PATCHING METHODS FOR THE ANALYSIS OF WAVE MOTION OVER A SUBMERGED PLATE[1]

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Several patching methods including point collocation, segment collocation and the Galerkin method for the analysis of wave motion over a submerged plate are described on the basis of the weighted residual approximation. Formulation of the problem also involves the conventional linear potential wave theory. Comparative computations of the reflection caused by and the fluid force acting on the plate show that the Galerkin method is most effective. The collocation methods are less efficient because they include a large number of high order terms which are only for the purpose of making the numerical problem solvable and have no appreciable contribution to promote the accuracy of results.

1. INTRODUCTION

It has been a usual practice to solve the problem of wave motion over a submerged obstacle by the linear potential wave theory. Although by this theory little can be known about the important phenomena such as wave breaking and the deformation of wave profile, the reflection and transmission caused by and the wave force acting on the obstacle, which are of particular interest in many engineering practices, can be understood to a large extent.

The governing equation of the linear potential wave theory is the Laplace equation in terms of the velocity potential. Since the Laplace equation can be analytically solved by separation of variables if the domain concerned has a simple geometry and the boundary conditions are expressed by linear combinations of the potential and its normal derivative, a problem whose domain can be divided into several subdomains with such simple geometry can then also be solved by patching the formal solutions obtained in each subdomain. Various patching methods which ensure the difference of the velocity potential on each side of a patching boundary to be minimum in some sense have been developed[1)-4)]. The present study is to carry out a comparative study of these methods and to find the principle underlying them.

2. BASIC FORMULATION

By assuming that the fluid is invicid and incompressible and its motion irrotational, a wave with small amplitude around a horizontally submerged impermeable plate, as shown in Fig. 1, can be described by a scalar function ϕ, the velocity potential with the time factor $e^{-i\sigma t}$ (σ is the angular frequency of the wave motion) separated. ϕ satisfies the following conditions[5)]:

$$\nabla^2\phi = 0 \tag{1}$$

$$\frac{\partial\phi}{\partial z}=\frac{\sigma^2\phi}{g} \qquad (z=0) \tag{2}$$

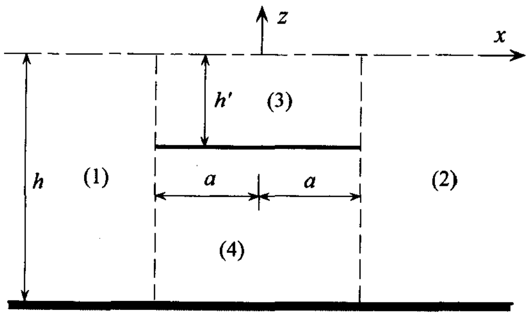

Fig. 1 Definition sketch of wave motion around a submerged plate.

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$$\frac{\partial \phi}{\partial z}=0 \qquad (z=-h;\ z=-h' \text{ and } -a<x<a) \tag{3}$$

$$\frac{\partial \phi}{\partial x}+\mathrm{i}k\phi=2\mathrm{i}k\phi_0 \qquad (x\to-\infty) \tag{4}$$

$$\frac{\partial \phi}{\partial x}-\mathrm{i}k\phi=0 \qquad (x\to+\infty) \tag{5}$$

where k is the incident wavenumber, h the total water depth, h' the submerged depth of the plate, a half of the plate length and g the gravitational acceleration; $\phi_0=q\cosh k(h+z)e^{ik(x+a)}$ with $q=gH_0/(2\sigma\cosh kh)$ is the incident wave potential (the incident wave height is H_0).

By introducing artificial inner boundaries at $x=-a$ and $x=a$, as also indicated in Fig. 1, the domain occupied by the fluid is divided into 4 single-connected rectangular subdomains. The formal solution of the velocity potential in each subdomain may then be directly obtained by separation of variables. If the potential in the reflection region ($-\infty<x<a$ and $-h\leq z\leq 0$), the transmission region ($a<x<\infty$ and $-h\leq z\leq 0$), the region above the plate ($-a<x<a$ and $-h'<z\leq 0$) and the region below the plate ($-a<x<a$ and $-h\leq z<-h'$) are denoted by ϕ_1, ϕ_2, ϕ_3 and ϕ_4, respectively, we have

