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Endogenous Technical Progress in a Vintage Type Model of CO$_2$ Emissions

Hans W. Gottinger

Abstract
A vintage model of technical progress is introduced in which fossil energy input required by a piece of capital is determined by the year in which it is purchased and cannot be changed after the capital is in place. A major outcome of such a model is that the long run pattern of technical progress is neutral with respect to the inputs, if energy use is proportional to output, prices are stable, and the research budget is constant or exponentially increasing. However, if CO$_2$ causes increases in the price of fossil fuels, through consideration of the shadow price or taxes, research will be induced to concentrate on energy because such research allows substitution away from energy.

1 INTRODUCTION
With a view toward generalizing the class of models on optimal energy use, as derived in Gottinger (1991), we develop a unique model in which prices influence the pattern of technical progress and furthermore non-neutral technical progress is possible. The present model has some more realistic features.

First, it is a vintage-type model; that is the fossil energy input re-
quired by a piece of capital (equipment) is determined by the year in which it is purchased and cannot be changed after the capital is in place (Solow, 1964 and 1988).

Second, research can be performed to improve the productivity of new capital or to reduce its energy requirements. The output required, the research budget, and cost of energy and capital are all exogenous. The objective is to minimize the cost of production.

The first push toward generalization involves the departure from neutrality of technical progress. The assumption of neutral technical progress is quite common. Neutral technical progress may be seen as a natural process, as a continuation of a past trend, or may be assumed for its simplicity.

If we look for economic factors as sources of explanation for the pattern of technical progress, we should take into account the different age structure of the capital stock. A machine in place usually has a limited capacity for changes in its mix of inputs.

In order to substantially change the ratio between the inputs to the production process, the capital stock must change. A vintage model of capital, in which the date of purchase determines the operating characteristics of capital, accounts for the inflexibility of capital.

In what follows, we develop two models which differ mainly in their fossil fuel requirements. In each model, a special or preferred ratio of technical coefficients exists and particular price patterns cause research to move the technology toward this ratio. We examine how this ratio changes under different price patterns. Finally, we examine a variation from these basic models which allows us to discuss CO₂ limits.

The remainder of the this paper is organized as follows:
We first develop several models which reasonably represent the influence of economic factors on technical change. We show the sensitivity of the optimal research pattern to assumptions about the relation of energy use to output and the research budget.

Such models could well form an essential part of policy-oriented energy models in which capital stock vintages generate major changes in technology and productivity (Scherago et al, 1990, Jorgenson, 1989; Ingham et al., 1987).

2 A VINTAGE MODEL

A basic feature of this model is that the portion of research devoted to each input, energy and capital, is an endogenous variable. As technical progress occurs, the characteristics of machines change. Each machine is distinguished by its vintage. T(t) is the age of the oldest machine or equipment in use at time t. M(t) is the life of a machine bought at time t. By definition,

\[ M(t) = T(t + M(t)) \] (1)

Let the scale parameters for the oldest equipment and the equipment in use be defined by:

\[ \frac{dT}{dt} = \gamma, \quad \frac{dM}{dt} = \lambda \] (2)

Y(k) is the production from all machines bought at time k. I(k) is the number of machines purchased at time k. F(k) is the energy used by machines of vintage k. The critical energy assumption made is that energy use is proportional to the output level.

\[ Y(k) = \min (A_1(k)I(k), A_2(k)F(k)) \] (3)
$A_1(k)$ and $A_2(k)$ are the Leontief type technical coefficients.

If the technology is used efficiently the conditions below hold:

$$Y(k) = \begin{cases} A_1(k)I(k), & k \geq t-T(t) \\ 0, & k < t-T(t) \end{cases} \quad (4)$$

$$F(k) = \begin{cases} \frac{Y(k)}{A_2(k)}, & k \geq t-T(t) \\ 0, & k < t-T(t) \end{cases} \quad (5)$$

Because, in this context, we emphasize the nature of technological progress, the production level $Y^*(t)$ and the total research budget $R^*(t)$ are assumed exogenous. The objective is to minimize all future discounted costs. The problem is described in the following equations:

$$J = \min Y, \tau, \theta \int_{t^*}^{\infty} \exp(-r(t-t^*)) \left\{ p_1(t) \frac{Y(t)}{S_1} + p_2(t) \int_{t-T(t)}^{\infty} F(k) \, dk \right\} \, dt \quad (6)$$

Future costs have two components. Machines sell at a price of $p_1$ and the first term in the bracket of (6) is the expenditure on machines at time $t$.

