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Author(s)	Adachi, Kenzo
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# Note on the Global Regularity for the $\bar{\partial}$ -Equation

Kenzō ADACHI

Department of Mathematics, Faculty of Education,  
Nagasaki University, Nagasaki 852, Japan

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## Abstract

Let  $\Omega$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary. The purpose of this paper is to give another proof of the following well known Kohn's global regularity theorem:

**THEOREM.** *Suppose that  $\alpha \in C_{(0,q)}^\infty(\bar{\Omega})$  is a smooth  $(0, q)$ -form with  $\bar{\partial}\alpha=0$ . Then there is a smooth solution  $v \in C_{(0,q-1)}^\infty(\bar{\Omega})$  to the equation  $\bar{\partial}v=\alpha$ .*

The proof proceeds for the most part along the proof of Kohn[5]. Instead of the estimate obtained by Kohn[5], we use the estimate obtained by Catlin[1].

1. Kohn solution in pseudoconvex domains. We denote by  $L_{(0,q)}^2(\Omega)$  the space of all  $(0, q)$ -forms  $f$  such that  $\int_\Omega |f|^2 dV < \infty$ . For  $\lambda \in C^\infty(\bar{\Omega})$ ,  $t$  real, and  $\phi, \psi \in L_{(0,q)}^2(\Omega)$ , we define

$$\langle \phi, \psi \rangle_{(t)} = \langle \phi, e^{-t\lambda} \psi \rangle$$

$$\|\phi\|_{(t)}^2 = \langle \phi, \phi \rangle_{(t)} = \int_\Omega |\phi|^2 e^{-t\lambda} dV,$$

where  $\langle, \rangle$  is the usual inner product in  $L_{(0,q)}^2(\Omega)$ .

For  $\phi \in C_{(0,q)}^\infty(\bar{\Omega})$ , we define

$$\partial\phi = \sum_K \sum_j \frac{\partial\phi_{jK}}{\partial z_j} dz^K, \quad \partial_t\phi = e^{t\lambda} \partial(e^{-t\lambda}\phi).$$

Let  $\rho$  be a defining function for the domain  $\Omega$ . Define the space  $D_q \subset C_{(0,q)}^\infty(\bar{\Omega})$  by

$$D_q = \left\{ \phi \in C_{(0,q)}^\infty(\bar{\Omega}) : \sum_{j=1}^n \phi_{jK} \frac{\partial\rho}{\partial z_j} = 0 \text{ on } \partial\Omega \text{ for all } K \right\}$$

Then we have, for  $\psi \in D_q$ ,  $\phi \in C_{(0,q)}^\infty(\bar{\Omega})$ ,

$$\langle \partial\psi, \phi \rangle = \langle \psi, \bar{\partial}\phi \rangle, \quad \langle \partial_t\psi, \phi \rangle_{(t)} = \langle \psi, \bar{\partial}\phi \rangle_{(t)}.$$

The Hilbert space adjoints of  $\bar{\partial}$  with respect to  $\langle, \rangle$  and  $\langle, \rangle_{(t)}$  are denoted by  $\bar{\partial}^*$  and  $\bar{\partial}_t^*$ , respectively. We define  $Q_t: D_q \times D_q \rightarrow C$  by

$$Q_t(\phi, \psi) = \langle \bar{\partial}\phi, \bar{\partial}\psi \rangle_{(t)} + \langle \partial_t\phi, \partial_t\psi \rangle_{(t)}$$

We denote by  $\tilde{D}_q$  the Hilbert space obtained by completing  $D_q$  under the norm  $\{Q_t(\phi, \phi) + \|\phi\|_{(t)}^2\}^{\frac{1}{2}}$ . Then we have for  $\phi, \psi \in \tilde{D}_q$ ,

$$\langle \phi, \psi \rangle_{(t)} = \langle \phi, e^{-t\lambda} \psi \rangle, \quad \bar{\partial}_t^* \phi = e^{t\lambda} \bar{\partial}^*(e^{-t\lambda} \phi).$$

We define

$$\mathbf{H}_t = \{\phi \in \tilde{\mathbf{D}} : \bar{\partial} \phi = \bar{\partial}_t^* \phi = 0\}.$$