$$\phi_1=q\Omega_0e^{-\omega_0(x+a)}+\sum_{n=0}^{\infty}A_n\Omega_ne^{\omega_n(x+a)} \tag{6}$$

$$\phi_2=\sum_{n=0}^{\infty}B_n\Omega_ne^{\omega_n(x+a)} \tag{7}$$

$$\phi_3=\sum_{n=0}^{\infty}(C_n\cosh\omega'_nx+D_n\sinh\omega'_nx)\Omega'_n \tag{8}$$

$$\phi_4=\sum_{n=0}^{\infty}(E_n\cosh\lambda_nx+F_n\sinh\lambda_nx)\Lambda_n \tag{9}$$

where A_n, B_n, C_n, D_n, E_n and F_n are unknown constants to be determined; $\{\omega_n\}=\{-ik, k_1\cdots, k_n, \cdots\}$, $\{\Omega'_n\}=\{\cos\omega_n(h+z)\}$, $\{\omega'_n\}=\{-ik', k'_1,\cdots,k'_n\cdots\}$, $\{\Omega'_n\}=\{\cos\omega'_n(h'+z)\}$, $\{\lambda_n\}=\{n\pi/(h-h')\}$, $\{\Lambda_n\}=\{\cos\lambda_n(h+z)\}$; $k, k_1,\cdots,k_n\cdots$ and $k'\ k'_1,\cdots, k'_n\cdots$ are the positive roots of the following dispersion equations:

$$\frac{\sigma^2}{g}=k\tanh kh=-k_n\tan k_nh=k'\tanh k'h'=-k'_n\tan k'_nh' \qquad (n=1,\ 2,\ \cdots) \tag{10}$$

The unknown constants involved in (6)−(9) must be determined so that the velocity potential and its normal derivatives of each order are continuous across the inner boundaries. Since it can be proved that its higher order derivatives are automatically continuous once the velocity potential and its first order derivative are continuous, the patching conditions at the inner boundaries can be specified as

$$\phi_1=[\phi_3:\phi_4] \quad \text{and} \quad \frac{\partial\phi_1}{\partial x}=\frac{\partial}{\partial x}[\phi_3:\phi_4] \qquad (x=-a) \tag{11}$$

$$\phi_2=[\phi_3:\phi_4] \quad \text{and} \quad \frac{\partial\phi_2}{\partial x}=\frac{\partial}{\partial x}[\phi_3:\phi_4] \qquad (x=a) \tag{12}$$

where the notation $[(\cdot):(\cdot)]$ implies that the value before the colon should be taken if $-h'<z\leq 0$ and the value after the colon should be taken if $-h\leq z<-h'$.

It is unlikely in general that (11) and (12) be exactly satisfied by properly choosing the values of the unknown constants. A procedure to ensure the difference of the velocity potential and its normal derivative on each side of the inner boundaries, $R_m(z)$ and $\partial R_m(z)/\partial x$, where the subscript m $(=1, 2)$ is designated to distinguish the boundaries at $x=-a$ and at $x=a$, to be minimum on $0\leq z\leq -h$ is, therefore, needed for a good approximation. The method of weighted residuals is applicable on this occasion, since, if a set of functions $\{w_i(z),\ i=1, 2, \cdots, N\}$ on $0\leq z\leq -h$ is available so that

$$\int_{-h}^{0}w_iw_jdz=\begin{cases}\Psi_i\neq 0 & (i=j)\\ 0 & (i\neq j)\end{cases}$$

where w_i are called the weighting functions, the relations

$$\int_{-h}^{0} w_i R_m dz=0 \quad \text{and} \quad \int_{-h}^{0} w_i \frac{\partial R_m}{\partial x} dz=0 \quad (m=1,2 \text{ and } i=1,2,\cdots,N) \tag{14}$$

lead to $R_m=0$ and $\partial R_m/\partial x=0$ in some weighted average sense.

By properly choosing the weighting functions $\{w_i\}$, (14) is expected to give rise to a set of linear algebraic equations in terms of the unknown constants involved in (6)—(9). The unknown constants and consequently the velocity potential may then be determined. There are different choices to the weighting functions, each produces a special patching method. In the following, we describe several cases of particular interest.

