Identifying Trough of Recent Recession in Japan

An Application of Stochastic Business Indicator

Shinji Yoshioka

Abstract

In Japan, the Indexes of Business Conditions calculated by the Cabinet Office of the Government of Japan is employed for assessing business cycle. The CI consists of three components, such as leading, coincident, and lagging indexes. The CI is calculated by composing month-to-month percentage changes in multiple economic indicators. On contrary, in the U.S., for observing business condition, a stochastic business indicator is mainly employed. This study applies the latter U.S. approach to estimate a latent stochastic business indicator for Japanese economy according to Stock and Watson using a state space model solved by Kalman filter. The estimated stochastic business indicator seems to fit quite well to existing Japanese official Indexes of Business Conditions. The estimated results appear to indicate that the trough month of the latest recession in Japan is March.

Key words: Business cycle, Recession, Stochastic business indicator, State space model, Kalman filter, Japan

JEL Classification: C, C, C, E, and O

1. Introduction

From April on, the Government of Japan has officially adopted a composite index, named Indexes of Business Conditions prepared by the Cabinet Office for assessing business cycles. This Indexes of Business Conditions hereafter, CI consists of three indexes, such as leading, coincident, and lagging indexes. Each Index includes following components:

Table Components of Indexes of Business Conditions

<table>
<thead>
<tr>
<th>Leading Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>L Index of Producer Inventory Ratio of Finished Goods Final Demand Goods</td>
</tr>
<tr>
<td>L Index of Producer Inventory Ratio of Finished Goods Producer Goods For Mining and Manufacturing</td>
</tr>
</tbody>
</table>

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According to CI, the Cabinet Office of the Government of Japan identifies the reference date of business cycle in Japan as follows:

1 Although the CI is one of main criteria, more generalized and broader approach is taken for the identification of the reference dates of business cycle in Japan.
Table  The Reference Dates of Business Cycles in Japan

<table>
<thead>
<tr>
<th>Peak By Month</th>
<th>Trough By Month</th>
<th>Peak By Quarter</th>
<th>Trough By Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun, 97 09</td>
<td>Oct, 97 10</td>
<td>Q, 97 3,</td>
<td>Q, 97 4,</td>
</tr>
<tr>
<td>Jan, 98</td>
<td>Nov, 97 11</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Jun, 98 09</td>
<td>Jun, 97 09</td>
<td>Q, 98 3,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Dec, 97 09</td>
<td>Oct, 97 10</td>
<td>Q, 97 3,</td>
<td>Q, 97 4,</td>
</tr>
<tr>
<td>Oct, 97 09</td>
<td>Oct, 97 10</td>
<td>Q, 97 3,</td>
<td>Q, 97 4,</td>
</tr>
<tr>
<td>Jul, 97 09</td>
<td>Dec, 97 09</td>
<td>Q, 97 3,</td>
<td>Q, 97 4,</td>
</tr>
<tr>
<td>Nov, 97 10</td>
<td>Mar, 98 11</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Jan, 98 09</td>
<td>Oct, 97 09</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Feb, 97 09</td>
<td>Feb, 97 09</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Jun, 97 09</td>
<td>Nov, 97 10</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Feb, 97 09</td>
<td>Oct, 97 10</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>May, 97 09</td>
<td>Jan, 98 10</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Nov, 97 11</td>
<td>Jan, 98 10</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
<tr>
<td>Oct, 98 09</td>
<td>n/f 1</td>
<td>Q, 98 1,</td>
<td>Q, 98 4,</td>
</tr>
</tbody>
</table>

Source: CAO

Although the Cabinet Office has not officially identified the trough of the recent recession after the peak of October, many economists regard that the recession ended in the first quarter in March. This is mainly because of the movement of CI, which is indicated as follows.

Figure  Development of CI

Note: The shadowed periods are of recession. Although CAO does not reveal the latest trough month, it is provisionally set in March according to a broad consensus among economists.

Source: Author based on Cabinet Office data
While the Japanese CI is calculated by composing month-to-month percentage changes in multiple economic indicators, The U.S. takes another approach, which employs a stochastic business indicator. The latter methodology uses a state space model to be solved by Kalman filter, in order to estimate a latent indicator. This study applies the latter U.S. approach to estimate a stochastic business indicator for Japan. Including this introductive section, this paper consists of four sections: the second section focuses on methodology of Japanese CI and stochastic business indicator; the third present data and estimation results; and, the final section briefly concludes the paper. This study is based on available data and information until March, and EViews is employed for estimation.

2. Methodology and Model

**Methodology of Indexes of Business Conditions**

Summarizing CAO, the Indexes of Business Conditions is calculated according to following four steps.

