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<td>経営と経済</td>
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このページは、NAOSITE: Nagasaki University's Academic Output SITEにアップロードされています。
A pile of $M$ coins is arranged so that each coin is heads up. We take the top coin, flip it over and put it back on top of the pile. We then take the top two coins, and holding them together turn them upside down and put them back on top of the pile. We continue, taking in turn the top three, four and so on until the whole pile is turned upside down. We then commence at the top again flipping the top stack of one, two, three coins and continue in this fashion until the pile is again all heads up. How many flips are required? (Newman: The Flippin’ Coins Problem, Mathematics Magazine, Vol. 54, No. 2, 51-59, 1981)

For $M=3$, the pile of coins will undergo these changes (brackets indicate the stack of coins flipped at each stage):

\[
\begin{array}{cccccccc}
H & T & T & T & H & H & T & H \\
H & H & H & T & T & T & H & H \\
H & H & H & H & H & T & T & H \\
\end{array}
\]

Thus the number of flips required for a pile of three coins is 9.

Newman proved that the number of flips for a pile of $M$ coins is of the form $Mk$ or $Mk-1$ ($k$ is a natural number) and that this number is bounded by $M^2$ ($M>1$). This means that a pile of heads can occur only after a move that flips over the whole pile of $M$ coins, or just prior to such a move. He proved these results by representing as binary decimals the state of the pile just after the pile flip,
A coin has the two-fold state of the flip (head and tail). We now consider a thing that has the three-fold state of the flip like a signal (red, blue, yellow). We denote the three-fold state by 0, 1 and 2: if it is flipped over, the state $a$ changes to the state $a + 1 \mod 3$.

A pile of $M$ signals is arranged so that the state of each signal is 0. Then we consider the number of flips required for that the state of the signals is again all 0.

For example, the number of flips for $M = 3$ is 9 as follows.

\[
\begin{array}{cccccccc}
0 & 1 & 1 & 2 & 1 & 0 & 1 & 2 \\
0 & 0 & 2 & 0 & 0 & 1 & 1 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 0 \\
\end{array}
\]

For $M = 4$, it is shown in Table 1. We show the number of flips for $M (1 \leq M \leq 36)$ in Table 2.

<table>
<thead>
<tr>
<th>NUMBER OF FLIPS</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
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<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
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<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>13</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
\begin{table}
\centering
\begin{tabular}{cccccc}
17 & 0 & 0 & 2 & 0 \\
18 & 1 & 1 & 2 & 0 \\
19 & 0 & 2 & 2 & 0 \\
20 & 1 & 0 & 0 & 1 \\
21 & 2 & 0 & 0 & 1 \\
22 & 1 & 0 & 0 & 1 \\
23 & 1 & 1 & 2 & 1 \\
24 & 2 & 0 & 2 & 2 \\
25 & 0 & 0 & 2 & 2 \\
26 & 1 & 1 & 2 & 2 \\
27 & 0 & 2 & 2 & 2 \\
28 & 0 & 0 & 0 & 1 \\
29 & 1 & 0 & 0 & 1 \\
30 & 1 & 2 & 0 & 1 \\
31 & 1 & 0 & 2 & 1 \\
32 & 2 & 0 & 1 & 2 \\
33 & 0 & 0 & 1 & 2 \\
34 & 1 & 1 & 1 & 2 \\
35 & 2 & 2 & 2 & 2 \\
36 & 0 & 0 & 0 & 0 \\
\end{tabular}
\caption{Table 1}
\end{table}

\begin{table}
\centering
\begin{tabular}{cccc}
\text{M (NUMBER IN PILE)} & \text{NUMBER OF FLIPS} \\
1 & 1 \\
2 & 4 \\
3 & 9 \\
4 & 36 \\
5 & 25 \\
6 & 36 \\
7 & 84 \\
8 & 32 \\
9 & 81 \\
10 & 60 \\
11 & 121 \\
12 & 120 \\
13 & 351 \\
\end{tabular}
\caption{Table 1 (continued)}
\end{table}
Table 2

From an examination of Table 2, the following conjectures are made.

Conjecture 1. The number of flips for a pile of M signals is of the form \( Mk \) (k is a natural number).

Conjecture 2. This number is bounded by \( 3M^2 \).

In this paper, we shall prove Conjecture 2, while Conjecture 1 is still an open question (Conjecture 1 is true for \( M \leq 200 \)).
AN EXTENSION OF THE FLIPPING COINS PROBLEM

We consider the permutation of the positions of the signals after the first pile flip. For $M = 3$, signals 1, 2 and 3 are in position 2, 3 and 1, respectively as shown in Table 3.

\[
\begin{array}{ccc}
1 & 1 & 2 \\
2 & 2 & 1 \\
3 & 3 & 3 \\
\end{array}
\]

Table 3

We denote by $\psi_M$ the permutation of the positions of the signals 1, 2, $\ldots$, $M$ after the pile flip, and by $\varphi_M$ the inverse of $\psi_M$. For $M = 3$, $\psi_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\varphi_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$. Newman showed that the order of the permutation $\varphi_M$ is the order of the cycle containing $M$. Since the order of the cycle is bounded by $M$, the order of $\varphi_M$ is so. We denote it by $t$, then the position of $M$ signals is the same after $t$-th pile flip. Although the state of $M$ signals after $t$-th pile flip may not be all 0, the state of $M$ signals changes to all 0 after $3t$-th pile flip. $3t$-th pile flip means $3tM$-th flip. Since $t \leq M$, Conjecture 2 is proved.

FACOM M-180 II AD in Information Processing Center, Nagasaki University was used in order to obtain Table 1 and 2.