the steel fibre tensile forces \( F_{t(1)} \) and \( F_{t(2)} \) as shown in Fig 3(a), expressions for the average normal fiber force and strain is first established.

Average normal fibre force and pull out strain

The derivations are made by considering an infinitesimal force \( dF \) as shown in Fig 3(a) and orientation of the fibers across a shear crack, Fig.4 In the derivation, two regimes are considered as illustrated by the stress diagram in Fig.3 (a). These are;

- Elastic range (fibers elastically strain) \( 0 \leq x \leq x_1 \)
- Pull out range (fibers pulling out) \( x_1 \leq x \leq \frac{w}{\psi} \)

Elastic Range

The force per fibre crossing the crack at right angles in the elastic range is determined as:

\[ F_x = E_f A_f \frac{w}{l_f} = E_f A_f \frac{x \psi_f}{l_f} \]  \hspace{1cm} (3)

Where, \( A_f \) and \( E_f \) are the area, elastic modulus, strain and fibre length, respectively.

From Fig.3 (d), the crack width \( w \) at any distance \( x \) is obtained \( w = x \tan \psi \geq x \psi \) where \( \psi \) is small. Thus eq. 3 becomes

\[ F_x = E_f A_f \frac{w}{l_f} = E_f A_f \frac{x \psi_f}{l_f} \]  \hspace{1cm} (4)

(Where \( l_f \) is the)

The fibres are randomly distributed (Fig.4). The average normal fibre force is determined as:

\[ F_{nv} = \frac{1}{\pi} \int_0^{\pi} F_x \sin \theta d\theta \]  \hspace{1cm} (5)
Substituting for the force per fibre from eq.4 and integrating:

\[ F_{p} = \frac{2}{\pi} E f A_f \frac{x \psi}{l_f} \]  \hspace{1cm} (6)

**Pull out range**

The average normal fibre force in the pull out range is determined as

\[ F_{p} = \frac{2}{\pi} E f A_f \varepsilon_{fp} \]  \hspace{1cm} (7)

Where \( \varepsilon_{fp} = \frac{x \psi}{l_f} \) is the fibre strain, equivalent to the effective pullout strain value after cracking.

**Fibre pull-out strain** \( \varepsilon_{fp} \)

The fibre pull-out strain is a function of the bond stress \( \tau_b \) and the fibre aspect ratio \( A_f \). To derive an expression for the pull out strain, an arbitrarily fibre pull out mechanism as shown in Fig.5 is considered.

The equilibrium of force between the concrete matrix and a fibre under tension will be:

\[ A_f \sigma_f = A_c \sigma_c \]  \hspace{1cm} (8)

Where \( A_f \) and \( A_c \) are the fibre and concrete areas respectively while \( \sigma_f \) and \( \sigma_c \) are the fibre and concrete stresses respectively. Assuming that the effect of the fibre compression force is negligible:

\[ A_f \sigma_f = A_c \sigma_c \]  \hspace{1cm} (9)

Equilibrium of forces at the fibre-concrete interface is expressed as:

\[ A_f \sigma_f = \tau_b \pi d_f l_f \]  \hspace{1cm} (10)

For fibre pull out to occur, the force in the fibre should exceed the interfacial shear force. This can be expressed by combining eqs. 9 and 10 as:

\[ A_f E_f \varepsilon_{fp} \geq \tau_b \pi d_f l_f \]  \hspace{1cm} (11)

\[ e_{fp} = \frac{\tau_b \pi d_f l_f}{A_f E_f} \]  \hspace{1cm} (12)

Experimental investigations have shown that the net fibre pull-out length is equal to \( l_f / 4 \)\(^7\), thus eq. 12 can be re-written as:

\[ e_{fp} = \frac{\sigma_f A_f}{E_f} \text{ or } e_{fp} = \frac{\sigma_f}{E_f} \]  \hspace{1cm} (13)