Energy costs $p_2$ per unit, and the second term is the cost of energy for all machines operating at time $t$.

We require that:

$$\int_{t-T(t)}^{\infty} Y(k) \, dk - Y^*(t) \geq 0 \quad (7)$$

The relation (7) states that the production from all machines must meet the output goal. Furthermore, research can increase the productivity of
machines and reduce their energy requirements.

\[
dA_1/dt = a_1 \theta R^*(t), \quad dA_2/dt = a_2(1 - \theta) R^*(t)
\]

(8)

where \( \theta \) designates the portion of research funds devoted to each input factor.

\[
\tau \leq 1 \quad \text{and} \quad \lambda + 1 \geq 0
\]

(9) implies that older machines are taken out before newer machines.

The first step in the analysis incorporates (7) into the objective function. This requires the use of the Lagrange multiplier \( L \). The problem is also changed from minimization to maximization and restated below:

\[
\text{Max}_{Y, \tau, \theta} \int_{t^*}^{\infty} \left[ e^{-r(t-t^*)} \left( -p_1 \frac{Y}{A_1} - p_2 \int_{t-T(t)}^{t} \frac{Y}{A_2} dk + L \int_{t-T(t)}^{t} Ydk - LY^* \right) dt - J \right]
\]

(10)

As stated in (10) the outer integral sums over time while the inner integrals sum over vintages operating in a given year.

An equivalent statement is possible with the outer integral being over vintages and the inner over years of operation. The equivalent form is given by

\[
-J = \text{Max}_{Y, \lambda, \theta} \int_{t^* - T(t^*)}^{t + M(t)} e^{-r(t-t^*)} \left[ -\Omega(t) \frac{Y}{A_1} - p_1 - \frac{Y}{A_2} \int_{t}^{t+M(t)} \Omega(x)e^{-r(x-t)} dx + \int_{t}^{t+M(t)} p_2(x) dx + Y \int_{t}^{t+M(t)} \Omega(x)e^{-r(x-t)} L(x) dx - \Omega(t)L(t)Y^* \right] dx
\]

(11)
The function $\Omega(t)$ gets rid of costs previous to $t^*$. It is defined by

$$\Omega(t) = \begin{cases} 1, & t \geq t^* \\ 0, & t < t^* \end{cases}$$  \hspace{1cm} (12)

To form the current value Hamiltonian (V. L. Smith, 1977) we use the adjoint variables $\eta$, $\mu_1$, and $\mu_2$:

$$H = F(Y, A_1, A_2, M) + \eta \lambda + \mu_1 a_1 \theta R^* + \mu_2 a_2 (1 - \theta) R^*$$  \hspace{1cm} (13)

3 THE ANALYSIS OF NECESSARY CONDITIONS

We derive the necessary conditions of the above vintage model and provide for each a suitable economic interpretation in the context of the proposed model.

For $t \geq t^*$ we have

$$\int_t^{t+M(t)} e^{-r(x-t)} L(x) \, dx - \frac{b_1}{A_1} - \frac{1}{A_2} \int_t^{t+M(t)} e^{-r(x-t)} p_2(x) \, dx \leq 0$$  \hspace{1cm} (1)

Equality holds in (1) if $Y(t)$ is greater than zero. $L$ can be interpreted as the current marginal value of a new unit of production at time $t$. When $Y(t)$ is greater than zero, the discounted value of a new unit of production over a machine’s lifetime, must equal the capital cost per unit of production plus the discounted lifetime energy costs per unit of production.

$$\eta \leq 0$$  \hspace{1cm} (2)

$\eta$ is the value of retiring newer units before old. A closer examination of the problem shows that $\eta$ always equals zero. The nature of technical progress, as described in Sec i (8), assures that $A_1$ and $A_2$ never decrease. New
technologies are better in every way than old; therefore, a newer technology never will be retired before an older. (If new machines were improved in one characteristic but worse in another, this would not necessarily be the case).

