Prime Characteristics of Turbidity Coefficient as Estimated from Direct Solar Radiation Measurements

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(Manuscript received 30 Nov. 1968)

Abstract

By means of the direct solar radiation measurements made during the IGY, the monthly and diurnal variations of turbidity coefficient are discussed. The former is classified into two types, and the latter is revealed to be complicated. A fairly well relation is found between the turbidity coefficient and the size of population in Japan. The relation between the precipitable water and the surface water vapor pressure is also discussed in this paper.

1. Introduction

For the purpose of investigating the contamination of the atmosphere with the aid of measurements of direct solar radiation, many investigators have developed various techniques. However, they have been essentially based on Linke's turbidity factor (1942) or Ångström's turbidity coefficient (1930). In order to study the hemispherical distribution of turbidity coefficient, Yamamoto, Tanaka and Arao (1968) developed a chart for estimating the coefficient, basing on the assumption that the size distribution of aerosol particles obeys the Junge distribution. In this chart there is a particularity in taking account of the attenuations not only by the Rayleigh scattering and water vapor but by ozone and carbon dioxide.

The chart was constructed as follows: The intensity of total direct radiation on the ground at the mean sun-earth distance, $I$, is given by

$$I = \int I_0 \ T_R \ T_{o_3} \ T_M \ T_{H2O} \ T_{CO2} \ d \lambda$$

(1)

where $I_0$ is the solar constant, $T_R$, $T_{o_3}$, $T_M$, $T_{H2O}$ and $T_{CO2}$ are transmissions due to the Rayleigh atmosphere, ozone, aerosols, water vapor and carbon
dioxide respectively. \( m \) is the air mass and \( \lambda \) is the wavelength in microns. As the transmissions except \( T_M \) have been examined or calculated by many investigators, with use of their results one can get the intensity of dust free radiation, \( Id.f \), that is

\[
Id.f = \int_0^\infty T_R \ T_a \ T_{H2O} \ T_{CO2} \ d\lambda
\]

Then the transmission due to aerosol particles is given by

\[
T_M = \frac{I}{Id.f}
\]

While owing to the assumption of the Junge distribution \( T_M \) is expressed as a function of \( \lambda \), i.e.

\[
T_M(\lambda) = \exp(-\beta \lambda^{-1})
\]

\( T_M \) is the function of \( m \), water vapor and ozone amounts, but it can be transformed to be the function of \( m \) and water vapor amount with the supposition that the mean ozone amount in the unit column is 0.30 cm. Consequently, the coefficient of turbidity \( \beta \) is given by the function of air mass and water vapor amount, if the intensity of total direct radiation is known. The detail explanation of the theoretical bases is not repeat here. While, because of the assumption of the Junge distribution, there is a little difference between this chart and others; e.g. one of them is Robinson's chart (1966) that was well constructed, and in which his coefficient of turbidity, \( B \), is given by

\[
T_M(\lambda) = \exp(-B \lambda^{-1.5})
\]

and ozone amount is 0.34 cm. The nature of turbidity studied with use of the data made during IGY is discussed in this paper.

2. Data and Procedures

The intensity of direct solar radiation and the water vapor amount in a vertical column are indispensable for utilizing the turbidity chart. The water vapor content per unit column in grams, \( w \), is given by

\[
w = \frac{1}{g} \int_0^{P_o} q \ dp
\]