3. DESCRIPTION OF VARIOUS PATCHING METHODS

3.1 Point Collocation

One possible choice to the weighting functions $\{w_i\}$ is the delta functions:

$$w_i=\delta(z;z_i) \quad (i=1,2,\cdots,N) \tag{15}$$

where z_i are the coordinates of a set of points specified on the inner boundaries. Substituting (15) into (14) we have

$$R_m(z_i)=0 \quad \text{and} \quad \frac{\partial R_m(z_i)}{\partial x}=0 \quad (m=1,2 \text{ and } i=1,2,\cdots,N) \tag{16}$$

It can be noted that (16) is simply to guarantee the continuity of the potential and its normal derivative at the specified points on the inner boundaries, the concept of point collocation, which has been applied to the analysis of wave motion over submerged plates by Yoshida et al.[4]

When a total number of $2N$ points for collocation are symmetrically specified at $x=-a$ and $x=a$, (16) can be written as

$$q\Omega_{0i}+\sum_{n=0}^{M}A_n\Omega_{ni}=\sum_{n=0}^{[M_1:M_2]}([C_n:E_n]\cosh[\omega'_n:\lambda_n]a-[D_n:F_n]\sinh[\omega'_n:\lambda_n]a)[\Omega'_{ni}:\Lambda_{ni}] \tag{17}$$

$$\sum_{n=0}^{M}B_n\Omega_{ni}=\sum_{n=0}^{[M_1:M_2]}([C_n:E_n]\cosh[\omega'_n:\lambda_n]a+[D_n:F_n]\sinh[\omega'_n:\lambda_n]a)[\Omega'_{ni}:\Lambda_{ni}] \tag{18}$$

$$\begin{aligned}-q\omega_0\Omega_{0i}+\sum_{n=0}^{M}A_n\omega_n\Omega_{ni}=&\sum_{n=0}^{[M_1:M_2]}(-[C_n:E_n][\omega'_n:\lambda_n]\sinh[\omega'_n:\lambda_n]a\\&+[D_n:F_n][\omega'_n:\lambda_n]\cosh[\omega'_n:\lambda_n]a)[\Omega'_{ni}:\Lambda_{ni}]\end{aligned} \tag{19}$$

$$\begin{aligned}-\sum_{n=0}^{M}B_n\omega_n\Omega_{ni}=&\sum_{n=0}^{[M_1:M_2]}([C_n:E_n][\omega'_n:\lambda_n]\sinh[\omega'_n:\lambda_n]a\\&+[D_n:F_n][\omega'_n:\lambda_n]\cosh[\omega'_n:\lambda_n]a)[\Omega'_{ni}:\Lambda_{ni}]\end{aligned} \tag{20}$$

$(i=1,2,\cdots,N)$

where the subscript i associated with the eigenfunctions Ω_n, Ω'_n and Λ_n denotes the values of these functions at $z=z_i$. An operation among (17), (18), (19) and (20) gives

$$q\Omega_{0i}+\sum_{n=0}^{M}(A_n+B_n)\Omega_{ni}=2\sum_{n=0}^{[M_1:M_2]}[C_n:E_n]\cosh[\omega'_n:\lambda_n]a\,[\Omega'_{ni}:\Lambda_{ni}] \tag{21}$$

$$-q\omega_0\Omega_{0i}+\sum_{n=0}^{M}(A_n+B_n)\omega_n\Omega_{ni}=-2\sum_{n=0}^{[M_1:M_2]}[C_n:E_n][\omega'_n:\lambda_n]\sinh[\omega'_n:\lambda_n]a\,[\Omega'_{ni}:\Lambda_{ni}] \tag{22}$$

$$q\Omega_{0i}+\sum_{n=0}^{M}(A_n-B_n)\Omega_{ni}=-2\sum_{n=0}^{[M_1:M_2]}[D_n:F_n]\sinh[\omega'_n:\lambda_n]a\,[\Omega'_{ni}:\Lambda_{ni}] \tag{23}$$

$$-q\omega_0\Omega_{0i}+\sum_{n=0}^{M}(A_n-B_n)\omega_n\Omega_{ni}=2\sum_{n=0}^{[M_1:M_2]}[D_n:F_n][\omega'_n:\lambda_n]\cosh[\omega'_n\lambda_n]a\,[\Omega'_{ni}:\Lambda_{ni}] \tag{24}$$

