UNDRAINED STRENGTH OF NORMALLY CONSOLIDATED CLAY MEASURED IN THE SIMPLE SHEAR TEST

by

Hidetoshi OCHIAI*

This paper discusses a method for calculating the "true" undrained strength ratio, $(c_u/p)_{ss}$, and the internal friction angle, ϕ'_{ss}, in terms of effective stress of normally consolidated clays measured in a simple shear test. The proposed method consists in assuming that the directions of the principal stress axes during the simple shear test are expressed by the relation proposed by Oda and Konishi [1]. Experimental results obtained from three types of clays confirm that the method gives more reasonable estimates of $(c_u/p)_{ss}$ and ϕ'_{ss} for normally consolidated clay compared to other methods. The method also supports Ladd's statement that $(c_u/p)_{ss}$ should generally be less than $(c_u/p)_{TC}$ determined from the triaxial compression test due to undrained strength anisotropy[2]. Finally, a significant disadvantage of the method proposed by Duncan and Dunlop[3] is discussed, and it is pointed out that the Duncan and Dunlop method may predict excessively high values of ϕ'_{ss} for normally consolidated clay.

INTRODUCTION

The undrained shear strength ratio, c_u/p, is an important index expressing the increasing rate of the undrained shear strength due to consolidation, in which c_u is defined as follows:

$$c_u=\frac{1}{2}(\sigma_1-\sigma_3)_{max}=\tau_{max} \qquad (1)$$

When the ratio, c_u/p, is determined by means of the laboratory shear tests, the tests should simulate the in situ modes of failure because most clays exhibit anisotropic strength behavior[2]. The simple shear test has some advantages over other tests in this regard, and the shearing resistance measured under the simple shear test conditions may provide a very useful measure of shear strength for stability analyses of field loading conditions. For example, Ladd[2] stated that the simple shear test gives a good to slightly conservative estimate of the average strength along a cincular arc failure for nonlayered soft clay. Nevertheless, it should be noted that the shear stress, τ_h, on the horizontal plane measured in the simple shear test is not equal to the maximum shear stress nor the shear stress on the failure plane. That is, $(\tau_h)_{max}$ is not equal to the undrained shear strength, c_u, of saturated clay defined by Eq. (1), and the ratio, $(\tau_h)_{max}/p$, therefore is not equal to the "true" undrained strength ratio, $(c_u/p)_{ss}$, of saturated clay measured in simple shear:

$$(\tau_h)_{max}/p \neq (c_u/p)_{ss} \qquad (2)$$

A method for calculating the undrained strength ratio in simple shear, $(c_u/p)_{ss}$, of normally consolidated clay by using the values of the normal stress (consolidation pressure), p, and the shear stress, τ_h, acting on the horizontal plane measured in the simple shear test will be newly proposed in this paper.

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*Department of Civil Engineering

A METHOD FOR CALCULATING THE UNDRAINED STRENGTH OF NORMALLY CONSOLIDATED CLAY MEASURED IN THE SIMPLE SHEAR TEST

When only the normal stress (consolidation pressure), p, is applied on the horizontal plane, the sample in the simple shear apparatus remaines at the state of K_0−consolidation. Under the initial stress conditions the vertical and horizontal effective stresses, σ_v' and σ_h', are equal to the major and minor effective principal stresses, σ_1' and σ_3', and they are expressed as follows:

$$\sigma_v'=\sigma_1'=p \quad , \quad \sigma_h'=\sigma_3'=K_0 p \tag{3}$$

in which K_0 is the coefficient of earth pressure at rest.