**Step** A formula is used for calculating the symmetric percent change of individual series as in the following:

\[ r_i = \frac{y_i - y_i^{\prime}}{y_i + y_i^{\prime}} \]

where \( r \) Symmetric percent change
\( y \) Individual series
\( i \) Number assigned to each indicator
\( t \) Time point

If the given time series is zero or a negative value, or is already in percentage form, simple arithmetic differences are calculated:

\[ r_i = y_i - y_i^{\prime} \]

Then, outliers are trimmed using the following formula:

\[ \psi = \begin{cases} -kQ - Q & \text{for } r_i < -kQ - Q \\ r_i & \text{for } -kQ - Q \leq r_i \leq kQ - Q \\ kQ - Q & \text{for } r_i > kQ - Q \end{cases} \]

where \( Q \) The first quartile in the interquartile range
\( Q \) The third quartile in the interquartile range

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2 For more detailed and precise information, see Note for Calculation of CAO.
The trend of individual series mean percent change is calculated by the trimmed month backward moving average as follows:

\[
\bar{\Delta_\text{t}} = \frac{\sum_{i=1}^{n} \Delta_\text{t} - \Delta_\text{t}}{n} \]

where \( \Delta_\text{t} \) Mean percent change

Next, percent change normalized by interquartile range is calculated by applying the following formula:

\[
\bar{Z}_\text{t} = \frac{\sum_{i=1}^{n} Z_\text{t} - \bar{Z}_\text{t}}{Q_i - \bar{Q}_i} \]

where \( Z_\text{t} \) Mean percent change

Composite percentage change is calculated by adding up trend composite mean percentage change, and the mean of percent change normalized by interquartile range composite percent change normalized by interquartile range. In this process, composite percent change normalized by interquartile range is multiplied by the mean of interquartile ranges composite interquartile range so that the levels of the trend component and the cyclical component coincide as follows:

\[
\bar{V}_\text{t} = \frac{\sum_{i=1}^{n} V_\text{t} + \bar{Q}_i - \bar{Q}_i}{n} \]

where \( V_\text{t} \) Composite percent change

Finally, the index is rebased so that the value for the reference year is equal to. The current reference year is

**Model of Stochastic Business Indicator**

On contrary of above methodology of CI in Japan, that of stochastic business indicator assumes a unique and latent index, which affects and reveals existing and observable indicator,
such as production, labor, income, and consumption, etc. Assuming that this unique and latent index and the error terms follow autoregressive AR process, the model of stochastic business indicator is mathematically represented in the following model

\[
y_i \equiv \bar{y}_i + \sum_{j=1}^{n} c_{ij} y_{i-j} + \sum_{j=1}^{m} e_{ij} u_{i-j} + \epsilon_i
\]

where

- \( y \): Observable business indicators
- \( c \): Unique and latent business indicator
- \( u, e, \epsilon \): Error term
- \( i \): Number of observable indicators
- \( n \): Number of lags of AR process for \( c \)
- \( m \): Number of lags of AR process for \( u \)

Using lag operator \( L \), above model can be expressed as follows

\[
y_i \equiv i + \sum_{j=1}^{n} c_{ij} y_{i-j} + \sum_{j=1}^{m} e_{ij} u_{i-j} + \epsilon_i
\]

Here, \( \bar{c} \) represents a lag polynomial of \( \bar{c} = c_0 - c_1 L - c_2 L^2 - \ldots - c_n L^n \) and \( \bar{e} \) does that of \( \bar{e} = e_0 - e_1 L - e_2 L^2 - \ldots - e_m L^m \). On the other hand, error term \( e \) is a scalar stochastic variable that follows \( e \sim N(\Omega, \bar{e}^2) \), and \( \epsilon \) is also a scalar stochastic variable that follows \( \epsilon \sim N(\Omega, \epsilon^2) \). Of course, \( O \) is a null matrix \( \bar{\Omega} \).

Since this model for stochastic business indicator includes latent variables, the equation system is represented as a state space model. According to Okusa, the generalized state space representation of the stochastic business indicator is as follows

\[
\begin{align*}
\mathbf{x} &= \begin{pmatrix}
c \\
\vdots \\
c - n \\
u \\
\vdots \\
u - m
\end{pmatrix} \\
\mathbf{y} &= Z \mathbf{x} + \epsilon
\end{align*}
\]
Transit equation

$\dot{X} = A \cdot X + B \cdot u$

Disturbance term

$\Theta \sim N(0, \Theta^2)$

where $Z = \begin{bmatrix} O_{1,n} & I \mid O_{1,m} \end{bmatrix}$

$X = \begin{bmatrix} O_{n,1} & O_{2,1} & \cdots & O_{n,m} & O_{n,1} & O_{2,1} & \cdots & O_{n,m} \\ I & O_{1,1} & \cdots & I & O_{1,1} & \cdots & I & O_{1,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O_{m,n} & O_{1,1} & \cdots & O_{m,n} & O_{1,1} & \cdots & O_{m,n} & O_{1,1} \end{bmatrix}$