Where \( A_f = \frac{l_f}{d_f} \) = fibre aspect ratio

For fibre pull-out equilibrium mechanism

\( F_{p(i)} = \int_{0}^{l_f} dF \)  \hspace{1cm} (14)

\[ N_f = \text{Number of fibres cross the crack} \]


\[ V_f = \frac{N_f A_f}{b d x} \]  \hspace{1cm} (15)

Substituting for \( F_{p(i)} \) from eq. 6 and \( N_f \) from eq. 15 above, the relation for \( F_{p(i)} \) in eq. 14 is obtained as:

\[ F_{p(i)} = \frac{2 E f b V_f \psi}{\pi d_f} \int_{0}^{l_f} x dx = \frac{E_f b V_f \psi}{\pi l_f} \]  \hspace{1cm} (16)

Assuming that at pull out stage, the strain in the fibre is equal to the pull out strain:

\[ e_f = e_{fp} \]  \hspace{1cm} (17)

Where \( \Delta l_f = \text{fibre pull-out displacement change} \) corresponding to the increase in the crack width. Substituting for \( x_f \) in eq. 16 from eq. 17, the force \( F_{p(i)} \) is expressed as:

\[ F_{p(i)} = \frac{E_f V_f b e_{fp}^2 l_f}{\pi \psi} \]  \hspace{1cm} (18)
With \( K = \frac{E_f V_f \varepsilon_y}{\pi} \)

Eq. 18 can be re-written as

\[ F_{f(t)} = K_1 b \varepsilon_y l_f \]  

(20)

(3) Force \( F_{f(t)} \)

Similarly from Fig.3, eq. 7 and 15, the expression for the pull out tensile force carried by the steel fibres is obtained as;

\[ F_{f(t)} = \frac{E_f V_f \varepsilon_y}{\pi} \left( \frac{w}{\psi} - x_1 \right) \]  

(21)

Noting that \( K_1 = \frac{E_f V_f \varepsilon_y}{\pi} \) from eq. 20 and

\[ x_1 = \frac{e_y l_f}{\psi} \]

From eq. 17 Eq. 21 is re-written as

\[ F_{f(t)} = 2 K_1 b \left( \frac{w}{\psi} - \frac{e_y l_f}{\psi} \right) \]  

(22)

(4) Dowel Force \( F_d \)

The expression for the dowel force \( F_d \) has been derived based on dowel bearing mechanism in concrete road pavements [9]. It is assumed that the relative shear displacement between the crack faces is in tandem with that of the reinforcement bar as shown in Fig 6. The dowel load is transferred to the supporting concrete across the crack through bearing and the interface bond between concrete and the anchored part of the re-bar. Equations applied on dowel bars on concrete road pavements [9, 10] are applied in this study. Where;

\[ \sigma_b = k y_d \]  

(23)

Where \( \sigma_b \) = bearing stress

\( y_d \) = deflection of the dowel bar (mm)

\( k \) = modulus of dowel support (N/mm²),

The value of modulus of dowel support is estimated from that suggested by Frigberg [10].

\( k = 6895 \) or \( 0.25 \sqrt{E_c} \)  

(24)

Referring to Fig.6 and applying eq.23 in the derivation of dowel force \( F_d \).

\[ F_d = \sigma_b \delta_b = k y_d \delta_b \]  

(25)

Re-writing eq.25 in terms of the area of the reinforcement bar and substituting for \( y_d = \frac{\delta}{2} = \frac{w \cos \alpha}{2} \)

from ig.6, Fig.2b, then

\[ y_d = \frac{\delta}{2} = \frac{w \cos \alpha}{2} \]  

(26)

\[ F_d = k w \cos \alpha \sqrt{\frac{A}{\pi}} \]  

(27)

However, from the geometry of Fig.2a the shear displacement in relation to the shear angle \( \gamma \) (equal to shear strain) is determined as;

\[ \delta = w \cos \alpha = a \tan \gamma \approx a \gamma \]  

(28)

\[ a = c \cos \alpha \]  

(29)

Combination of eq. 28 and 29 yields the expression for the general crack width as;

\[ w = c \gamma \]  

(30)