Let  $H_t : L^2 \rightarrow \mathbf{H}_t$  be the projection. By Hörmander[3], we have

**LEMMA 1.** *Let  $\Omega$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary and let  $\lambda$  be a  $C^\infty$  strictly plurisubharmonic function in  $\bar{\Omega}$ . Then for  $t > 0$  we have*

$$(1.1) \quad t \|f\|_t^2 \leq c(\|\bar{\partial}_t^* f\|_t^2 + \|\bar{\partial} f\|_t^2),$$

where  $f \in \tilde{\mathbf{D}}_q$ .

Proof. By Hörmander[3], we have for  $f \in \mathbf{D}_q$

$$\begin{aligned} \|\bar{\partial}_t^* f\|_t^2 + \|\bar{\partial} f\|_t^2 &= t \sum_K \sum_{j,k} \int_{\Omega} f_{jK} \bar{f}_{kK} \frac{\partial^2 \lambda}{\partial z_j \partial \bar{z}_k} e^{-t\lambda} dV \\ &\quad + \sum_J \sum_j \int_{\Omega} \left| \frac{\partial f_j}{\partial \bar{z}_j} \right|^2 e^{-t\lambda} dV + \sum_K \sum_{j,k} \int_{\partial \Omega} f_{jK} \bar{f}_{kK} \frac{\partial^2 \rho}{\partial z_j \partial \bar{z}_k} e^{-t\lambda} dS \\ &\geq tc \int_{\Omega} |f|^2 e^{-t\lambda} dV = tc \|f\|_t^2. \end{aligned}$$

Taking the limit, we have lemma 1.

We set

$$\square_t = \bar{\partial} \bar{\partial}_t^* + \bar{\partial}_t^* \bar{\partial}.$$

Then we have

$$\text{Dom}(\square_t) = \{\phi \in \tilde{\mathbf{D}} : \bar{\partial} \phi \in \text{Dom}(\bar{\partial}^*), \bar{\partial}_t^* \phi \in \text{Dom}(\bar{\partial})\}.$$

For  $\phi \in \text{Dom}(\square_t)$ , we have

$$\|\phi\|_t^2 \leq c(\|\bar{\partial} \phi\|_t^2 + \|\bar{\partial}_t^* \phi\|_t^2) = c \langle \square_t \phi, \phi \rangle_{(t)} \leq c \|\square_t \phi\|_t \|\phi\|_t.$$

Therefore we have

$$\|\phi\|_t \leq c \|\square_t \phi\|_t \quad \text{for } \phi \in \text{Dom}(\square_t).$$

Hence  $R\square_t$  is closed and isomorphic to  $(\mathbf{H}_t)^\perp$ . For  $\alpha \in (\mathbf{H}_t)^\perp$ , there exists a unique  $\phi \in (\mathbf{H}_t)^\perp$  such that  $\alpha = \square_t \phi$ . We define  $N_t \alpha = \phi$ . If  $\alpha \in \mathbf{H}_t$ , we define  $N_t \alpha = 0$ . Then the bounded operator  $N_t : L^2 \rightarrow L^2$  satisfies the following.

$$N_t \square_t = \square_t N_t = I - H_t, \quad N_t \bar{\partial} = \bar{\partial} N_t, \quad N_t \bar{\partial}_t^* = \bar{\partial}_t^* N_t.$$

Hence if  $\alpha \in L^2_{(0,q)}(\Omega)$  satisfies that  $\bar{\partial} \alpha = 0$  and  $H_t \alpha = 0$ , then  $v = \bar{\partial}_t^* N_t \alpha$  is a solution of the equation  $\bar{\partial} v = \alpha$ . Now we prove the following.