$(i=1,2,\cdots,N)$

It is obvious that (21)−(24) may be separated into two independent groups: the group formed by (21) and (22), from which (A_n+B_n), C_n and E_n can be solved, and the group formed by (23) and (24), from which (A_n-B_n), D_n and F_n can be solved. For numerical solutions, the infinite series in (6), (7), (8) and (9) have been truncated. During the truncation, however, we require the highest order terms retained in all the series to be of the same order, that is, $\omega_M \sim \Omega'_{M_1} \sim \lambda_{M_2}$, for an efficient computational procedure. This principle of equivalent highest order terms retained in all series can be realized through properly evaluating M, M_1 and M_2. Since $\omega_n \sim n\pi/h$, $\omega'_n \sim n\pi/h'$ and $\lambda_n \sim n\pi/(h-h')$ when n is large, we need only to require

$$\frac{M_1}{M_2} \simeq \frac{h'}{h-h'} \quad \text{and} \quad M=M_1+M_2+1 \tag{25}$$

It is now evident that only the value of M, which is related to the accuracy of results, should be specified; M_1 and M_2 are determined from M and the depth h and h'.

It is worthwhile to be pointed out that considering the equivalent highest order terms retained in all series gives rise to numerical advantages because it is the highest order evanescent terms that worsen the property of the matrix for solving the unknowns. To get a solution precisely, the high order evanescent terms which do not promote the accuracy of the final solution must be avoided if possible.

It is known that the number of unknowns involved in each group formed by (21) and (22) and by (23) and (24) is $M+M_1+M_2+3$, which equals to $2(M+1)$ if (25) is considered. On the other hand, the number of simultaneous equations for solving the unknowns is $2N$, twice the number of the points specified on each inner boundary for collocation. When $N=M+1$ (in which M_1+1 points must be located on $-h' \leq z \leq 0$ and M_2+1 points on $-h \leq z \leq -h'$), the system is closed and the solution uniquely determined.

3.2 Segment Collocation

The weighting functions $\{w_i\}$ may also take the following form:

$$w_i=\begin{cases} 1 & (z \in [z_{i1},\ z_{i2}]) \\ 0 & (z \notin [z_{i1},\ z_2]) \end{cases} \qquad (i=1, 2, \cdots, N) \tag{26}$$

where $[z_{i1}, z_{i2}]$ represent segments on the inner boundaries with their lower and upper vertical coordinates being z_{i1} and z_{i2}, respectively. As both inner boundaries at $x=-a$ and at $x=a$ are symmetrically discretized into segments, substitution of (26) into (14) yields

$$q\bar{\Omega}_{0i}+\sum_{n=0}^{M} A_n\bar{\Omega}_{ni}=\sum_{n=0}^{[M_1:M_2]}([C_n:E_n]\cosh[\omega'_n:\lambda_n]a-[D_n:F_n]\sinh[\omega'_n:\lambda_n]a)[\bar{\Omega}'_{ni}:\bar{\Lambda}_{ni}] \tag{27}$$

$$\sum_{n=0}^{M} B_n\bar{\Omega}_{ni}=\sum_{n=0}^{[M_1:M_2]}([C_n:E_n]\cosh[\omega'_n:\lambda_n]a-[D_n:F_n]\sinh[\omega'_n:\lambda_n]a)[\bar{\Omega}'_{ni}:\bar{\Lambda}_{ni}] \tag{28}$$

$$-q\omega_0\bar{\Omega}_{0i}+\sum_{n=0}^{[M_1:M_2]} A_n\omega_n\bar{\Omega}_{0i}=\sum_{n=0}^{[M_1:M_2]}(-[C_n:E_n][\omega'_n:\lambda_n]\sinh[\omega'_n:\lambda_n]a$$
$$+[D_n:F_n][\omega'_n:\lambda_n]\cosh[\omega'_n:\lambda_n]a)[\bar{\Omega}'_{ni}:\bar{\Lambda}_{ni}] \tag{29}$$

$$-\sum_{n=0}^{M} B_n\omega_n\bar{\Omega}_{ni}=\sum_{n=0}^{[M_1:M_2]}([C_n:E_n][\omega'_n:\lambda_n]\sinh[\omega'_n:\lambda_n]a$$
$$+[D_n:F_n][\omega'_n:\lambda_n]\cosh[\omega'_n:\lambda_n]a)[\bar{\Omega}'_{ni}:\bar{\Lambda}_{ni}] \tag{30}$$