Where, \( c \) = the general length of the crack path

The crack width is expressed from eq.30 in terms of fibre pull out strain and initial yield shear strain \( \gamma_y \) as;

\[ w = c \gamma \]  

(31)

Where \( c \gamma_y = e_y l_f \) is the initial yield crack width

32

The actual net fibre pull out length is given as \( 0.25 l_f \) [7], however this length is reduced during gradual pull out of the fibre. Thus the remaining effective length expressed in terms of the yield shear strain ratio after substitution of \((c \gamma)\) from 33 becomes

\[ l_f' = l_f - w = 0.25 \delta y \frac{1}{\gamma_y} - 4 e_y \gamma_y \]  

(33)

Re-writing the term \((w)\) in eq 27 in terms of eq. 33 the expression for the dowel force will be;

\[ F_d = k e_y l_f \gamma_y \cos \alpha \sqrt{\frac{A}{\pi}} \]  

(34)

Fig. 6 Relative deflections of Reinforcement bar and the crack faces

(5) Bar Tensile Force \( F_s \)

The tensile force acting on the re bar can be assessed in a similar manner as that of the fibre. An effective pull out length from the shortest anchored side from the crack face is assumed. The tensile force
The theoretical prediction of shear strength evolution in steel fibre reinforced concrete beams without stirrups is derived based on the equilibrium of external and internal forces previously derived in section 2.1.

(1) Horizontal Equilibrium

\[ F_{t1} + F_{t2} - F_{v} + F_{a} \sin \alpha + F_{w} = (F_{cv} + F_{f}) \cos \alpha \]  

(38)

From eq.37, \((F_{cv} + F_{f})\) is substituted in eq. 38 to obtain;

\( (F_{t1} + F_{t2}) - F_{a} \sin \alpha + F_{cos} = \frac{Q}{2} \cos \alpha \)  

(39)

(2) Vertical Equilibrium

\[ \frac{Q}{2} - (F_{t1} + F_{t2} - F_{a} \cos \alpha - F_{w} \sin \alpha - F_{c} \cos \alpha) = 0 \]

Substituting for shear stress \(\tau_b\) and \(l_{ef}\), respectively, are substituted in the above.

\[ F_{t} = 2\pi \tau_{b} l_{ef} \]

Where \(\tau_{b}\) = Bond shear strength and

\[ l_{ef} = l_{w} - w = l_{w} - \frac{Y}{Y_{f'}} \]

2.2 Shear Strength Predictive Eq.

The overall shear strength predictive relation is derived based on the equilibrium of external and internal forces previously derived in section 2.1.

(3) Moment equilibrium about point O (Fig.3, 2)

\[ \frac{Q \cos \alpha}{2 \psi} = \frac{1}{2} \left( \varepsilon - \frac{w}{\psi} - F_{t2} \right) \left( \alpha + \frac{1}{2} \left( \frac{w}{\psi} \right)^{2} \right) - \frac{2}{3} F_{t2} \alpha - \frac{1}{2} F_{v} \sin \alpha - \frac{1}{2} F_{w} \psi = 0 \]

Force relation given in eq.s 1, 2 20, 22, 34 and 36 respectively, are substituted in the above.

\[ \frac{Q w}{2 \psi} \cos \alpha - \frac{1}{2} \left( \varepsilon - \frac{w}{\psi} - F_{t2} \right) \left( \alpha + \frac{1}{2} \left( \frac{w}{\psi} \right)^{2} \right) - \frac{2}{3} F_{t2} \alpha - \frac{1}{2} F_{v} \sin \alpha - \frac{1}{2} F_{w} \psi = 0 \]

Reducing further to

\[ \left( \frac{w}{\psi} \right)^{2} \left( \frac{Q \cos \alpha}{2 \psi} - \frac{\sigma_{y}}{2} + \frac{1}{2} K \left( \varepsilon - \frac{w}{\psi} \right)^{2} - K_{f} - \frac{\sigma_{f}}{2} \right) + \frac{\sigma_{y}}{2} \left( Q_{f} + c_{f} \sigma_{y} - F_{v} - F_{t2} \right) = \varepsilon - \frac{w}{\psi} \]