**PROPOSITION 1.** *Let  $\Omega$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary. If  $\alpha \in L^2_{(0,q)}(\Omega)$  is a  $\bar{\partial}$  closed form, then  $v = \bar{\partial}_t^* N_t \alpha$  is a solution of  $\bar{\partial} v = \alpha$ .*

Proof. By Hörmander[4], there exists  $\beta \in L^2_{(0,q-1)}(\Omega)$  such that  $\alpha = \bar{\partial} \beta$ . Let  $f \in \mathbf{H}_t$ , then we have  $\bar{\partial}_t^* f = e^{t\lambda} \bar{\partial}^*(e^{-t\lambda} f) = 0$ . Thus we have  $\bar{\partial}^*(e^{-t\lambda} f) = 0$ . On the other hand we have

$$\langle \alpha, f \rangle_{(t)} = \langle \alpha, e^{-t\lambda} f \rangle = \langle \bar{\partial} \beta, e^{-t\lambda} f \rangle = \langle \beta, \bar{\partial}^*(e^{-t\lambda} f) \rangle = 0$$

Thus we have  $\alpha \in \mathbf{H}_t^\perp$ , which proves that  $v = \bar{\partial}_t^* N_t \alpha$  is a solution of  $\bar{\partial} v = \alpha$ .

We consider the complex

$$L^2_{(0,q-1)}(\mathcal{Q}) \xrightarrow{\bar{\partial}_{0,q-1}} L^2_{(0,q)}(\mathcal{Q}) \xrightarrow{\bar{\partial}_{0,q}} L^2_{(0,q+1)}(\mathcal{Q}).$$

We set  $T = \bar{\partial}_{0,q-1}$  and  $S = \bar{\partial}_{0,q}$  and denote by  $T^*$  and  $S^*$  adjoint operators of  $T$  and  $S$  with respect to  $\langle, \rangle_{(t)}$ , respectively. Then we have the following.

**PROPOSITION 2.** *Let  $\mathcal{Q}$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary and let  $f$  be a  $C^\infty(0, q)$ -form in  $\bar{\mathcal{Q}}$  with  $\bar{\partial}f=0$ . Then it holds that*

$$(1.2) \quad \|N_t f\|_t^2 \leq c(t) \|f\|_t^2.$$

Proof. It follows from lemma 1 that

$$t \|N_t f\|_t^2 \leq c \|T^* N_t f\|_t^2, \quad t \|T^* N_t f\|_t^2 \leq c \|T T^* N_t f\|_t^2 = c \|f\|_t^2.$$

Thus we obtain (1.2).

**2. The estimates.** The results in this section are essentially given by Catlin[1]. But we give the proof for the reader's convenience. For  $\phi \in C^\infty_{(0,q)}(\bar{\mathcal{Q}})$ , define

$$\|\phi\|_{m,t} = \sum_{|\alpha| \leq m} \|D^\alpha \phi\|_t.$$

Here  $\alpha$  refers to  $\frac{\partial^{|\alpha|}}{\partial x^{\alpha_1} \dots \partial x^{\alpha_n}}$ . We use some convenient notation. If  $A$  and  $B$  are functions on a set of parameters  $S$ , we use the notation  $A \lesssim B$  to mean that for some  $c > 0$ ,  $|A(\sigma)| \leq c |B(\sigma)|$  for all  $\sigma \in S$ . Further, we denote by  $A^m$  a tangential differential operator of order  $m$ . Then we have the following.

**PROPOSITION 3.** *For large  $t$ , we have*

$$(2.1) \quad \|u\|_{m,t}^2 \lesssim \|\square_t u\|_{m,t}^2 + c_m(t) \sum_{j=0}^{m-1} \|u\|_{j,t}^2,$$

where  $u \in \text{Dom}(\square_t) \cap C^\infty_{(0,q)}(\bar{\mathcal{Q}})$ .