$(i=1, 2, \cdots, N)$

where

$$\bar{\Omega}_{ni}=\int_{z_{i1}}^{z_{i2}}\Omega_n dz=\frac{1}{\omega_n}(\sin\omega_n z_{i2}-\sin\omega_n z_{i1}) \tag{31}$$

$$\bar{\Omega}'_{ni}=\int_{z_{i1}}^{z_{i2}}\Omega'_n dz=\frac{1}{\omega'_n}(\sin\omega'_n z_{i2}-\sin\omega'_n z_{i1}) \tag{32}$$

$$\bar{\Lambda}_{ni}=\int_{z_{i1}}^{z_{i2}}\Omega_n dz=\frac{1}{\lambda_n}(\sin\lambda_n z_{i2}-\sin\lambda_n z_{i1}) \tag{33}$$

Following the procedure described for point collocation, (27)—(30) can be rearranged into two independent groups and finally solved. The unknown constants A_n, B_n, C_n, D_n, E_n and F_n and, consequently, the velocity potential are thus determined.

3.3 The Galerkin Method

Since they satisfy the orthogonality condition (13), the base functions for expressing the velocity potential in the subdomains can also be chosen as the weighting functions $\{w_i\}$:

$$w_i=[\Omega'_i : \Lambda_i] \quad (i=1,\ 2,\ \cdots) \tag{34}$$

or

$$w_i=\Omega_i \quad (i=1,2,\cdots) \tag{35}$$

This is the Galerkin method.

Since the two weighting function families (34) and (35) do not result in appreciable differences of the solutions, we consider only one of them, say, (34). Substituting (34) into (14) we obtain

$$q\{\Omega_0;\Omega'_i\}+\sum_{n=0}^{M}A_n\{\Omega_n;\Omega'_i\}=(C_i\cosh\omega'_i a-D_i\sinh\omega'_i a)\{\Omega'_i;\Omega'_i\} \tag{36}$$

$$q\{\Omega_0;\Lambda_i\}+\sum_{n=0}^{M}A_n\{\Omega_n;\Lambda_i\}=(E_i\cosh\lambda_i a-F_i\sinh\lambda_i a)\{\Lambda_i;\Lambda_i\} \tag{37}$$

$$\sum_{n=0}^{M}B_n\{\Omega_n;\Omega'_i\}=(C_i\cosh\omega'_i a+D_i\sinh\omega'_i a)\{\Omega'_i;\Omega'_i\} \tag{38}$$

$$\sum_{n=0}^{M}B_n\{\Omega_n;\Lambda_i\}=(E_i\cosh\lambda_i a+F_i\sinh\lambda_i a)\{\Lambda_i;\Lambda_i\} \tag{39}$$

$$-q\omega_0\{\Omega_0;\Omega'_i\}+\sum_{n=0}^{M}A_n\omega_n\{\Omega_n;\Omega'_i\}=(-C_i\omega'_i\sinh\omega'_i a+D_i\omega'_i\cosh\omega'_i a)\{\Omega'_i;\Omega'_i\} \tag{40}$$

$$-q\omega_0\{\Omega_0;\Lambda_i\}+\sum_{n=0}^{M}B_n\omega_n\{\Omega_n;\Lambda_i\}=(-E_i\lambda_i\sinh\lambda_i a+F_i\lambda_i\cosh\lambda_i a)\{\Lambda_i;\Lambda_i\} \tag{41}$$

$$-\sum_{n=0}^{M}B_n\omega_n\{\Omega_n;\Omega'_i\}=(C_i\omega'_i\sinh\omega'_i a+D_i\omega'_n\,\mathrm{conh}\,\omega'_i a)\{\Omega'_i;\Omega'_i\} \tag{42}$$

$$-\sum_{n=0}^{M}B_n\omega_n\{\Omega_n;\Lambda_i\}=(E_i\lambda_i\sinh\lambda_i a+F_i\lambda_n\,\mathrm{conh}\,\lambda_i a)\{\Lambda_i;\Lambda_i\} \tag{43}$$