\[(\mu_1, a_1 - a_2) R^* < 0, \quad \theta = 0\]
\[(\mu_1, a_1 - a_2) R^* = 0, \quad 0 \leq \theta \leq 1\]  \hspace{1cm} (3)
\[(\mu_1, a_1 - a_2) R^* > 0, \quad \theta = 1\]

\(\mu_1\) and \(\mu_2\) can be interpreted as the values of improvements in capital and energy use, respectively. (3) assures that research is devoted to the input which provides the greatest value, and that research is only done on both inputs when equally valuable.

\[
\frac{d\eta}{dt} - r\eta + e^{-rM(t)} Y(t) \left[ L(t) + M(t) \frac{p_z(t + M(t))}{A_2(t)} \right] \leq 0
\]  \hspace{1cm} (4)

Equality holds in (4) if \(M\) is greater than zero. \(M\) equal to zero practically does not occur, because purchasing and never using a machine cannot be optimal.

\[
\frac{d\mu}{dt} r\mu_1 + \frac{p_1}{(A_1)^2} Y = 0
\]  \hspace{1cm} (5)

\[
\frac{d\mu_2}{dt} r\mu_2 + \frac{Y}{(A_2)^2} \int_{t}^{t + M(t)} e^{-r(x-t)} p_2(x) dx = 0
\]  \hspace{1cm} (6)

(5) and (6) state that the values of research on capital and energy use decrease at the social discount rate and increase with the marginal value of an improvement in the technical coefficient. Since \(\eta\) equals zero, \(\frac{d\eta}{dt}\) equals zero and (4) can be simplified and rewritten as
\[ L(t+M(t)) = \frac{p_2(t+M(t))}{A_2(t)} \quad \text{or} \quad L(t) = \frac{p_2(t)}{A_2(t-T(t))} \quad (7) \]

\( L(t) \) is the current marginal value of a new unit of production. (7) states that a machine is retired when its operating cost per unit of production equals the current marginal value of a new unit of production. \( L(t) \) is an important quantity; but to investigate it further, additional assumptions are required. If \( Y(t) \) is greater than zero, (1) holds with equality and can be restated as:

\[ \int_{t}^{t+M(t)} e^{-\alpha} L(X) dx - \frac{1}{A_1} e^{-\alpha} \int_{t}^{t+M(t)} e^{-r(x+t)} p_2(x) dx = 0 \quad (8) \]

By taking the time derivative of (8) and using (7) to simplify the result, we derive

\[ L(t) = \frac{1}{A_1} \left[ P_1 (r+A_1^*) \frac{dP_1}{dt} \right] + \frac{1}{A_2} \left[ P_2 + A_2^* \int_{t}^{t+M(t)} e^{-r(x-t)} p_2(x) dx \right] \quad (9) \]

(9) simply states that the marginal value of a new unit of production must equal its marginal cost.

The first parenthesis on the right-hand side is divided by \( A_1 \) to put capital related costs on a per unit of output basis. The first term within this parenthesis shows the capital cost being amortized at the discount rate plus the rate of capital improvement. The next term is the capital savings lost by purchasing now rather than later. The second pair of large parentheses contain energy related costs and are placed on a per unit basis by dividing by \( A_2 \). Inside are the current energy cost plus the savings in lifetime energy costs which are lost by purchasing now rather than later. An interesting point is
that \( L(t) \) does not depend on \( Y^*(t) \), except that \( Y^*(t) \) must be great enough since \( Y(t) \) is greater than zero.

Necessary conditions (3) - (6) determine the research balance. Here we examine the case in which both technical coefficients improve. When this occurs the value of research in capital and energy must be equal over a period of time. If \( \mu_1 \) equals \( \mu_2 \) over time, then the change of each with time must also be equal:

\[
\frac{d\mu_1}{dt} = \frac{d\mu_2}{dt}
\]

Using (5) and (6) this condition is met only if \( a_1 \) times the marginal change in costs when \( A_1 \) is improved equals \( a_2 \) times the marginal change in costs when \( A_2 \) is improved.

\[
a_1 \frac{p_1}{(A_1)^2} Y = a_2 \frac{Y}{(A_2)^2} \int_{t}^{t+M(t)} e^{-r(x-t)} p_2(x) dx
\]

(11) is perhaps the most important derivation in this part, and has significant implications. It simply states that the value of research in each input must be equal if we commit research funds to each, \( a_1 \) and \( a_2 \) measure the effectiveness of research in improving the efficiency of capital and energy, respectively.