Proof. Suppose that  $A^m_b$  is supported in the interior of  $\mathcal{Q}$ . Then we have in view of (1.1) and the method of Kohn[5],

$$(2.2) \quad t \|A^m_b u\|_t^2 \lesssim Q_t(A^m_b u, A^m_b u) \lesssim \|\square_t u\|_{m,t}^2 + \sum_{|\beta|=m} \|D^\beta u\|_t^2 + c(t) \|u\|_{m-1}^2.$$

The above inequality is still valid for every tangential differential operator  $A^m_b$ . Since  $\square_t$  is elliptic and the second order terms are independent of  $t$ ,  $\frac{\partial^2 u}{\partial r^2}$  is written in the following form

$$\frac{\partial^2 u}{\partial r^2} = c_1 \square_t u + c_2 \frac{\partial}{\partial r} A^1_b u + c_3 A^2_b u + c(t) (c_4 A^1_b u + c_5 \frac{\partial}{\partial r} u + c_6 u).$$

Therefore we obtain for  $2 \leq k \leq m$

$$\|A^{m-k}_b \frac{\partial^k u}{\partial r^k}\|_t^2 \lesssim \|\square_t u\|_{m,t}^2 + \|A^{m-1}_b \frac{\partial u}{\partial r}\|_t^2 + \|A^m_b u\|_t^2 + c(t) \sum_{j=0}^{m-1} \|u\|_{j,t}^2.$$

On the other hand we have

$$\begin{aligned} \|\frac{\partial}{\partial r} A^{m-1}_b u\|_t^2 &\lesssim \|A^m_b u\|_t^2 + Q_t(A^{m-1}_b u, A^{m-1}_b u) \\ &\lesssim \|A^m_b u\|_t^2 + \|\square_t u\|_{m-1}^2 + \|u\|_{m-1,t}^2 + \sum_{|\beta|=m-1} \|D^\beta u\|_t^2 + c(t) \|u\|_{m-2,t}^2. \end{aligned}$$

Thus we have

$$(2.3) \quad \|A_b^{m-k} \frac{\partial^k}{\partial r^k} u\|_i^2 \lesssim \|\square_t u\|_{m,t}^2 + \|A_b^m u\|_i^2 + \sum_{|\beta|=m-1} \|D^\beta u\|_i^2 + c(t) \|u\|_{m-2,t}^2.$$

Together with (2.2) and (2.3), we obtain

$$\begin{aligned} t \|u\|_{m,t}^2 &\lesssim t \|A_b^m u\|_i^2 + \sum_{k=0}^m t \left\| \frac{\partial^k}{\partial r^k} A_b^{m-k} u \right\|_i^2 \\ &\lesssim \|\square_t u\|_{m,t}^2 + \|u\|_{m,t}^2 + c(t) \sum_{j=0}^{m-1} \|u\|_{j,t}^2. \end{aligned}$$

For sufficiently large  $t$ , we obtain the desired inequality.

Now we are going to prove the following.

**PROPOSITION 4.** *Let  $\Omega$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary. Then we have*

$$(2.4) \quad \|T^* u\|_{m,t}^2 + \|Su\|_{m,t}^2 \leq c_m(t) (\|\square_t u\|_{m,t}^2 + \|u\|_i^2),$$

where  $u \in \text{Dom}(\square_t) \cap C^\infty_{(0,q)}(\bar{\Omega})$ .

Proof. We assume that  $A_b^m$  is supported in the interior of  $\Omega$ . Then we have

$$\begin{aligned} &\|A_b^m T^* u\|_i^2 + \|A_b^m Su\|_i^2 \\ &= (A_b^m \square_t u, A_b^m u) + ([T, A_b^m] T^* u, A_b^m u) + (A_b^m T^* u, [A_b^m, T^*] u) \\ &\quad + ([S^*, A_b^m] Su, A_b^m u) + (A_b^m Su, [A_b^m, S^*] u) \\ &\lesssim \|\square_t u\|_{m,t}^2 + \|u\|_{m,t}^2 + c(\varepsilon) \|u\|_{m,t}^2 + \varepsilon \|T^* u\|_{m,t}^2 + \varepsilon c(t) \sum_{j=0}^{m-1} \|T^* u\|_{j,t}^2 \\ &\quad + c(\varepsilon) \sum_{j=0}^{m-1} \|u\|_{j,t}^2 + \varepsilon \|Su\|_{m,t}^2 + \varepsilon c(t) \sum_{j=0}^{m-1} \|Su\|_{j,t}^2. \end{aligned}$$

