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A Study on the CAE System for Linkage Mechanism

by

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The basic study on the CAE system for the 2-dimensional linkage mechanism is developed. An arbitrary linkage mechanism can be solved by applying the solutions of the basic 4-link mechanism (it is called a unit here). The basic formulas are deduced. Based on this concept, a CAE system is constructed. Using this system, the Paucellier mechanism is analyzed. The reasonable solutions are obtained. Validity and effectiveness of the CAE system are verified through this example.

1. Introduction

Computer aided engineering (CAE)\(^1\) has successfully been in practical use in the fields with respect to machine design, mechanical analysis, computer simulation and so on. As compared with these fields, design of mechanism seems to be behind in computerization. In these days, the mechanical aspects as well as the kinematic and geometrical views should be taken into consideration for the design of mechanism. However, the computer system which is succeeded in synthesizing these subjects in practical use is not still very few, although each of them has been developed individually accompanying development of the robotics\(^2\).

From these points of view, this study aims at constructing the versatile CAE system through their systematic combination with the intention of applying it to the mechanism design of ordinary machines. For this purpose, it is required that the machine elements is treated as elastic bodies and the analyses in regard to these subjects are carried out simultaneously. The present system employs the matrix and vector method for the kinematic analyses\(^3\)\(^,\)\(^4\) and the boundary element method\(^5\) for the mechanical analyses of strength and dynamics.

The basic study of the CAE system for the linkage mechanism, as the first step, will be developed here. Especially, this report will focus on the problems of 2-dimensional motion analysis of the closed linkage mechanism. Based on this investigation, some typical analyses using the present system will be illustrated.

2. CAE System for Linkage Mechanism

It is considered that the CAE system for linkage mechanism is generally expected to discharge the functions as listed, for example, in table 1. The motion analyses of mechanism are performed through the matrix operation for 3-dimensional mechanism and vector operation for 2-dimensional problems. Although the 2-dimensional analysis

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can be contained in the 3-dimensional case, it is remarkably efficient to design special device for the 2-dimensional one. Since the mechanical analyses require many times for calculation, efficient equipment must be needed. In this study, the boundary element method (BEM) will be chosen because of its powerful effectiveness. In the practical system, the techniques for the computer graphics, in addition to these analyses, should be put into practice for the pre and post processor. However, they are beyond the purposes in this study.

Figure 1 shows an example of the CAE system which combines organically motion analysis with mechanical analysis utilizing the BEM. At every moment, posture or location, velocity (or angular velocity) and acceleration (or angular acceleration) of every elements are obtained through the motion analysis. These values are utilized in the next step of solving the equation of motion. This calculations are executed by the BEM and provides the values of torque, force, stress, deflection and so on. If the motion is continued then go to the next time step, otherwise go to the next step of transient problems. This is also calculated by the BEM. After that, the design is checked. If the present design bill satisfies us then stop, otherwise return and try again.

Based on this concept, 2-dimensional motion analysis of the linkage mechanism is discussed in this report, leaving the details of the BEM analysis to another report. The details will be shown in the following chapter.

3. Two-dimensional Motion Analysis
3. 1 Basic Equations

In the first instance in this study, 2-dimensional motion analysis of the closed linkage mechanism will be discussed. As mentioned in the previous chapter, motion analysis must provide the location, velocity and acceleration of every link. For this purpose, it is sufficient to obtain the angle, angular

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**Fig. 1** A Typical Example of the CAE System
A Study on the CAE System for Linkage Mechanism

velocity and angular acceleration. It is known that such problems of the planar mechanism can be solved through the planar triangular method. In this study, this excellent method is employed with a slight extension. Our method could be named as 4-links method. When the linkage mechanism has no sliding pair, it can be divided into 4-links mechanism (here, let us call it a unit). The 4-links method utilizes this fact, and therefore, it can’t be applied directly to the mechanism which contains the sliding pair. If an arbitrary unit is solved in general way, all the linkage mechanism can be solved by applying the said solution to each unit one after another. So, the kinematic solutions of a unit will be derived at first in the followings.

Suppose that a unit (namely, 4-rods linkage) a-b-c-d with $r_a$, $r_b$, $r_c$ and $r_d$ in length, respectively, is arbitrarily located on the complex plane as shown in figure 2. Let all the lengths be known and the motion of link a and d be defined (that is, $a_0$, $b_0$, $c_0$, $d_0$ and $t_0$ is given). In order to decide the location of every link, it is sufficient to find the angles of another two links b and c.