Of course, the state variable $\Theta$ is an $n \times m$ vector. According to usual definition, $I_k$ means a unit matrix with $k$ rows and columns, and $O_{k,1}$ represents a null matrix with $k$ rows and 1 columns. $\Theta$ means a diagonal matrix with its elements of $\Theta = diag(\Theta_1, \Theta_2, \cdots, \Theta_m)$ while $h$ is a diagonal element of $H$. is a parameter of $n$-degree lag polynomial for the latent index. $\Theta$ is a diagonal matrix with elements of $\Theta$, which is a parameter of $m$-degree lag polynomial for the error term $u$.

Since above-mentioned general form of the state space model quoted from Okusa is quite complicated, this study assumes following three points, which seem adequately plausible, to simplify the model according to existing literatures, including Stock and Watson and Okusa.

- The observable indicators are taken from production, labor, income, and consumption, i.e., $i$.
- The unique and latent business indicator $c$ is subject to AR process, i.e., $n$.
- The error term $u$ is subject to AR process, i.e., $m$.

Above model for stochastic business indicator will be transformed into following simplified state space model system.

Observation Equations

$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \end{bmatrix} = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \Theta_4 \end{bmatrix} + \begin{bmatrix} c \theta_1 \\ \Theta_1 \theta_2 \\ \Theta_1 \theta_3 \\ \Theta_1 \theta_4 \end{bmatrix}$
Transit Equations

\[
\begin{pmatrix}
  c \\
  u_0 \\
  u_1 \\
  u_2 \\
  u_3
\end{pmatrix} =
\begin{pmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 \\
  u_0 & u_1 & u_2 & u_3 & u_4 \\
  u_0 & u_1 & u_2 & u_3 & u_4 \\
  u_0 & u_1 & u_2 & u_3 & u_4 \\
  u_0 & u_1 & u_2 & u_3 & u_4
\end{pmatrix}
\begin{pmatrix}
  \epsilon \\
  u_1 \\
  u_2 \\
  u_3 \\
  u_4
\end{pmatrix}
\]

This simplified state space model will be solved with Kalman filter presented at Kalman. In this study, further explanation for state space models and Kalman filter will be out of target. For comprehensive information on application of a state space model to econometric field, Harvey is one of the most useful literatures if necessary. Apart from Kalman’s original paper, Meinhold and Singpurwalla, Snyder and Forbes, Snyder and Forbes, and Grewal and Andrews will provide further information on Kalman filter and its algorithm. Some relevant internet sites, including Kalman Filter of the Department of Computer Science at the University of North Carolina, where the reprint of Kalman is uploaded, are also helpful.

3. Data and Estimation Results

According to the assumption and the model presented in the previous section, following actual and observable data are employed:

- **Production** • Index of Industrial Production • Mining and Manufacturing • published by the Ministry of Economics, Trade and Industry, seasonally adjusted series.
- **Employment** • Index of Non-Scheduled Worked Hours • establishments with employees or more • published by the Ministry of Health, Labor and Welfare, seasonally adjusted series.
- **Income** • Real Wage Index of Total Cash Earnings • establishments with employees or more • published by the Ministry of Health, Labor and Welfare, seasonally adjusted series.
- **Consumption** • Retail Commercial Sales Value of Monthly Report on the Current Survey of Commerce published by the Ministry of Economics, Trade and Industry, adjusted to real term by Consumer Price Index published by the Statistics Bureau, and seasonally adjusted by X with a default option by author.

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3  http://www.cs.unc.edu/~welch/kalman
4  All data are monthly and available from January to December.
First of all, Augmented Dickey-Fuller (ADF) unit root tests based on Dickey and Fuller are completed in order to check the data generating process of relevant above four data. Table reports the test results.

Table Results of ADF Tests

<table>
<thead>
<tr>
<th></th>
<th>log level</th>
<th>log difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t-Statistic</td>
<td>p-value</td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Lag length are decided according to Akaike Information Criteria based on Akaike under the condition of maximum months. P-value is measured at a one-sided basis on MacKinnon.

Source: Author

According to the results of ADF tests, log first-order differential series reject the existence of unit root for all relevant data such as production, employment, income, and consumption, while log level series do not. Hence, log differential series will be employed for estimation. Table reports their descriptive statistics. All data are available from February to December.

