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Analysis of Arbitrarily Shaped Dielectric Lens Antenna by Ray Tracing Method

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Introduction

The authors have reported the three dimensional ray tracing method for the analysis of dielectric lens antenna with arbitrarily shaped inner and outer surface [1], [2]. In these papers, however, the multiple reflections are not considered. In this paper, the ray tracing method is applied to the dielectric lens antenna. The primary radiator of this lens antenna is the H-sectional horn fed by the rectangular waveguide [3]. The multiple reflection of ray is considered.

Formulation

Figure 1 shows the dielectric lens antenna and the coordinate system for the ray tracing method [1], [2]. $i_x$, $i_y$, and $i_z$ are the unit vectors of the Cartesian coordinate system. Let the source point on the primary radiator be $P_0(x_0, y_0, z_0)$. The equivalent electric and magnetic currents $J_i(x_0, y_0)i_y$ and $M_i(x_0, y_0)i_z$ are assumed on the source point $P_0$. The inner surface of lens is $S_1$, the outer surface of lens is $S_2$ and the reference plane of lens antenna is $S_3$. The points of the ray on $S_1$, $S_2$, and $S_3$ are defined as $P_1$, $P_2$, and $P_3$, respectively. $R_v = P_{v-1}P_v$ is the distance between two points $P_{v-1}$ and $P_v$ ($v=1,2,3$), and $i_v$ is the unit vector from $P_{v-1}$ to $P_v$.

Snell's law is expressed as follows:

$$k_1 \sin \theta_1 = k_2 \sin \theta_2.$$  (1)

Where $k_1$ and $k_2$ are the propagation constants within the outside and inside region of lens, and $\theta_1$ and $\theta_2$ are the incident and the refractive angles, respectively. Applying Eq. (1) to the ray tracing method, we obtain

$$i_{v+1} = \frac{1}{n'} i_v + n_i \left[ \frac{1}{n'} \left( 1 - \frac{1}{n^2} \left( i_v \cdot n_i \right)^2 \right) - \frac{1}{n'} i_v \cdot n_i \right].$$  (2)

Where $n$ is the refractive index of lens.

$$n = \frac{k_2}{k_1} = \sqrt{\frac{\mu_2}{\mu_1} \frac{\varepsilon_2 - j \left( \sigma_2 / \omega \right)}{\varepsilon_1 - j \left( \sigma_1 / \omega \right)}}, \quad n' = \begin{cases} \frac{n}{\nu + 1} & (\nu = 1) \\ \frac{1}{n} & (\nu = 2) \end{cases}$$  (3)

Applying Eq. (2) to the ray tracing method, the points $P_1$, $P_2$, and $P_3$ on $S_1$, $S_2$, and $S_3$ can be obtained straightforwardly.
Figure 1. Coordinate system for ray tracing method

Fresnel’s transmission coefficients of parallel and perpendicular polarization are given by the following expressions.

(a) Parallel polarization

\[ T_{TM} = \frac{2\mu_1 n \cos \theta_1}{\mu_1 n^2 \cos \theta_1 + \mu_1 \sqrt{n^2 - \sin^2 \theta_1}} \quad (5) \]

(b) Perpendicular polarization

\[ T_{TE} = \frac{2\mu_2 \cos \theta_1}{\mu_1 \cos \theta_1 + \mu_1 \sqrt{n^2 - \sin^2 \theta_1}} \quad (6) \]

By using Eqs. (1), (3) - (5), the incident electric field at \( P_1 \) is expressed as follows.

\[ E_{\mathbf{R}_1}^i = \left( E_{\mathbf{R}_1}^x i_x + E_{\mathbf{R}_1}^y i_y + E_{\mathbf{R}_1}^z i_z \right) \psi \quad (7) \]

\[ E_{\mathbf{R}_1}^x(x_1, y_1) = \frac{k_1^2}{4\pi R_1^2} (x_1 - x_0) J_y(x_0, y_0) \quad (8) \]

\[ E_{\mathbf{R}_1}^y(x_1, y_1) = \frac{k_1^2}{4\pi R_1^2} \left[ \frac{k_1^2}{R_1^2} (y_1 - y_0)^2 - j\omega \mu_0 \right] J_y(x_0, y_0) + \frac{jk_1}{R_1} (z_1 - z_0) M_z(x_0, y_0) \quad (9) \]

\[ E_{\mathbf{R}_1}^z(x_1, y_1) = \frac{k_1^2}{4\pi R_1^2} \left( y_1 - y_0 \right) \left( z_1 - z_0 \right) J_y(x_0, y_0) - \frac{jk_1}{R_1} (y_1 - y_0) M_z(x_0, y_0) \quad (10) \]

\[ \psi = \exp(-jkR_t) \quad (11) \]

