A numerical Research on the Primary Scattering of the Sun's Ray in the High Atmosphere (II)

by Takao SATO
Nagasaki University
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Abstract

The author has investigated the primary scattering at 75 levels from 1 km to 38 km height, the interval between neighboring two levels being 500 m. Moreover the total solar energy is divided into twelve wavelength ranges, each of which has the partial energy equal to one twelfth of the total.

Let \( T_0 \) be each level point above explained and draw a line passing through \( T_0 \) and making an angle \( \theta_1 \) with a line passing \( T_0 \) and the earth's centre \( O \). For convenience sake the line thus defined will be hereafter called \( \theta_1 \) line. Let \( T_4 \) be the intersecting point of \( \theta_1 \) line with the earth's surface or its atmospheric upper limit, and \( T_1, T_2, T_3 \) be three points which divide the line section between \( T_0 \) and \( T_4 \) into four equal lengths. Hereafter the above five points will be called in general \( T_n \). The author has researched and compared the primary scattering intensity which originates at \( T_n \) and reaches the level point \( T_0 \) for 30°, 60° and 90° of the Sun's altitude \( h \), determined each of two positions of the level point which take respectively max. and minimum value for each combination of the wavelength domains, \( \theta_1 \), \( h \), and \( T_n \).

Introduction

When we elevate high and higher in the atmosphere, the blue sky becomes dark and darker as a well known phenomenon. The scattering problem in the high atmosphere is not yet fully solved even if we restrict it in the primary scattering only. The author has adopted the earth's atmosphere of 40 km thickness, composed of \( 4.10^8 \) numbers of concentric homogeneous spherical shell of 10 cm thickness with its centre at the earth's center.

In the case of the primary scattering, the phase function has simple and rigorous expression with no ambiguity (Ref. 1).
1. The relation between the number of level point and its height

Let us now denote 1 km level above the ground by No. 1, and 38 km by No. 75. As the thickness of the adjacent two levels is 500 m, No. \( q \) corresponds to the height of \((q + 1)/2\) in the unit of km. By this method we can know No. and the height of all level points up to 75.

2. Some explanations

   a) Transmission coefficient in the high atmosphere. The discussion is the same as that explained in Ref. 2).

   b) Extinction effect and twelve wavelength ranges. In this paper we have used the same values indicated in Table 1 in Ref. 3). Hence the detailed explanation may be here omitted. But we must explain the value of transmissivity for each \( \lambda_i \) corresponding to \( \lambda_i' \) in that Table i.e. the value of \( P_{s_{i}}^{M_{o}} \) in Ref. 2), \( M_{o} \) being the total mass of vertical air column on a unit area from sea level to the upper atmospheric limit (i.e. 40 km level) and \( P_{s_{i}} \) corresponding to \( P_{s} \) in Table 1 in Ref. 3). The notation \( m \) is in this paper the sum of the mass traversed by the solar ray in reaching \( T_{m} \) from the upper atmospheric limit and that traversed by the primary scattered ray from \( T_{n} \) to \( T_{o} \). \( m/M_{o} \) is naturally positive. The author has calculated and made a Table giving the value of \( P_{s_{i}}^{m/M_{o}} \) for \( m/M_{o} = 0.01, 0.02, \ldots, 4.99, 5.00 \), and for larger than 5.00 \( m/M_{o} \) is 5.05, 5.10, 5.15 \( \ldots \). This table is called \( P \) table.

   The value of transmissivity for any traversed mass which is not given in \( P \) table may be approximated with permissible error by the value for the nearest \( m/M_{o} \).

   c) Relation between the value of \( \theta_{i} \) and the height. The value of \( \theta_{i} \) of tangent from any level point to the earth's surface is specified by \( \theta'_{i} \). This is larger than 85° for the height indicated by \( q \leq 47 \), and smaller than 85° for \( q \geq 48 \), so that we must introduce two different selections about \( \theta_{i} \) for the above two ranges of \( q \), because this introduction is necessary to obtain the horizontal scattering intensity at the level point now in question in Ref. 4). We have in conclusion selected for this purpose \( \theta_{i} = 0°, 30°, 60°, 65°, 70°, 75°, 80°, (85° + \theta'_{i})/2, \theta'_{i}, (90° + \theta'_{i})/2, 90°, 95°, 100°, 105°, 110°, 115°, 120°, 150°, 180° \) for \( q \leq 47 \), and \( q \geq 48 \) we have used \( 0°, 30°, 60°, 65°, 70°, 75°, 80°, (80° + \theta'_{i})/2, \theta'_{i}, 85, 87.5, 90, 95, 100, 105, 110, 115, 120, 150, 180 \), arranged respectively in order of the magnitude.

   The denser division near \( \theta'_{i} \) is attributed to the fact that the direction near the tangent has much influence on the evaluation of scattering intensity.

3. Numerical Research and Chart on the primary scattering

The author has calculated the Value (hereafter being denoted by \( S_{i} \)) of the primary scattering intensity received at each level point from a whole cone.
of one steradian with its axis at ($\theta$, $\Lambda$) and its vertex at the point in (1/12) $I_0/D^2$ unit in the partial wavelength domain, and in $I_0/D^2$ unit in the total domain. Here $\Lambda$ is the azimuth of $\theta$, line with respect to the vertical plane passing through the Sun’s centre, $I_0$, the solar constant, $D$ the distance of the earth from the Sun in Astronomical Unit. In this paper $S_1$ are graphically represented for $\theta$, from $0^\circ$ to $75^\circ$.

Let us take $q$ the number of the level point as abscissa and $S_1$ as ordinate. The value of $S_1$ on the figure is the multiplication of the following three quantities: $10^{-n}$ (n being 2, 3 etc) on the ordinate line, the number given at the side of the line, $1/12$, $I_0/D^2$ (or $I_0/D^2$) in the total domain, which is denoted by $T$ in the figure. We must add the next explanation. (1), (2), ……(12) is the notation of the partial wavelength domain $\lambda_1$, $\lambda_2$……$\lambda_{12}$.

References

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