Table Descriptive Statistics

<table>
<thead>
<tr>
<th>Data Description</th>
<th>Production</th>
<th>Employment</th>
<th>Income</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum Sq. Dev</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Taking first-order differential series, an observation will be missed.
### Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>Production</th>
<th>Employment</th>
<th>Income</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Employment</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Income</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Consumption</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
</tbody>
</table>

Source: Author’s calculation

Based on above model and data, the unique and latent business indicator is estimated. Table reports the estimation results. The parameters for production and employment are rela-

### Table Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Parameter</th>
<th>Std. Error</th>
<th>t-statistics</th>
<th>R² adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>constant</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td></td>
<td>SWI</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Employment</td>
<td>constant</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td></td>
<td>SWI</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Income</td>
<td>constant</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td></td>
<td>SWI</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td>Consumption</td>
<td>constant</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
<tr>
<td></td>
<td>SWI</td>
<td>ø</td>
<td>ø</td>
<td>ø</td>
</tr>
</tbody>
</table>

Source: Author’s estimation

### Figure Stochastic Business Indicator and Index of Business Conditions

Note: Same as Figure

Source: Cabinet Office data and author’s estimation
tively large, while those for income and consumption are small. This fact indicates the sensitivity to business cycle, of course.

Figure depicts the estimated stochastic business indicator \( \text{SWI} \) compared with the coincident index of Indexes of Business Conditions \( \text{CI} \) calculated by the Cabinet Office, the Government of Japan.

Although the estimated stochastic business indicator \( \text{SWI} \) does not show clear cyclical movements in the first half of the 1990s, after the bubble economy that began in late 1980s, the SWI indicates distinct cycles. To check properties, CI and SWI are decomposed to cycle and trend series using Hodrick-Prescott filter based on Hodrick and Prescott. The smoothness parameter \( \lambda \) is set at 1600 according to wide and common consensus among economists. Defining the \( \text{GAP} \) as percentage ratio of cycle series to trend, i.e., \( \text{GAP} = \frac{\text{Cycle}}{\text{Trend}} \), the estimated SWI movement during the estimation period fits quite well to CI as Figure depicts.

**Figure** \( \text{GAP of Stochastic Business Indicator and Index of Business Conditions} \)

![GAP of Stochastic Business Indicator and Index of Business Conditions](image)

Note: The unit of vertical axis is percent of cycle series to trend.

Same as Figure.

Source: Author’s estimation

Here, Table compares four kinds of peak and trough months of Japan’s business cycle, identified by the official reference dates of the Government of Japan (CAO), turning points of the Indexes of Business Conditions (CI), the reference chronology of the turning points of the OECD Composite Leading Indicator (OECD), and the turning points of the estimated stochastic business indicator in this study (SWI). They are not necessarily coincident, but siz-

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6 See [http://www.oecd.org/document/49/0,3746,en_2649_34831_26300982_1_1_1_1,00.html](http://www.oecd.org/document/49/0,3746,en_2649_34831_26300982_1_1_1_1,00.html) Accessed on March 2023.
ably close to each other, including the trough of the latest recession. While the OECD Composite Leading Indicator points to April 2015, both CI and SWI identify March 2015 as the trough, which appears quite plausible and acceptable among Japanese economists.

### Table 1 Peak and Trough Months of Business Cycles in Japan after 2015

<table>
<thead>
<tr>
<th></th>
<th>Peak Month</th>
<th>Trough Month</th>
<th>Peak Month</th>
<th>Trough Month</th>
</tr>
</thead>
</table>

Note: CAO’s peak month of October 2015 for the latest recession is provisional.

The OECD Composite Leading Indicator identifies a recession with its peak as June 2014 and its trough as August 2015, but it is not included in the table.

Source: CAO, CI, OECD, SWI

4. Conclusion

This study has successfully estimated the stochastic business indicator based on Stock-Watson methodology and has identified the trough month of the latest recession in Japan as March 2015, which many economists will support. The results are too consistent to coincident Index of Business Conditions calculated by the Government of Japan.

Concerning identification of business cycle turning point, this study focuses on Indexes of Business Conditions of the Government of Japan, which is based on observable indicators, and stochastic approach, suggested by Stock and Watson. The latter approach is also employed for many economists. Melo V. et al. adopt for Colombian economy, Picchetti and Toledo estimate Brasirian industrial index, Lemoine applies to the UK, French, German and the Euro-zone business cycles. Additionally, many other methodologies are also explored. Kim and Nelson propose a Bayesian approach based on a Markov-switching model. Yoshioka utilizes GDP gap estimated with a state space model for business cycle dating, Yoshioka employs Markov Regime Switching model. Fukuda and
Onodera also propose a new index of coincident economic indicators in Japan to improve the forecast performance. Relating only to business cycle dating, there are plenty of literatures, including Harding and Pagan, Artis et al, and Chauvet and Hamilton, which propose a quarterly real-time GDP based recession probability index.

Finally, it is noteworthy to stress that identification of business cycle is essentially important for the macroeconomic stabilization policy.

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