The electric field on the reference plane \( S_3 \) is obtained as,

\[ E_{\mathbf{R}_3}(x_3, y_3) = \left( k_{i x} E_{\mathbf{R}_3}^x(x_3, y_3) + k_{i y} E_{\mathbf{R}_3}^y(x_3, y_3) + k_{i z} E_{\mathbf{R}_3}^z(x_3, y_3) \right) \psi \quad (12) \]

\[ E_{\mathbf{R}_3}(x_3, y_3) = \left\{ \frac{k_{i x}}{R_1} E_{\mathbf{R}_1}^x(x_3, y_3) + \frac{k_{i y}}{R_2} E_{\mathbf{R}_2}^y(x_3, y_3) + \frac{k_{i z}}{R_3} E_{\mathbf{R}_3}^z(x_3, y_3) \right\} \times \exp \left\{ -j(k_{i x} R_1 + k_{i y} R_2 + k_{i z} R_3) \right\} \quad (13) \]

\[ R_1 = \frac{P_1 P_t}{R_2} \quad R_2 = \frac{P_2 P_t}{R_3} \quad R_3 = \frac{P_3 P_t}{R_1} \]

\( K_{i x}, K_{i y}, K_{i z} \) denote the variables defined by the normal vectors and the transmission coefficients at the points \( P_1 \) and \( P_2 \). The transmission coefficient at each point depends on the mode of incident ray. In the calculation of the electric field, the multiple reflections are also...
The radiation field is calculated by the surface integral on the reference plane $S_3$. Replacing the coordinate origin to the point of intersection of $S_3$ and the z-axis, the electric field is expressed by

$$
E(r, \theta, \phi) = \frac{1}{4\pi} \int_{S_3} \left\{ -j\omega\mu_0\psi(n' \times H) + (n' \times E') \times \nabla'\psi + (n' \cdot E')\nabla'\psi \right\} dS' \\
= \exp\left(-\frac{jkr}{r}\right) D(\theta, \phi) \tag{14}
$$

$$
D(\theta, \phi) = \frac{jk}{4\pi} \int_{S_3} \left[ \left\{ E^x_{\phi}(x, y) \cos\phi + E^y_{\phi}(x, y) \sin\phi \right\} \hat{t}_x - \left\{ E^y_{\phi}(x, y) \cos\phi \right\} \hat{t}_y \right] \exp\left\{ jk \left( x \sin\theta \cos\phi + y \sin\theta \sin\phi \right) \right\} dS' \tag{15}
$$

**Numerical and experimental results**

Figure 2 shows the structure of dielectric lens antenna [3]. The primary radiator is the H-sectral horn fed by the rectangular waveguide. The width of H-sectral horn is $W = 4.5\, \text{mm}$. The dielectric lens of relative permittivity 2.1 is located at the aperture of horn antenna. Figure 3 shows the example of the locus of ray. Figure 4 shows the electric field radiation patterns at frequency 59.6 GHz. The calculated patterns are compared with the measured results [3]. Figure 5 shows the frequency characteristics of electric field radiation patterns.

![Figure 2. Geometry of dielectric lens antenna](image)

![Figure 3. Locus of ray](image)

(a) without multiple reflections  
(b) multiply reflected rays
Conclusion
The three dimensional ray tracing method for the dielectric lens antenna has been presented. The multiple reflections have be considered in this method. The radiation pattern of the dielectric lens antenna has be calculated by the ray tracing method. This ray tracing method is applicable to the dielectric lens antenna with nonsymmetrical inner and outer surface. Now the radiation patterns of the dielectric lens antenna with the matching layers on the lens surfaces are calculating.

References