The marginal change in costs when the efficiency of an input is improved measures the value of changing \( A_1 \) and \( A_2 \).

(11) can be rearranged to give an expression in terms of the ratio of the technical coefficients. The ratio of the technical coefficients is determined by the ratio of the lifecycle costs times the ratio of the research efficiencies.
\[
\left[ \frac{A_2}{A_1} \right]^2 = \frac{a_2}{a_1} \int_{t}^{t+M(t)} e^{-r(x-t)} \frac{\dot{p}_2(x)}{\dot{p}_1} \, dx
\]  (12)

4 NEUTRAL TECHNICAL PROGRESS

Improving the performance of the economy while not changing the ratio of the technical coefficients is the Leontief technology equivalent of neutral technical progress. The following proposition underlines the existence of such a ratio:

**Proposition 1**

Given the vintage model 2(1) to 2(9), if \( p_1 \) and \( p_2 \) are constant and \( R^*(t) = R^* \exp(\gamma t) \) there is a unique ratio of technical coefficients \( A_1^* \) to \( A_2^* \) such that \( \theta \) and \( M \) are constant along an optimal path. The ratio is expressed in

\[
\left[ \frac{A_2^*}{A_1^*} \right]^2 = \frac{a_2}{a_1} \frac{p_2}{\dot{p}_1} \frac{(1 - e^{-rM})}{r} \]  (1)

**Proof**

If \( A_1 \) and \( A_2 \) are invested in at a constant ratio, each will change at the rate \( \gamma \). By substituting from 3(7) into 3(9):

\[
e^{\gamma t} = \frac{A_1}{A_2} \frac{\dot{p}_1}{\dot{p}_2} (r + \gamma) + 1 + \gamma \frac{(1 - e^{-rM})}{r} \]  (2)

This equation can be solved for the ratio of \( A_2 \) to \( A_1 \) and substituted into 3 (12) to yield.
Assuming \( M \) is constant, \( M = T \). At \( M = 0 \), the left-hand side of (3) is zero and the first derivative of the left-hand side with respect to \( M \) is negative. At \( M = \infty \), the left-hand side of (3) is infinite. The second derivative of the left-hand side with respect to \( M \) is positive for all values of \( M \) greater than zero. From this we can conclude that there is a single, strictly positive value of \( M \) which satisfies (3).

Once the lifetime which satisfied (3), \( M^* \), is found, (3) can be solved for the ratio of \( A_1 \) to \( A_2 \).

To prove that the necessary conditions are also sufficient, we would need to first show that \( F(Y, A_1, A_2, M) \) in (11) is concave in both control and state variables. We have examined the matrix which determines concavity of \( F(Y, A_1, A_2, M) \), and found that concavity depends on parameter values.

5  INDUCED PRICE CHANGES

The existence of a \( \text{CO}_2 \) problem adds on additional cost to the use of fossil fuels. In previous models Gottinger(1991), this is captured through a shadow price, \( q \). In this analysis we examine the response of the above equilibrium to an increase in the price of fossil fuel, which might be caused by the \( \text{CO}_2 \) induced shadow price. More precisely, we examine an increase in the ratio between the fossil fuel and capital prices. We find that an increase in the relative price of an input results in increased research in that input and a corresponding fall in the purchases of that input relative to the other input. The calculation is complicated because the optimal machine life
changes when prices change. We first derive a relationship between the change in the equilibrium ratio of technical coefficients, $A_2^*/A_1^*$, to the change in the constant machine lifetime, $M^*$. Rearranging terms in 4 (1), we derive

$$\frac{p_1}{A_1^*} = \frac{a_2}{a_1} \frac{A_1^*}{A_2^*} \frac{p_2}{A_2^*} = \frac{1 - e^{-rM^*}}{r}$$

If we substitute from (1) and 3 (7) into 3 (9) and rearrange terms:

$$e^{rM^*} = \frac{a_2}{a_1} \frac{A_1^*}{A_2^*} \frac{(1 - e^{-rM^*})}{r}(r + \gamma) + \left[ 1 + \gamma \frac{(1 - e^{-rM^*})}{r} \right]$$