The principal stress axes in the simple shear test rotate during the progressive increase of shear stress, τ_h, on the horizontal plane. Oda and Konishi[1] proposed the following relationship between the inclination angle, ψ, an angle between the major principal stress axis and the vertical direction, and the effective stress ratio, τ_h/σ_n', acting on the horizontal plane:

$$\tau_h/\sigma_n'=\chi\cdot\tan\psi \tag{4}$$

in which σ_n' is the normal effective stress on the horizontal plane and χ is a material constant which is expressed by the following equation[4),5),6)]:

$$\chi=1-K_0$$
$$=\sin\phi_{cv}=\frac{2\sin\phi_\mu}{1+\sin\phi_\mu} \tag{5}$$

in which ϕ_{cv} is the internal friction angle at the critical void ratio and ϕ_μ is the interparticle friction angle. The normal effective stress, σ_n', on the horizontal plane is related to the initial normal stress (consolidation pressure), p, as follows:

$$\sigma_n'=\beta p \tag{6}$$

in which β is a quantity related to the pore water pressure, u, during the undrained simple shear test and is expressed as follows:

$$\beta=(p-u)/p \tag{7}$$

Using Eqs. (4), (5) and (6), the major and minor effective principal stresses, σ_1' and σ_3', at any stage of the simple shear test are expressed in the following forms[4),6)]:

$$\sigma_1'=\sigma_n'\frac{(1-K_0)+(\tau_h/\sigma_n')^2}{1-K_0}$$
$$=p\frac{\beta^2(1-K_0)+(\tau_h/p)^2}{\beta(1-K_0)} \tag{8}$$

$$\sigma_3'=K_0\sigma_n'=K_0\beta p \tag{9}$$

Therefore, the minor effective principal stress, σ_3', is determined from only the normal effective stress, σ_n', independent of the shear stress, τ_h. In the case of the drained simple shear test under the constant normal stress, p, when $\sigma_n'=p(\beta=1)$, the minor effective principal stress, σ_3', is a constant value ($=K_0p=K_0\sigma_n'$) but the horizontal effective stress, σ_h', is not constant. (There exists a significant

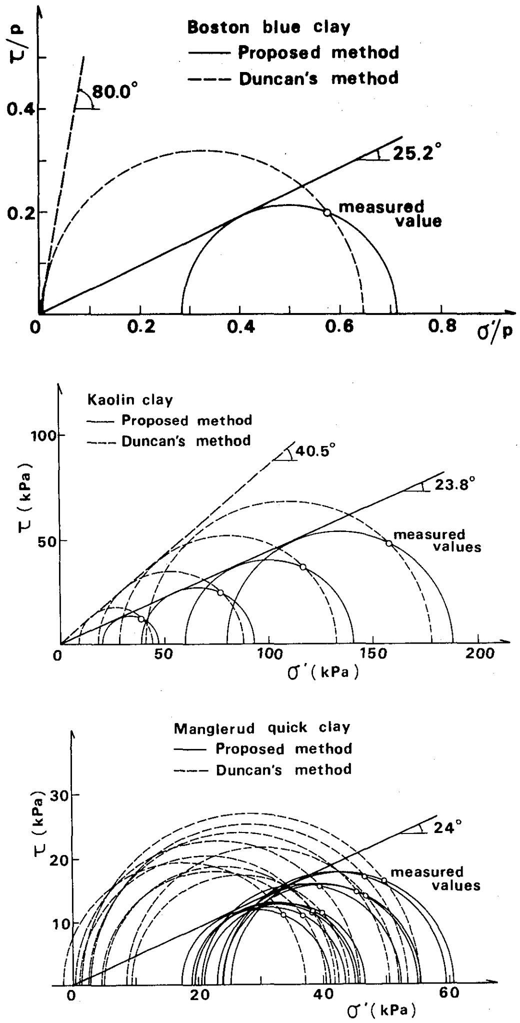

Fig. 1 Mohr stress circles in terms of effective stress based on the two methods.

difference between the proposed method and the Duncan and Dunlop method[3].) Konishi[7] has experimentally confirmed that σ_3' is nearly constant for the case of the drained simple shear test having a constant normal stress on the two-dimensional assembly of photoelastic rods. And the minor principal stress, σ_3, in terms of total stress is expressed from Eqs. (7) and (9) as follows:

$$\sigma_3 = \sigma_3' + u = K_0 p + (1 - K_0) u \tag{10}$$

The undrained shear strength, $(c_u)_{ss}$, of normally consolidated clay measured in the simple shear test can be easily obtained from Eqs. (8) and (9) as follows:

$$(c_u)_{ss} = \frac{1}{2}(\sigma_1' - \sigma_3')_{\max} = p\frac{\beta^2(1-K_0)^2 + ((\tau_h)_{\max}/p)^2}{2\beta(1-K_0)} \tag{11}$$