(40)

The expression for \(\frac{w}{\psi}\) is obtained from eq. 38 as:

\[ w = \left( \frac{Q_{f} + c_{f} \sigma_{y} - F_{v} - F_{t2}}{2k_{f} - K_{f} \frac{\varepsilon - \frac{w}{\psi}}{w} + \sigma_{f} - \sigma_{y}} \right) \]  

(41)

Substituting eq.38 and determining the approximate solution, eq.41 becomes;

\[ Q = \frac{3K_{f}}{2} \left( 3 - \left( \frac{\varepsilon - \frac{w}{\psi}}{w} \right)^{2} - \frac{k_{f} \left( 2 - \frac{\varepsilon - \frac{w}{\psi}}{w} \right)}{\sigma_{f} - \sigma_{y}} \right) \]

(43)

Substitute for \(Q_{f}, F_{v}, F_{t2}\) from eq.41 and with \(c = \frac{a}{\cos \alpha}\) (Fig.1). From the shear span to depth ratio, \(a = d \beta\). Thus eq.43 becomes;

\[ Q = \frac{3K_{f}}{2} \left( 3 - \left( \frac{\varepsilon - \frac{w}{\psi}}{w} \right)^{2} - \frac{k_{f} \left( 2 - \frac{\varepsilon - \frac{w}{\psi}}{w} \right)}{\sigma_{f} - \sigma_{y}} \right) \]

(44)

To account for the influence of the shear span to depth ratio in shear, eq.44 is rewritten as follows

\[ Q = \frac{K_{f}}{2} \left( 3 - \left( \frac{\varepsilon - \frac{w}{\psi}}{w} \right)^{2} - \frac{k_{f} \left( 2 - \frac{\varepsilon - \frac{w}{\psi}}{w} \right)}{\sigma_{f} - \sigma_{y}} \right) \]

(45)

Substitution for \(w, F_{v}\) and \(F_{t2}\) from eqs.31, 34 and 36 respectively, with simplification of the dowel contribution part (last term), the shear strength equation (in Newton) is then given as;

\[ V = \frac{d k_{f}}{2} \left( 3 - \left( \frac{\varepsilon - \frac{w}{\psi}}{w} \right)^{2} - \frac{k_{f} \left( 2 - \frac{\varepsilon - \frac{w}{\psi}}{w} \right)}{\sigma_{f} - \sigma_{y}} \right) \]

(46)

It can be seen that eq.46 follows the traditionally applied principle of superposition and can simply be written as

\[ V = V_{f} + V_{c} + V_{d} \]

(47)
In order to make evolution predictions, the shear strain ratio in eq.46 must be applied incrementally (ie \( \gamma = 0, 1, 2, 3 \) etc) in the prediction analysis.

The yield shear strain is determined theoretically based on the relation given by Gere and Timoshenko [8], however since this is not the maximum value, the factor applied in the given equation has been assumed to be equal to 1.2.

\[
G = \frac{E}{2(1+\nu)} \quad \text{(49)}
\]

\( A_c \) = Cross sectional area of the concrete beam \( \nu \) = Poisson ratio

2.3 Determination of deflections

Deflections at mid span of the beam are obtained by combination of moment-curvature relations [8] and moment-deflection relations [12]. The curvature ratio relationship in beams before and after cracking is given as follows [8]:

\[
\kappa = \kappa_y = \frac{1}{\rho} \quad \text{(50)}
\]

\( \kappa, \kappa_y \) is the curvature in elastic bending and at yielding respectively, \( \rho \) is the plastic moment, \( M_y \) is the general moment between the yield and plastic moment, respectively. That is \( M_y < M \leq M_r \).