$(i=1,\ 2,\ \cdots)$

where

$$\{\Omega_n;\Omega'_i\}=\int_{-h'}^{0}\Omega_n\Omega'_i\,dz=\frac{\sin(\omega_n h-\omega'_i h')}{2(\omega_n-\omega'_i)}+\frac{\sin(\omega_n h+\omega'_i h')}{2(\omega_n+\omega'_i)}-\frac{\omega_n\sin\omega_n(h-h')}{2(\omega_n^2-\omega'^2_i)} \tag{44}$$

$$\{\Omega_n;\Lambda_i\}=\int_{-h}^{-h'}\Omega_n\Lambda_i\,dz=\frac{\sin(\omega_n-\lambda_i)(h-h')}{2(\omega_n-\lambda_i)}+\frac{\sin(\omega_n+\lambda_i)(h-h')}{2(\omega_n+\lambda_i)} \tag{45}$$

$$\{\Omega'_i;\Omega'_i\}=\int_{-h'}^{0}\Omega'_i\Omega'_i\,dz=\frac{h'}{2}\left(1+\frac{\sin 2\omega'_i h'}{2\omega'_i h'}\right) \tag{46}$$

$$\{\Lambda_i;\Lambda_i\}=\int_{-h}^{-h'}\Lambda_i\Lambda_i\,dz=\begin{cases}h-h' & (i=0)\\ \dfrac{h-h'}{2} & (i\geq 1)\end{cases} \tag{47}$$

Upon eliminating C_i, D_i, E_i and F_i from (36)—(43) we have

$$\sum_{n=0}^{M} (\omega'_i \sinh \omega'_i a + \omega_n \cosh \omega'_i a) \{\Omega_n ; \Omega'_i\} (A_n + B_n) = -q (\omega'_i \sinh \omega'_i a - \omega_0 \cosh \omega'_i a) \{\Omega_0 ; \Omega'_i\} \tag{48}$$

$$\sum_{n=0}^{M} (\lambda_i \sinh \lambda_i a + \omega_n \cosh \lambda_i a) \{\Omega_n ; \Lambda_i\} (A_n + B_n) = -q (\lambda_i \sinh \lambda_i a - \omega_0 \cosh \lambda_i a) \{\Omega_0 ; \Lambda_i\} \tag{49}$$

$$\sum_{n=0}^{M} (\omega'_i \cosh \omega'_i a + \omega_n \sinh \omega'_i a) \{\Omega_n ; \Omega'_i\} (A_n - B_n) = -q (\omega'_i \cosh \omega'_i a - \omega_0 \sinh \omega'_i a) \{\Omega_0 ; \Omega'_i\} \tag{50}$$

$$\sum_{n=0}^{M} (\lambda_i \cosh \lambda_i a + \omega_n \sinh \lambda_i a) \{\Omega_n ; \Lambda_i\} (A_n - B_n) = -q (\lambda_i \operatorname{conh} \lambda_i a - \omega_0 \sinh \lambda_i a) \{\Omega_0 ; \Lambda_i\} \tag{51}$$

$(i=1,\ 2,\ \cdots)$

where (48) and (49) form a set for solving (A_n+B_n) while (50) and (51) form a set for solving (A_n-B_n). Once A_n and B_n are determined, C_n, D_n, E_n and F_n can be directly evaluated from (36), (37), (38) and (39). Closure of the set for solving (A_n+B_n) and (A_n-B_n) requires, however, a careful consideration of i. Since the smaller i, the more essential the related equation, those equations with i larger than a specified value may then be all neglected. If i in (48) and (50) is taken up to M_1 while its value in (49) and (51) is up to M_2, the condition $M=M_1+M_2+1$ must then be satisfied. Another relation between M_1 and M_2 may be determined from the condition of equivalent highest order terms for all the base functions.

4. COMPUTATION OF WAVE TRANSFORMATION AND WAVE FORCE

Once the velocity potential expressed by (6), (7), (8) and (9) are determined, the reflection coefficient K_R and transmission coefficient K_T, which are defined by the ratios of reflected and transmitted wave heights to the incident wave height, respectively, can be readily evaluated:

$$K_R = |B_0|/q \quad \text{and} \quad K_T = |C_0|/q \tag{52}$$

Since it is known that K_R and K_T are correlated through

$$1 - K_R^2 - K_T^2 = 0 \tag{53}$$

we need only to present the results for K_R.