Therefore, the undrained strength ratio, $(c_u/p)_{ss}$, of normally consolidated clay measured in the simple shear test is expressed in the following form:

$$(c_u/p)_{ss} = \frac{\beta^2(1-K_0)^2 + ((\tau_h)_{\max}/p)^2}{2\beta(1-K_0)} \tag{12}$$

The internal friction angle, ϕ_{ss}', in terms of effective stress measured in the simple shear test is expressed as follows:

$$\sin\phi_{ss}' = \left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}\right)_{\max} = \frac{\beta^2(1-K_0)^2 + ((\tau_h)_{\max}/p)^2}{\beta^2(1-K_0^2) + ((\tau_h)_{\max}/p)^2} \tag{13}$$

The experimental results of normally consolidated Boston blue clay[2),8)], kaolin clay[9] and Manglerud quick clay[10], and the values of $(c_u/p)_{ss}$ and ϕ_{ss}' calculated for these clays from Eqs. (12) and (13) are listed in Table 1. Mohr stress circles in terms of effective stress on these clays based on the proposed method are shown in Fig. 1. Table 1 and Fig. 1 also include the calculated values of $(c_u/p)_{ss}$ and ϕ_{ss}' and Mohr stress circles in terms of effective stress based on the Duncan and Dunlop method. As shown in Table 1, the values of $(c_u/p)_{ss}$ calculated from Eq. (12) on these clays are about 7%, 10% and 12 % larger than the values of $(\tau_h)_{\max}/p$. For Boston blue clay, although the value of $(\tau_h)_{\max}/p$ is slightly less than the average strength along the circular arc of an embankment failure[2], it seems that the value of $(c_u/p)_{ss}$ calculated by the proposed method gives the average strength. And the values of $(c_u/p)_{ss}$ of these clays calculated from Eq. (12) are less than the $(c_u/p)_{TC}$ values determined by means of the triaxial compression test, as shown

Measured values						Proposed Method		Duncan's Method	
p (kPa)	$(\tau_h)_{max}$ (kPa)	u (kPa)	$(\tau_h)_{max}/p$	$\sigma_n'/p = \beta$	K_o	$(c_u/p)_{ss}$	ϕ_{ss}' (degs.)	$(c_u/p)_{ss}$	ϕ_{ss}' (degs.)
Boston blue clay (Ladd, 1973, 1979)									
-	-	-	0.200	0.575	0.50	0.213	25.2	0.320	80.0
Kaolin clay (Ohara and Matsuda, 1978)									
49	11.76	10.78	0.24	0.78	0.51	0.27	23.8	0.35	41.3
98	24.50	22.54	0.25	0.77		0.27	24.1	0.35	41.3
147	36.26	32.34	0.25	0.78		0.27	23.8	0.35	40.8
196	47.04	41.16	0.24	0.79		0.27	23.8	0.34	38.7
Manglerud quick clay (Bjerrum and Landva, 1966)									
58.80	10.78	22.34	0.18	0.62	0.51	0.21	23.2	0.30	53.1
58.80	9.60	18.23	0.16	0.69		0.21	22.0	0.29	40.7
58.80	11.37	21.17	0.19	0.64		0.22	23.4	0.31	51.7
66.64	11.17	20.68	0.21	0.69		0.23	23.5	0.32	46.0
66.64	11.37	29.30	0.17	0.56		0.19	23.5	0.30	72.4
65.66	10.29	32.14	0.16	0.51		0.18	23.8	0.29	-
67.62	14.70	28.42	0.22	0.58		0.23	26.1	0.33	80.8
75.46	16.37	29.40	0.22	0.61		0.23	25.1	0.33	64.7
73.50	14.60	29.40	0.20	0.60		0.21	24.2	0.32	64.5
85.26	16.17	36.65	0.19	0.57		0.20	24.3	0.31	72.6

Table 1 The experimental results of three normally consolidated clays and the values of $(c_u/p)_{ss}$ and ϕ_{ss}' calculated by the two methods.