Estimation of the mid span deflection due to elastic bending is obtained based on the relations given in [12]. Based the relations, elastic deflection in beam under bending is given as:

\[
\delta_e = \frac{1}{\rho} \left( \xi \right)^2 \quad \text{(51)}
\]

\( \xi = \frac{4\varphi^2 - 8\varphi + 1}{48\varphi} \) and \( \varphi = a/l_e \)

For small deflections, moment curvature relationship in elastic bending can be determined as

\[
\frac{1}{\rho} = \frac{M}{EI} \quad \text{(52)}
\]

It is assumed in this study that at onset of yield, the elastic curvature limit is equal to yield curvature, therefore

\[
\frac{1}{\rho} = \frac{1}{\rho_y} = \kappa_y \quad \text{(53)}
\]

Combination of eq.51 to 54 yields the relation for the determination of deflections (eq.54) from elastic to inelastic bending (after cracking). By inspection of eq.60, the curvature ratio range is found to be within a ratio not exceeding 1.73.

\[
\delta_y = \frac{2\xi, M}{EI} \left( \frac{3 - \left( \kappa / \kappa_y \right)^2}{2} \right) \quad \text{(54)}
\]

The deflections due to bending from eq. 54 are then added to shear deflections estimated from the relation between the shear strains and the shear displacement from eq. 28. Where the shear displacements are obtained as follows:

\[
\delta_y = a\gamma \quad \text{(55)}
\]

\[
= a\gamma \left( \frac{\gamma}{\gamma_y} \right) \quad \text{(55)}
\]

\[
= a\frac{1}{2} \frac{Q}{GA_y} \left( \frac{\gamma}{\gamma_y} \right) \quad \text{(55)}
\]

The total deflections are then estimated as follows

\[
\delta = \delta_e + \delta_y \quad \text{(56)}
\]

3. Verification of shear strength formula eq. (46)

3.1 Basis of verification

Validity of the derived eq.46 was checked against experimental results obtained from a total of 12 test beams. Geometry and reinforcement details similar to those used in the experiments (Fig.7) were used in the theoretical predictions. The tensile and compressive strengths applied were obtained from concrete cylinder tests; however the bond strength was estimated based on the value (4.15 Mpa) proposed by Narayanan R, et al [2]. Other properties are as shown in Table1. Twelve simply supported beams under bending- shear (Fig.7) were tested in the experiments. Tests on the specimens were done using a 300kN capacity universal testing machine.

Controlled loading was applied on the specimens while the full field deformations (displacements) were monitored and recorded using a set of optical measurements equipment system (ESPI) comprising a Desk top computer (PC) and CCD camera (ESPI sensor) equipped with laser beam sensors as shown in Fig.8.
Table 1 Parameters applied in analysis

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<td>$k \text{ (N/mm}^2\text{)}$</td>
<td>6895</td>
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Fig. 7 Test set up and measurement points

(a) ESPI camera and specimen  
(b) Processor

Fig. 8 Full filed optical equipment and set up

3.2 Verification results and Discussions

1) Analytical predictions

Fig. 9 shows the theoretical predictions from eq. (47) for fibrous beams. As depicted in these figures, the strength evolution behavior is approximately linear in the initial stages beyond which a non linear behavior is observed. Complete deformation behavior in which increase in the shear strength commensurate with the fibre content is predicted well. The reduction in shear strength with increase in the shear span to depth ratio, a phenomenon commonly observed in practice is also predicted well.

Fig. 9 Theoretical prediction eq. (46)

2) Experimental and theoretical comparisons

Fig 10 shows comparisons between experimental and theoretical results. There is generally a very good correlation between the theoretical predictions and the experimental results. Both results also indicate an increase in strength in the fibrous beams over non fibrous beams. A decrease in the strength with increase in shear span depth ratio is also observed in both theoretical and experimental results. In Fig 10a, ductility representation after yield is observed to be limited in the ESPI results as compared with the theoretical predictions. This is because the failure was predominantly shear in which deformation after failure at mid span could not be detected well by the ESPI method due to presence of rigid displacement.
Fig. 10 Experimental and theoretical comparison

4. Conclusions and Recommendations
The key assumptions made and validity of the derived theoretical model has been confirmed through comparison with experiments. The comparison showed that the model is consistently in agreement and conservative in all the cases considered. However, there is need for more experimental data to evaluate the model in detail particularly in prototype beams specimens.

Acknowledgement
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