Vector equation with regard to the quadrilateral ABCD is given by the following equation (see Fig. 2)

$$r_a e^{i \theta_a} + r_c e^{i \theta_c} = r_b e^{i \theta_b} + r_d e^{i \theta_d}. \tag{1}$$

In regard to the triangles ABD and BCD, next equations are valid with the help of an additional vector $p$, respectively.

$$r_a e^{i \theta_a} = r_b e^{i \theta_b} + p e^{i \theta_b}. \tag{2}$$

$$r_c e^{i \theta_c} = r_d e^{i \theta_d} + p e^{i \theta_d}. \tag{3}$$

Dividing Eq.(2) by $e^{i \theta_b}$, we can obtain

$$p^2 = r_a^2 - 2r_a r_b \cos (\theta_a - \theta_b) + r_b^2. \tag{4}$$

From Eq. (2), $\theta_b$ can be got using p as follows;

$$\begin{bmatrix} \cos \theta_b, \sin \theta_b \end{bmatrix} = \begin{bmatrix} (r_a \cos \theta_a - r_b \cos \theta_b)/p, (r_a \sin \theta_a - r_b \sin \theta_b)/p \end{bmatrix}. \tag{5}$$

In a similar way, we can get from Eq. (3)

$$\cos (\theta_b - \theta_c) = (r_c^2 - r_b^2 - p^2)/2r_b p. \tag{6}$$

After $\theta_b$ is decided, $\theta_c$ is obtained through the next equation.

$$\begin{bmatrix} \cos \theta_c, \sin \theta_c \end{bmatrix} = \begin{bmatrix} (p \cos \theta_b - r_b \cos \theta_c)/r_c, (p \sin \theta_b - r_b \sin \theta_c)/r_c \end{bmatrix}. \tag{7}$$

Using Eqs. (6) and (7), absolute location of all links of this unit is determined at every step. In much the same way as mentioned above, the solutions in the other cases that the defined rods are given in different way can also be easily obtained. It is sufficient that only three cases in total are considered.

In regard to velocity, there are many quantities to be concerned, namely, angular velocity of each link and velocity of a specified point, for instance. Here let us show the angular velocities of another

![Fig. 2 Basic 4-links Mechanism](image-url)
two links (b and c) and velocity of point C.

By differentiating Eq. (1) with respect to time and using a little transformation, \( \dot{\theta}_c \) is given in the form which does not include \( \theta_c \) as follows (in order to simplify the expression, let the lengths of the rod vectors be constant):

\[
\dot{\theta}_c = \frac{\langle \dot{\theta}_a \rangle \sin(\theta_c - \theta_a) - \dot{\theta}_a \sin(\theta_c - \theta_a) \rangle}{\langle \theta_c \rangle \sin(\theta_c - \theta_c) / \langle \theta_c \rangle .}
\]

In the same manner, \( \dot{\theta}_b \) can be got as follows:

\[
\dot{\theta}_b = \frac{\langle \dot{\theta}_a \rangle \sin(\theta_c - \theta_c) - \dot{\theta}_a \sin(\theta_c - \theta_a) \rangle}{\langle \theta_c \rangle \sin(\theta_c - \theta_c) / \langle \theta_c \rangle .}
\]

For the velocity (and also for the acceleration) of a specified point (here, suppose point C), it is convenient to consider in the absolute field. Position vector of point C is given using point vector OA as follows (needless to say, it depends on the way of assignment):

\[
OC = OA + \langle r_a \rangle e^{i\theta} + \langle r_c \rangle e^{i\theta}.
\]

The velocity vector of the point C is obtained by differentiating the above with respect to time so far as the motion of point A is defined. Its lengthy expression is omitted here.

By differentiating Eq. (1) twice with respect to time, the angular accelerations of link b and c are given by the following equations (in order to simplify the expressions here, let \( \dot{\theta}_b, \dot{\theta}_a \) and \( \dot{\theta}_a \) be zero):

\[
\ddot{\theta}_b = \left\{ \langle r_a \rangle \ddot{\theta}_a^2 - \langle r_b \rangle \ddot{\theta}_b^2 \cos(\theta_c - \theta_b) - \langle r_b \rangle \ddot{\theta}_b^2 \cos(\theta_c - \theta_b) / \langle \theta_c \rangle \sin(\theta_c - \theta_c) \right\}.
\]

\[
\ddot{\theta}_a = \left\{ \langle r_a \rangle \ddot{\theta}_a^2 - \langle r_c \rangle \ddot{\theta}_c^2 \cos(\theta_c - \theta_c) + \langle r_c \rangle \ddot{\theta}_c^2 \cos(\theta_c - \theta_c) / \langle \theta_c \rangle \sin(\theta_c - \theta_c) \right\}.
\]

The acceleration of point C is obtained including the Coriolis’ component by differentiating Eq. (10) twice with respect to time. The lengthy equation is omitted, too.