in Fig. 2. (The values of $(c_u/p)_{TC}$ are about 0.33 for Boston blue clay, 0.34 for kaolin clay and 0.30 for Manglerud quick clay.) This result is in agreement with Ladd's statement that the rotation of the principal planes to produce a horizontal failure surface, as in the simple shear test, reduces the strength considerably, i. e. $(c_u/p)_{ss}$ should generally be less than $(c_u/p)_{TC}$ due to undrained strength anisotropy[2]. Indeed, the values of ϕ'_{ss} of these clays calculated from Eq. (13) are reasonable values for normally consolidated clay.

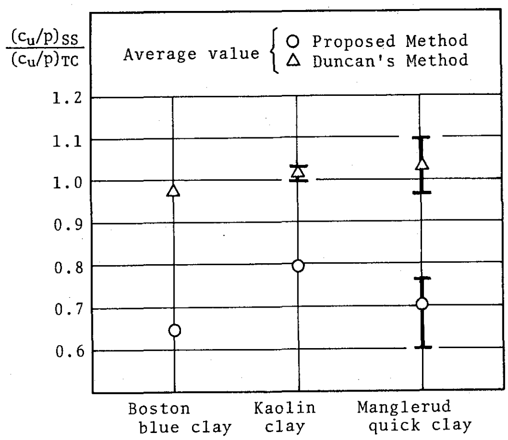

Fig. 2 The ratio of $(c_u/p)_{ss}$ to $(c_u/p)_{TC}$ of three normally consolidated clays based on the two methods.

ON THE DUNCAN AND DUNLOP METHOD

Duncan and Dunlop[3] proposed the following equations based on the assumption that the simple shear specimen may be considered to be an element of soil subjected to pure shear :

$$(c_u)_{ss}=\tau_{\max}=\sqrt{\frac{p^2(1-K_0)^2}{4}+(\tau_h)^2_{\max}} \quad (14)$$

$$(c_u/p)_{ss}=\sqrt{\frac{(1-K_0)^2}{4}+((\tau_h)_{\max}/p)^2} \quad (15)$$

The values of $(c_u/p)_{ss}$ of the above-mentioned three clays calculated from Eq. (15) are listed in Table 1. These calculated values of $(c_u/p)_{ss}$ are considerably larger than those calculated from Eq. (12), and these values calculated from Eq. (15) are nearly in agreement with the values of $(c_u/p)_{TC}$ of the same clays determined by means of the triaxial compression test as stated by Duncan and Dunlop[3]. This conclusion, however, contradicts Ladd's statement earlier.

Furthermore, one should look at what the Duncan and Dunlop method would predict for the Mohr stress circle in terms of effective stress. Under the pure shear conditions, taking Eq. (14) into consideration, the major and minor principal stresses, σ_1 and σ_3, are expressed in the following forms :

$$\left.\begin{aligned}\sigma_1&=\frac{1}{2}(1+K_0)p+\sqrt{\frac{p^2(1-K_0)^2}{4}+(\tau_h)^2}\\ \sigma_3&=\frac{1}{2}(1+K_0)p-\sqrt{\frac{p^2(1-K_0)^2}{4}+(\tau_h)^2}\end{aligned}\right\} \quad (16)$$

The co-ordinate of the center of the Mohr stress circle expressed by Eq. (16) is $\left(\frac{1}{2}(1+K_0)p,\ 0\right)$, which coincides with the center of the Mohr stress circle under the initial stress conditions, when only the normal stress (consolidation pressure), p , is applied on the horizontal plane. These circles, therefore, are concentric circles. It should be noted here that even though the normal stress, p , on the horizontal plane is constant during the undrained simple shear test, the normal effective stress, σ_n, is not constant ($\sigma'_n=p-u=\beta p$) due to the generation of pore water pressure, u . The co-ordinate of the center of the Mohr stress circle in terms of effective stress therefore is $\left(\frac{1}{2}(1+K_0)p-u,\ 0\right)$, and this co-ordinate does not coincide with the center of the Mohr stress circle under the initial stress conditions. This then indicates that the principal stresses expressed by Eq. (16) are those expressed not in terms of effective stress but in terms of total stress. The internal friction angle, ϕ'_{ss} , in terms of effective stress is therefore expressed in the following form :

$$\sin\phi_{ss}=\left(\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3-2u}\right)_{\max}=\frac{\sqrt{(1-K_0)^2+4((\tau_h)_{\max}/p)^2}}{(1+K_0)-2(1-\beta)} \quad (17)$$