Thus we have

$$(2.5) \quad \|A_b^m T^* u\|_i^2 + \|A_b^m Su\|_i^2 \lesssim \|\square_t u\|_{m,t}^2 + \varepsilon (\|T^* u\|_{m,t}^2 + \|Su\|_{m,t}^2) \\ + c(\varepsilon) \sum_{j=0}^{m-1} \|u\|_{j,t}^2 + \varepsilon c(t) \sum_{j=0}^{m-1} (\|T^* u\|_{j,t}^2 + \|Su\|_{j,t}^2).$$

(2.5) is still valid for every  $A_b^m$ . Since the complex  $\bar{\partial} \oplus \bar{\partial}_t^*$  is elliptic, and its first order terms are independent of  $t$ ,  $\frac{\partial}{\partial r} T^* u$  can be written in terms of the components of  $\bar{\partial} T^* u$  of  $A_b^1 T^* u$  and of the components of  $T^* u$ . Further, coefficients of the first order terms are independent of  $t$ . Therefore  $\frac{\partial^2}{\partial r^2} T^* u$  can be written in terms of the components of  $\frac{\partial}{\partial r} A_b^1 T^* u$ ,  $\frac{\partial}{\partial r} T^* u$ ,  $\sum_{|\beta| \leq 1} D^\beta \square_t u$ , and  $A_b^1 T^* u$ . Further, coefficients of the second order terms are independent of  $t$ . Thus we have

$$\begin{aligned} \|A_b^{m-k} \frac{\partial^k}{\partial r^k} T^* u\|_i^2 &= \|A_b^{m-k} \frac{\partial^{k-2}}{\partial r^{k-2}} \frac{\partial^2}{\partial r^2} T^* u\|_i^2 \\ &\lesssim \|A_b^{m-k+1} \frac{\partial^{k-1}}{\partial r^{k-1}} T^* u\|_i^2 + c(t) (\|\square_t u\|_{m-1,t}^2 + \|T^* u\|_{m-1,t}^2) \\ &\lesssim \|A_b^m T^* u\|_i^2 + \|A_b^{m-1} \frac{\partial}{\partial r} T^* u\|_i^2 + c(t) \|T^* u\|_{m-1,t}^2. \end{aligned}$$

By Kohn[5], we have

$$\begin{aligned} \left\| \frac{\partial}{\partial r} A_b^{m-1} T^* u \right\|_i^2 &\lesssim Q_t(A_b^{m-1} T^* u, A_b^{m-1} T^* u) + \|A_b^m T^* u\|_i^2 \\ &\lesssim \|\square_t u\|_{m,t}^2 + \|T^* u\|_{m-1,t}^2 + c(t) \|T^* u\|_{m-2,t}^2. \end{aligned}$$

Hence we have

$$(2.6) \quad \left\| A_b^{m-k} \frac{\partial^k}{\partial r^k} T^* u \right\|_i^2 \lesssim \|A_b^m T^* u\|_i^2 + \|\square_t u\|_m^2 + c(t) \|T^* u\|_{m-1}^2.$$

Similarly we have

$$(2.7) \quad \left\| A_b^{m-k} \frac{\partial^k}{\partial r^k} S u \right\|_i^2 \lesssim \|A_b^m S u\|_i^2 + \|\square_t u\|_{m,t}^2 + c(t) \|S u\|_{m-1,t}^2.$$

Therefore we have, together with (2.6) and (2.7)

$$\begin{aligned} \|T^* u\|_{m,t}^2 + \|S u\|_{m,t}^2 &\lesssim \|\square_t u\|_{m,t}^2 + c(\varepsilon) \sum_{j=0}^{m-1} \|u\|_{j,t}^2 + \varepsilon (\|T^* u\|_{m,t}^2 + \|S u\|_{m,t}^2) \\ &\quad + \varepsilon c(t) \sum_{j=0}^{m-1} (\|T^* u\|_{j,t}^2 + \|S u\|_{j,t}^2). \end{aligned}$$

We have, for  $\varepsilon > 0$  small

$$(2.8) \quad \|T^* u\|_{m,t}^2 + \|S u\|_{m,t}^2 \lesssim \|\square_t u\|_{m,t}^2 + c(t) \sum_{j=0}^{m-1} (\|u\|_{j,t}^2 + \|T^* u\|_{j,t}^2 + \|S u\|_{j,t}^2).$$

Repeating (2.8), we have the desired inequality.