3. 2 Constructing the System

The basic formulas deduced in previous section can be valid for an arbitrary unit. By applying them to all the units one by one, any linkage problems can be solved. It is realized that, in this manner, general system for two dimensional motion analysis can be constructed. The calculation flow of the system may become specifically as illustrated below:

1. data input,
2. calculate each unit one by one,
3. check the moving,
4. check locus of specified points,
5. check angular velocity and angular acceleration,
6. check velocity and acceleration of specified points.

The system will require the input data in regard to such as rods numbering, connectivity of rods for units, driven rod, fixed rod, specifying points to draw their locus and so on. Based on this concept, a temporarily general system has been constructed in this study. In the next chapter, an concrete example will be given.

4. Examples

4. 1 Model

In order to verify the availability of the basic formulas and the system construction, one example is supplied to the examination. See figure 3. The model shown here is the Paucellier mechanism which is devised to get a straight locus of a specific point. Note that this point is Q. The length of link 3 and link 6 is 220 and others 80. The link 1 is driven at a constant speed and the link 4 is fixed. Here, the motion of this mechanism, velocity and acceleration are naturally analyzed. Moreover, as a practical application of the CAE system, the

![Fig. 3 The Paucellier Mechanism](image-url)
influences of the length error of each link are investigated.

4.2 Motion

The present model has three units, namely, unit 1 (1234), unit 2 (1564) and unit 3 (2785). The analysis is practically put forward as follows. At first, a unit of which two links are defined is chosen and calculated using the basic formulas. In this example, both of unit 1 and unit 2 is possible. Secondly, another unit of those becomes the object. It goes without saying that only unsolved link is calculated. After these units are solved, two links in the 3rd unit turn to be defined. The last unit, at present, also can be solved. Notice that, in the present procedure, the basic formulas are valid for every unit by setting the links systematically in correspondence to the basic unit. The movement of this mechanism is illustrated in figure 4. When a proper ratio of lengths of all links is given, the point Q draws strictly a straight line perpendicular to the link 4.

4.3 Velocity and Acceleration

The motion analysis gives the values of all links at every step. Then the angular velocity, angular acceleration, velocity of any point and its acceleration can be calculated. Figure 5 typically shows the angular velocities of link 5 and link 8. The
angular accelerations are shown in figure 6. From the results, a designer can perceive, for example, that these angles change at a remarkably high rate around 240 degree. These changes can be observed on the screen of computer.

Figure 7 and figure 8 show the velocity and acceleration of the point Q, respectively. As shown here, a designer can also get easily the data of velocity and acceleration of any specific point. It is noticed that validity and effectiveness are verified through these solutions.

4. 4 Influence of Errors in Measurement

For the purpose of any other applications of the CAE system, influences of manufacturing error of links on the moving of the mechanism are investigated (The model used is the same with the one in figure 3). Suppose that a link has error in measurement of length. This error must yield the deviation of the locus drawn by the point Q. In this study, the error of 1% in length is given to each link one by one and their effects on the said deviation is investigated.

Figure 9 (a)-(e) shows the trace error. This trace is normalized by the distance between O and Q at driven angle of 180 degree. A designer will receive the information, for example, 1. link 7 (link 8, too) has the smallest effect, 2. link 3 (link 6, too) has the largest effect and so on. Although this calculation is nothing but one example, it should be noted that the CAE system can be utilized some specific purposes, as illustrated here.

5. Conclusions

The basic study on the CAE system for the 2-dimensional linkage mechanism is developed. Conclusion can be summarized as follows.

1. An arbitrary linkage mechanism can be divided into 4-linkage mechanism (it is called a unit here).
2. An arbitrary linkage mechanism can be solved by applying the solutions of the basic unit.
3. The basic formulas with respect to an arbitrary unit in general way are deduced.
4. Based on the concepts above, a CAE system is constructed.
Fig. 9 (a) Trace Error

Fig. 9 (b)

Fig. 9 (c)

Fig. 9 (d)
5. Using this system, the Paucellier mechanism is analyzed and reasonable solutions in regard to the moving, angular velocity, angular acceleration, velocity of a specific point and its acceleration are obtained. This assures the validity of the basic formulas.

6. For one example of applications of the CAE system, influence of error in measurement of link length on the moving is investigated. Through the analyses, validity and effectiveness of the CAE system are verified.

References
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