where \( q \) is the specific humidity, \( p \) is the atmospheric pressure in mb, \( g \) is the gravity acceleration in cm per sec\(^2\) and \( P_o \) is the surface pressure at the station. In real case, \( q \) was computed for each station at the seven standard levels, i.e. the levels of surface, 1000, 850, 700, 600, 500, 400, and 300mb, and at several significant levels from surface to 700mb. The vertical integration required to compute \( w \) was performed numerically applying the trapezoidal rule on the assumption that \( q = 0.011 g/\text{kg} \) at 100 mb and that the water vapor amount contained between 0 to 100 mb is 0.0038 grams. This assumption will be admitted to be fairly well on the average. In the aerological
data there were sometimes no humidity values at the levels of 300 or 400 mb. In such cases, the least standard level having the data both of temperature and humidity was connected with the level of 100 mb for the trapezoidal rule. The value of saturation water vapor pressure required to compute \( q \) was derived from Keyes formula (1947) in which that is expressed as a function of temperature only. Adopting this formula, we can get rid of much troubles in looking up the table one by one. The greater the optical path length is, the less the intensity of direct radiation becomes, and the atmosphere turns into unstable condition. In this point of view, the data such as the air mass was greater than five were removed as a rule. All the stations that had made the direct solar radiation measurements were made to be subjects of the investigation, however, those far away from aerological stations were excluded. The number of stations studied here was 75 and that of days that should be analysed for each station amounted to about 150 on the average. The work of computing the value of \( w \) were so great that the calculations were performed with use of HITAC-5020 Computer at Computation Center of Tokyo University.

3. Results and Discussion

3-1 Monthly variation of \( \beta \)

The data of IGY had not enough number of observations to establish a monthly average because of the bad condition of weather, however, the monthly variation of \( \beta \) is still one of the most important problems in the studies of atmospheric turbidity. It has been already revealed by Ångström (1961) that the turbidity coefficient shows a rather pronounced maximum in early summer. Kitaoka and Matuoka (1948) also investigated the monthly variation of turbidity factor in and near Japan, and mentioned that the maximum appears in March to May and the minimum in July to September in general. Setting the maximum aside, the minimum seems to be uncertain because it is feared that the amount of water vapor were over estimated, as later discussed. Then, in order to study the monthly variation of \( \beta \), stations, having enough number of observations to form a monthly average for each month, were picked out from all the stations analysed here. In practice the selection was carried out, as a rule, under the condition that the number of observations must not be less than five for each month. This might seem to be mild, but it is rather severe, because only 14 stations could be picked out from 75. In particular, out of 12 stations in Japan only one agreed to the condition. The stations selected in this way could be classified into two types with the manner of variation. In Fig. 1, the shape of variation considered to be the most natural variation are presented. It is evident in this figure that the maximum of \( \beta \) appears in spring or early summer, and that \( \beta \) decreases gradually from month
to month until it comes down to the minimum in autumn or early winter. Another type, as shown in Fig. 2, is that along with the peak of spring one more peak appears in summer. The second peak sometimes exceeds the first one, and this tendency seems to be concerned with the station situated in a large city; e.g. Tokyo and Dresden, although they had not enough data, showed the second type and both had the maximum value of monthly average in the second peak. However, the classification into two types would not be always proper because of various influences on $\beta$; those of precipitation, weather condition, number of observations, air mass and so on. Particularly the effect
of rain or snow would change the monthly average as discussed in the previous paper. Therefore, there are some of irregular type of variation, but most of them are belonging to the neutral type.

Generally speaking, it is certainly verified that the natural variation of $\beta$ shows a remarkable maximum in spring or early summer, and no more peak in other seasons. Furthermore, it is an interest fact that two peaks appear in spring and summer at the stations that might be rather contaminated with artificial sources.

3-2 Diurnal variation of $\beta$

It is not easy to study the diurnal variation of turbidity because of the rarity of days in which the total direct radiation measurements can be made throughout the daytime. Since the most of stations made three times measurements at most, these were not suitable to estimate diurnal variation.
Influence of water vapor amount on $\beta$. Owing to the variation of $w$ shown in lower part, $\beta$ is forced to vary as above. Here the relation between $w$ and $\varepsilon_s$ is adopted as $w=1.5\varepsilon_s$.

Fig. 3 (b) Same as Fig. 3 (a) but the fluctuation of $w$ is twice times as large as it.

Fortunately, the successive measurements throughout the day were made in Germany, hence the study were executed with use of the data of Gotha, Dresden and Heiligendamm. The opportunities that the measurements could be made in all the day were so rare that every station had the data of 10 days or so in a year and half.