The total wave force acting on the plate, $\mathbf{F}$, may be computed through integrating the dynamic pressure p over the surface of the plate. Since its direction is either vertically upward or downward, $\mathbf{F}$ can be represented by

$$F = \int_{-a}^{a} (p|_{z=-h'_-} - p|_{z=-h'_+}) dx \tag{54}$$

where $z=-h'_+$ and $z=-h'_-$ represent the upper and lower surfaces of the plate, respectively. A positive value of F indicates that the direction of the total force is upward and a negative value downward. The dimensionless coefficient for the description of the magnitude of F is introduced as:

$$K_F = \frac{|F|}{2a\rho g H_0} \tag{55}$$

When the linearized Bernoulli equation

$$p = i\rho\sigma\phi \tag{56}$$

is invoked, K_F can be expressed as

$$K_F = \frac{\sigma}{2agH_0} \left| \int_{-a}^{a} (\phi_4 - \phi_3)|_{z=-h'} dx \right| = \frac{2\sigma}{gH_0} \left| \sum_{n=0}^{M_1} \frac{C_n \sinh \omega'_n a}{\omega'_n a} - \sum_{n=0}^{M_2} (-1)^n \frac{E_n \sinh \lambda_n a}{\lambda_n a} \right| \tag{57}$$

Shown in Figs. 2 and 3 are the variations of the reflection coefficient K_R and the force coefficient K_F versus the dimensionless plate length $2a/L$ (L is the incident wavelength). The total water depth h=20cm, the submerged depth of the plate h'=6cm and the incident wave period T=8s are fixed. The integer M related to the truncation of the infinite series is 30 for the method of point collation and of segment collocation but is 10 for the Galerkin method.

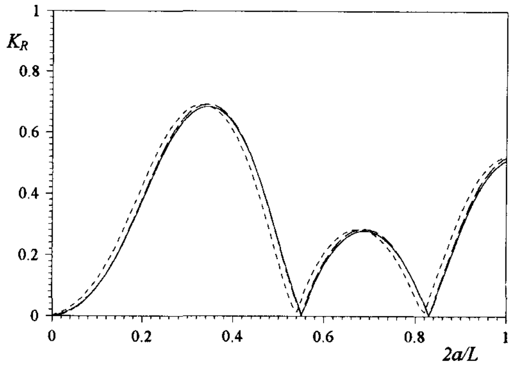

Fig. 2 Variation of reflection coefficient with the dimensionless plate length computed by point collocation (---), segment collocation (--) and the Galerkin method (—).

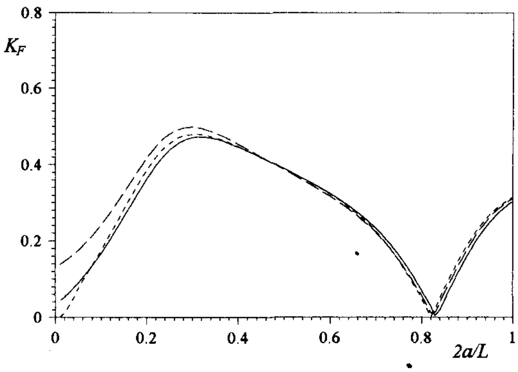

Fig. 3 Variation of force coefficient with the dimensionless plate length computed by point collocation (---), segment collocation (--) and the Galerkin method (—).

In Figs. 4, 5, and 6, we demonstrate the solutions of the reflection coefficient obtained by point collocation, segment collocation and the Galerkin method with different value of M. It is noted that the variation of the reflection coefficient obtained by point collocation and by segment collocation is still appreciable as M increases from 20 to 30 while that obtained by the Galerkin method is already very insignificant as M increases from 10 to 15. At M= 10, the solutions obtained by the collocation methods are nearly meaningless while the solution by the Galerkin method is almost unassailable from the engineering point of view. It is then implied that the number of the points and the segments in the collocation methods is much more influential to the accuracy of solution than the number of terms retained in the series, and inclusion of a large number of high order evanescent terms in the series is thus only for the closure of the linear system. This causes inefficiency in the collocation methods. For point collocation, it has been suggested that the terms with the order higher than necessary be neglected and the resulted linear equation system, in which the number of equations is larger than that of unknowns, be solved by the method of least squares[4]. This suggestion seems, however, not to be successful as expected. The reason is probably that not every equations established at the points with different z coordinates play equally important role in determining the velocity