Now we are going to prove the main estimate.

**PROPOSITION 5.** *Let  $\Omega$  be a bounded pseudoconvex domain in  $C^n$  with smooth boundary and let  $\alpha$  be a  $\bar{\partial}$  closed  $(0, q)$ -form in  $\bar{\Omega}$ . Then  $v = \bar{\partial}_i^* N_t \alpha$  is a solution of the equation  $\bar{\partial} v = \alpha$  and satisfies*

$$(2.9) \quad \|v\|_{m,t}^2 \leq c_m(t) \|\alpha\|_{m,t}^2.$$

*Proof.* In view of proposition 1,  $T^* N_t \alpha$  is a solution of  $\bar{\partial} v = \alpha$ . It follows from (2.1) that

$$\|N_t \alpha\|_i^2 \leq c(t) \|\alpha\|_i^2.$$

We obtain from (2.4)

$$\|T^* N_t \alpha\|_{m,t}^2 \leq (\|\square_t N_t \alpha\|_{m,t}^2 + \|N_t \alpha\|_i^2) \leq c_m(t) \|\alpha\|_{m,t}^2,$$

which completes the proof of proposition 5.

**3. Proof of the theorem.** Let  $H_j$  be the Sobolev space of order  $j$ . It suffices to find a sequence of solutions  $u_j \in H_j$  such that

$$(3.1) \quad \|u_{j+1} - u_j\|_j \leq 2^{-j}.$$

Suppose that  $u_1, \dots, u_{m-1}$  have already been chosen to satisfy (2.9) and (3.1) and we have a solution  $\tilde{u}_m$  satisfying (2.9). For each  $P \in \partial\Omega$  there is a coordinate neighborhood  $U$  and an  $n$ -tuple  $(a_1, \dots, a_n)$  such that for  $\varepsilon$  sufficiently small  $\Phi_\varepsilon(z) = (z_1 + \varepsilon a_1, \dots, z_n + \varepsilon a_n) \in \Omega$  whenever  $(z_1, \dots, z_n) \in U \cap \bar{\Omega}$ .

Let  $\{U_j\}$ ,  $j=1, \dots, \nu$ , be a covering of  $\partial\Omega$  by open coordinate neighborhoods of the above type and denote the corresponding transformations by  $\Phi_\varepsilon^j$ . Let  $U_0$  be an open subset of  $\Omega$  such that  $\{U_j\}$  is a covering of  $\bar{\Omega}$  and  $\{\rho_j\}$  a  $C^\infty$  partition of unity on  $\bar{\Omega}$

subordinate to  $\{U_j\}$ . We set  $h = \tilde{u}_m - u_{m-1}$  and  $h(\Phi_\epsilon^j(z)) = h_\epsilon^j(z)$ . Define

$$V_\epsilon(z) = \sum \rho_j(z) h_\epsilon^j(z) - w_\epsilon(z),$$

where we determine  $w_\epsilon$  later. Suppose  $\bar{\partial} V_\epsilon(z) = 0$ . Then we have

$$\bar{\partial} w_\epsilon = \sum \bar{\partial} \rho_j h_\epsilon^j = \sum \bar{\partial} \rho_j (h_\epsilon^j - h).$$

We can choose  $w_\epsilon \in H_m$  such that

$$\|w_\epsilon\|_m \leq c_m(t) \|\bar{\partial} \rho_j (h_\epsilon^j - h)\|_m.$$

Hence we have  $\lim_{\epsilon \rightarrow 0} \|w_\epsilon\|_m = 0$ . Further we have  $\lim_{\epsilon \rightarrow 0} \|h - V_\epsilon\|_{m-1} = 0$ . We take  $u_m = \tilde{u}_m - V_\epsilon$ . Then we have

$$\|u_m - u_{m-1}\|_{m-1} = \|\tilde{u}_m - V_\epsilon - u_{m-1}\|_{m-1} = \|h - V_\epsilon\|_{m-1} < 2^{-m},$$

which completes the proof of the theorem.

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