Fig. 3 shows Mohr stress circles during the undrained simple shear test derived from the Duncan and Dunlop method and from the new method proposed in this paper. In the case of the

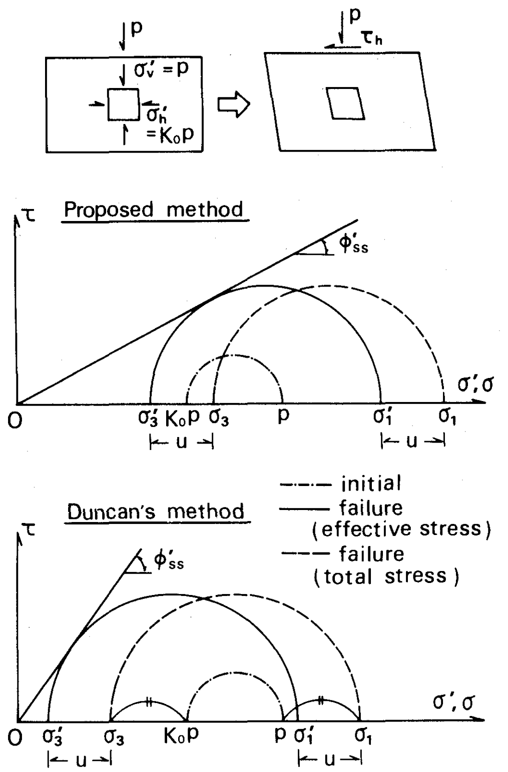

Fig. 3 Mohr stress circles in the consolidated undrained simple shear test derived from the two methods.

drained simple shear test under the constant normal stress, p , when $\sigma'_n=p(\beta=1)$, because Mohr stress circles during the test are concentric circles in the Duncan and Dunlop method, the horizontal effective stress, σ'_h, is a constant value $(=K_0p=K_0\sigma'_n)$ and the minor effective principal stress, σ'_3 , is not constant, while σ'_3 is a constant value $(=K_0p=K_0\sigma'_n)$ and σ'_h is not a constant value in the proposed method as stated above. The values of ϕ'_{ss} calculated from Eq. (17) are listed in Table 1, and Mohr stress circles in terms of effective stress are shown in Fig. 1. As shown in Table 1 and Fig. 1, the Duncan and Dunlop method may predict excessively high values of ϕ'_{ss} for normally consolidated clays. It is therefore not appropriate to employ the Duncan and Dunlop method for estimating the undrained shear strength of normally consolidated clays. Duncan and Dunlop also employed Eq. (15) to explain the liquefaction phenomena of sand under the simple shear conditions[3], but it is thought that the method would not be appropriate for the same reason.

CONCLUDING REMARKS

For some types of field loading conditions such as a circular arc failure for soft clay and a layer of saturated sand subjected to horizontal ground motion by an earthquake, the simple shear test provides a measure of shearing resistance which may be very useful for stability analyses. Nevertheless, the test has a disadvantage where the measured stresses are usually the normal and shear stresses acting on the horizontal plane only, and the stresses are not known for any other plane within the specimen. If we can quantitatively determine the values of the principal stresses at any stage of the simple shear test by using the values of the stresses measured on the horizontal plane, the test results may be widely used with satisfactory accuracy. In this paper, a method for calculating the undrained shear strength of normally consolidated clay by using the expressions of the principal stresses in the simple shear test derived in a previous paper by the author[4] is newly proposed, and its validity is confirmed by the experimental results of three clays. It is believed that the proposed method may be applied to other normally consolidated clays and may give a more reasonable estimate of the undrained shear strength of normally consolidated clays measured in the simple shear test.

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