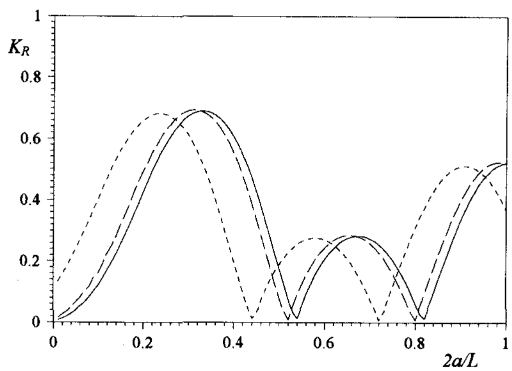

Fig. 4 Reflection coefficient obtained b y point collocation with M=10 (---), M=20 (--) and M=30 (—).

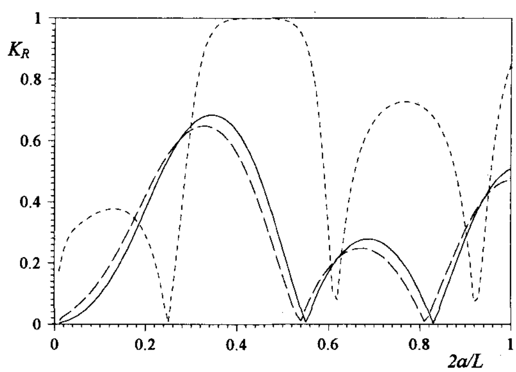

Fig. 5 Reflection coefficient obtained by segment collocation with M=10 (---), M=20 (--) and M=30 (—).

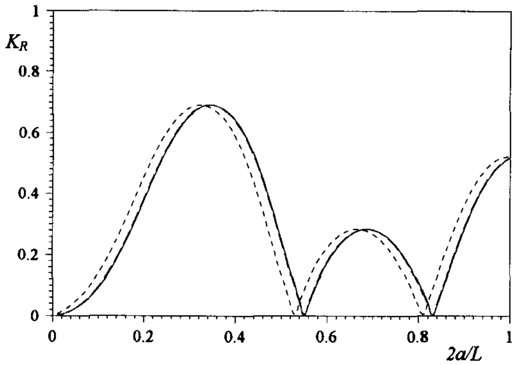

Fig. 6 Reflection coefficient obtained by the Galerkin method with M=5 (---), M=10 (--) and M=15 (—).

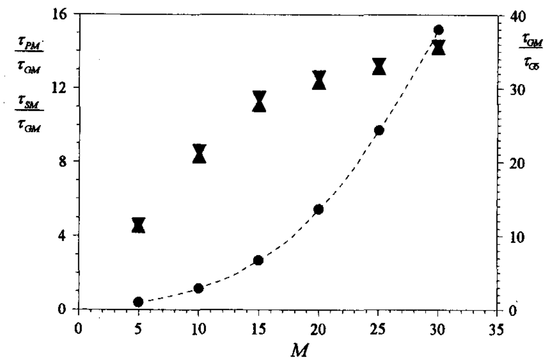

Fig. 7 Comparison of the computer time required by different methods (▲: τ_{PM}/τ_{GM}; ▼: τ_{SM}/τ_{GM}; •: τ_{GM}/τ_{G5})

potential since the wave motion exponentially decays as depth increases.

In Fig. 7 we show a comparison of the computer time required by different methods. It is noted that the computer time needed by point collocation (τ_{PM}) and that needed by segment collocation (τ_{SM}) are more than 8 times of that needed by the Galerkin method (τ_{GM}) at M=10. As M increases this ratio further increases. If the increase of the computer time required by the Galerkin method (τ_{GM}) with M (Fig. 7) is also taken into consideration, the superiority of the Galerkin method becomes evident.

5. CONCLUSIONS

We have studied three patching methods, *i.e.*, point collocation, segment collocation and the Galerkin method, for the analysis of wave motion over a submerged plate. The problem is formulated by the conventional linear potential wave theory and has been converted to minimize the error functions along the patching boundaries. The weighted residual approximation has been applied for the minimization. Comparative computations of the reflection caused by and the fluid force acting on the plate show that the Galerkin method is most effective. The collocation methods are less efficient because they include a large number of high order terms which are only for the purpose of making the numerical problem solvable and have no appreciable contribution to promote the accuracy of results.

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