We further rearrange (2) so that $A_1^*/A_2^*$ and $M^*$ each appear in separate terms.

$$\frac{a_2}{a_1} \frac{A_1^*}{A_2^*} (r + \gamma) + \gamma - r \frac{(e^{-rM^*} - 1)}{(1 - e^{-rM^*})} = 0$$

By taking the total derivative of (3), we can find the direction of change in the optimal lifetime when the optimal ratio of technical coefficients changes. The total derivative is:

$$rG[1 - e^{-rM^*}]^2 dM^* - \frac{a_2}{a_1} (r + \gamma) d\left[ \frac{A_1^*}{A_2^*} \right] = 0$$

$G$ is defined by

$$G = \gamma e^{rM^*} + re^{-rM^*} - (r + \gamma)e^{(r-\gamma)rM^*}$$

At $M^*$ equal to zero, $G$ is zero and the derivative of $G$ with respect to $M^*$ is
positive; therefore, $G$ is positive. From (4) we can then conclude that the optimal ratio of technical coefficients and the lifetime change in the same direction.

We complete the analysis of the impact of price changes by taking the total derivative

$$Y = F^v S^u K^{1-r}, \quad M \leq M_c$$

$$0, \quad M > M_c$$

The total derivative is

$$d\left[\frac{\bar{b}_1}{\bar{p}_2}\right] = \left[\frac{2a_2}{a_1} \frac{A_1^*}{A_2^*} \left(1 - e^{-rM^*}\right) \right] x \left[\frac{A_1^*}{A_2^*}\right] - \left[\frac{2a_2}{a_1} \frac{A_1^*}{A_2^*} e^{-rM^*}\right] dM^* = 0 \quad (6)$$

The multipliers of $d\left(\frac{A_1^*}{A_2^*}\right)$ and $dM^*$, the changes in the ratio of technical coefficients and the lifetime respectively, have the same sign. From (6) we then can conclude the following rule: when the ratio of the equipment price, $P_1$, to the fossil fuel price, $P_2$, increases, the level of research in capital relative to fossil fuel increases. Conversely, when the fossil fuel price goes up, relatively more research is being done on the efficiency of fossil fuel use. The research (and development) allows the economy to substitute equipment for energy in production.

6 CONSIDERATIONS OF MODEL WORKABILITY

Up to this point we have been concerned with model responses to specific patterns of shadow price increases. But it is difficult to examine even simple changes with this model. We look for an equilibrium when carbon dioxide causes the fossil fuel price to rise along an approximately exponential path. We find that the research response in this model to such a price rise is com
plex and cannot be described in simple terms.

It is natural to examine three possible equilibrium investment patterns: (i) balanced investment, (ii) all investment in $A_1$, and (iii) all investment in $A_2$.

If we examine the equation governing the balanced growth path, 3 (12), we see that a balanced growth path with constant lifetimes and exponentially increasing fossil fuel costs is not possible. According to 3 (9) and 3 (7) lifetimes cannot vary in any simple way so that a balanced growth path is possible. Putting all our research in $A_1$ also cannot satisfy the necessary conditions. With all research in $A_1$, 3 (9) and 3 (7) give two different expressions for $L$. Neither can all research in $A_2$ produce a simple solution. From 3 (9), when all investment is in $A_2$, the lifetime can only be constant if the rate of increase in research and the price is the same. If these rates are the same and the lifetime is constant, $\mu_1$ is constant and $\mu_2$ decreases at an exponential rate. The value of research in capital must eventually become higher than the value of research in fossil fuel. If all research is in $A_2$ and the lifetime is not constant, we cannot show that the necessary conditions are not satisfied, but any solution would be quite complex.

In order to accommodate some of these limitations, we will change the model in two important ways.

First, we assume that energy use is proportional to the number of machines in use and not the level of production. In the previous model, if no research was done on energy use but if machines were made twice as productive, each machine would consume twice as much energy. We now move to a situation, where machines may be made more productive without increasing the energy consumption per machine. In this modified model, the energy use of each vintage is:
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\[ F(k) - I(k) / A_2(k) \]  \hspace{1cm} (1)

Second, we assume that over time the rate of growth in research funds, R, and the price of fossil fuels, P₂, is the same. This seems to be reasonable if the research budget and the harm due to CO₂ are both proportional to an exponentially growing economy.

