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The Group of Isometries of a Metric Manifold

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Abstract

The group $I(M)$ of isometries of a connected metric manifold $M$ is a Lie group with respect to any topology finer than the pointwise topology. And if moreover the manifold $M$ is the coset space of a topological group with a two-sided invariant metric, then $I(M)$ is the total space of a principal fibre bundle over $M$ with fibre and group $I_a(M)$ (the isotropy subgroup of $I(M)$ at an arbitrarily fixed point $a$ of $M$).

Introduction. Myers and Steenrod [7] have proved that for a Riemannian manifold $M$ with a finite number of connected components, the group $I(M)$ of isometries of $M$ is a Lie transformation group with respect to the pointwise topology. We have tried to get similar conclusion to this for a connected metric manifold, and we have got the results as stated in the above abstract.

1. To become a locally compact topological transformation group.

In this paper the isometric group of a metric space $X$ means the set of all distance-preserving surjections of the space $X$ onto itself, and is denoted by $I(X)$. It is a group of homeomorphisms on $X$. Now we consider for what space $X$ the isometry group $I(X)$ with the compact-open topology is locally compact? In this connection we use the following generalization of a theorem of Dantzig and van der Waerden [1] (see Corollary in [5]).

PROPOSITION A. Let $X$ be either a compact metric space or a locally compact metric space with a finite number of connected components. Then the isometry group of $X$ is a locally compact topological transformation group under any topology finer than the pointwise topology.

REMARK. Similar generalization of the theorem of Dantzig and van der Waerden is found on page 46 of [3].
2. To become a Lie group.

We have proved in [4] the following proposition.

**Proposition B.** If a locally compact transformation group acting effectively on a connected metric manifold is locally Lipschitzian, then it is necessarily a Lie group.

Here the definition of “locally Lipschitzian” is as follows: a topological transformation group $G$ acting on a space $Y$ with a metric $\rho$ is locally Lipschitzian, if for any neighborhood $U$ of each point $a$ in $Y$ there exist a neighborhood $V$ of the identity of $G$ and a neighborhood $U_a$ of the point $a$ as follows:

1) $V(U_a) \subset U$, and
2) $\rho(g(x), g(y)) \leq c \cdot \rho(x, y)$ for all $g \in V$ and all $x, y \in U_a$, where $c$ is a constant.

Let $M$ be a connected metric manifold. Since the isometry group $I(M)$ of $M$, with any jointly continuous topology, is locally Lipschitzian, we have the following theorem from Propositions A and B.

**Theorem.** The isometry group of a connected metric manifold is a Lie group with respect to any topology finer than the pointwise topology.

3. To become the total space of a principal fibre bundle.

Now we consider a homogeneous space as a base space.

Let $G$ be a topological group with a two-sided invariant metric $\rho$, and $X = G/H$ be the left (or right) coset space of $G$ by a closed subgroup $H$. Then we can define a left (resp. right) invariant metric $\rho'$ on $X$ as follows:

$$\rho'(x_i, x_2) = \rho(g_iH, g_2H) \quad \text{for} \quad x_i \in X \quad \text{and} \quad g_i \in \pi^{-1}(x_i) \quad (i = 1, 2).$$

We say $\rho'$ the induced metric from $\rho$.

We have proved in [6] the following proposition.

**Proposition C.** Let $G$ be a topological group with a two-sided invariant metric $\rho$. Give the left (or right) coset space $X = G/H$ the induced metric from $\rho$. If $X$ is connected, locally Euclidean, and has a local cross-section, then the isometry group of the space $X$ is the total space of a principal fibre bundle over $X$ with respect to the compact open topology.

Using this we have the following.

**Corollary.** Let $G$ be a connected Lie group with two-sided invariant metric. Give the left or right coset space $X = G/H$ the induced metric. Then the isometry group $I(X)$ of the space $X$ is a Lie group and also the total space of a principal fibre bundle over $X$ with respect to the compact-open topology.

**Remark.** Examples of topological groups which has a two-sided metric are compact groups, Abelian groups, discrete groups, and product groups of such groups. H. Freudenthal [2] determined the type of a 2nd countable locally compact connected...
group with a two-sided invariant metric. It is the direct product of the group of translations of a Euclidean space and a compact connected group.

References