We conclude that given the above assumptions, technical progress is neutral when the cost of energy rises exponentially. In this regard, the model appears less realistic than the previous model. However, this model's behaviour may give an indication of the performance of similar, more realistic, but more complex models.

7 SPECIFIC MODEL CHARACTERISTICS

The necessary conditions are quite similar to those of the previous model. The major difference is that the output per unit of energy input is the product of \( A_1 \) and \( A_2 \). The Lagrangean equations are replaced by

\[ L(t+M(t)) = \frac{P_2(t+M(t))}{A_1(t)A_2(t)} \]  \hspace{1cm} (1)

\[ L(t) = \frac{1}{A_1(t)} \left[ \dot{p}_1 (r + A_1^*) - \frac{dp_1}{dt} \right] + \frac{1}{A_1(t)A_2(t)} \]

\[ \left[ P_2 + (A_1^* + A_2^*) \int_t^{t+M} e^{-r(x-t)} P_2(x) dx \right] \]

(1) states that a machine is retired when the value of production from a new machine equals the operating cost per unit of production of a machine to be retired. (1) can be interpreted just as 3 (7). \( L \) is again the present marginal value of a new unit of production.
The first pair of parentheses on the right-hand side of (2) contains the capital costs per unit of output, and the second pair contains the energy costs per unit of output.

**Proposition 2**

Given the energy requirement in 6 (1), if $p_1$ is constant and $p_2$ and $R^*(t)$ grow at the rate $r$, there is a unique ratio of technical coefficients, $A^*_1 / A^*_2$, such that $\theta$ and $M$ are constant along an optimal path.

The ratio is expressed as:

$$\frac{A^*_2}{A^*_1} = \left[ \frac{a_2}{a_1} \frac{A^*_2}{A^*_1} \right] \frac{p_2(0) (e^{(r-r)M^*} - 1)}{A_2(0) p_1}$$

**Proof**

The proof is very similar to Proposition 1. (3) is derived by equating the rates of change of the value of research in each technical coefficient. The equation which allows us to solve for the constant lifetime, $M^*$, is:

$$e^{rM^*} = \frac{A_2(0)}{p_2(0)} p_1 (r+\gamma) + \frac{2\gamma}{r} (1 - e^{-rM^*}) + 1$$

These two models illustrate the difficulties under which the relationship of energy to production is specified in order to determine the proper research response to increasing CO$_2$.

**8. CONCLUSIONS**

By referring to the step model of CO$_2$ impacts, as in Gottinger(1991), we
observe that in such a model a critical CO₂ level imposes a long-run limit on fossil fuel use, and this limit is reflected in the shadow price of fossil fuel.

We assume that fossil fuels have no cost except for the shadow price.

In our reference model the level of output was constant and exogeneously specified. The level of fossil fuel use continually falls along an exponential path as efficiency in fuel use is gained. Therefore, in such a model, long-run limitations on fossil fuel use are irrelevant.

In the two models presented here such limitations are relevant. Let us analyse this property more closely.

In the first model, demand Y*, and the research budget, R*, grow at the same rate.

The solution is very similar to the initial model presented. We gain one unknown in the shadow price, but the added condition that emissions be less than or equal to the absorption rate times the critical CO₂ level allows us to solve for this unknown. A balanced research policy with a fixed ratio of knowledge about capital and fossil fuel use satisfies the necessary conditions. The shadow price in this model is constant.

In the second model rather than meeting a set production goal, we use the utility maximization framework. The constant relative risk aversion utility function is assumed. Output can only be raised by adding additional machines, there is no research on improved machine productivity. The level of research on the efficiency of fossil fuel use is an endogenous variable. Maximization is over the investment in new machines, the level of research on fossil fuel use, and the machine lifetimes. The economy, consumption, production, investment, research, all grow at a single internally determined rate. The shadow price rises at the rate of growth of utility which is the growth rate times one minus the consumption elasticity of utility.
The main conclusion of this part of the paper is that economic factors may have dramatic effects on the pattern of technical development. We have found that the neutral pattern of technical development, often assumed in energy environmental economic studies can be the outcome of several different sets of economic assumptions. A second major conclusion is that the type of impact is highly dependent on which model structure is considered most appropriate.

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