Another question in studying the diurnal variation is the influence of the variation of water vapor amount. The water vapor amount can be calculated only once in the daytime, because the aerological measurement is ordinarily made at interval of 12 hr. Therefore, there is no choice but to assume that
Characteristics of Turbidity Coefficient as Estimated from Direct Solar Radiation Measurements

Fig. 4 Diurnal variation of $\beta$ for Heiligendamm. Black circles show individual turbidity coefficients and those of white instantaneous surface pressures of water vapor.

the water vapor amount calculated from the aerological measurement is constant in all the daytime.

In order to examine the fluctuation of turbidity due to the variation of water amount, two models concerning the variation of water vapor amount were established as shown in the lower parts of Figs. 3(a) and (b). Model-A is approximately corresponding to the variation of surface pressure from nine to 12 mb and Model-B from eight to 14 mb. The fluctuation examined here are shown in the upper parts of them, indicating that values of $\beta$ that should be 0.05, 0.10 and 0.15 respectively throughout the day in which $w$ is fixed to be 1.5 grams are obliged to vary owing to the variation of $w$. These figures were
constructed with the presumption that the day was the autumnal equinox day and the station was situated in latitude 40°N, and that \( w \) was calculated to be 1.5 grams from the aerological measurement made at nine in L. A. T. Since the presumption of water vapor amount was essentially based on the normal condition of the atmosphere, it can be acceptable as the average. Inferring from our data, it is highly probable that Model-A is more practical than B, therefore, the width of the fluctuation of \( \beta \) is less than 0.01 in the ordinary condition.

The diurnal variation of \( \beta \) are shown in Figs. 4 - 6, in which the corresponding surface pressures of water vapor \( e_s \) are also expressed for reference. Fig. 4 show the variations at Heiligendamm, where is situated in a rural
Prime Characteristics of Turbidity Coefficient as Estimated from Direct Solar Radiation Measurements

Fig. 6 Same as Fig. 4 but for Dresden.

district, hence there is little fluctuation in consideration of the influences of water vapor amounts. Furthermore, of special interest is the strong resemblance between Figs. 4 (a), (b) and the first type of monthly variation (Fig. 1). In Fig. 5 are shown the successive three day's variations at Gotha. It is interest that the values of $\beta$ are generally increasing with time, and that the high values, reaching at dusk, turn back to the low in the next morning. This may mean that $\beta$ is forced to increase with certain artificial effect in daytime but it becomes less in nighttime by the action of condensation and falling of aerosol particles due to the lower temperature and more calm condition of the atmosphere. Fig. 6 (a) also shows the same variation as above,
however, a type decreasing again before dusk is shown in Fig. 6(b) and that of smaller fluctuation in Fig. 6(c). The diurnal variation of $\beta$ in Dresden could be classified with three types shown here, although they were full of variety.

Next, the data which had three times measurements in a day were picked out from Japanese stations, and the number of times of the time at which $\beta$ had the largest value of the three were presented in Table 1. From this table a conclusive explanation could not be derived, but it seems that the maximum tends to shift to the afternoon as the annual mean value of $\beta$ increases. Especially, in Tokyo and Tateno where is in the suburbs of Tokyo, the number of times at 15h are very larger than others. On the other hand, that of 9h is much prominent at the stations of more than a half. This may lead to the following conclusion; If any diurnal variation of $\beta$ is defined in a rural district, it may show a small peak in the morning and decrease gradually as seen in Fig. 4(a).

Table 1  Frequency of times in which the maximum value of $\beta$

<table>
<thead>
<tr>
<th>Stations</th>
<th>Annual mean value of $\beta$</th>
<th>9h</th>
<th>12h</th>
<th>15h</th>
<th>Total of days in which the three ordinary measurements could be done</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nemuro</td>
<td>0.070</td>
<td>5</td>
<td>13</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>Sapporo</td>
<td>0.108</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Miyako</td>
<td>0.093</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>Akita</td>
<td>0.074</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>Sendai</td>
<td>0.099</td>
<td>15</td>
<td>9</td>
<td>4</td>
<td>28</td>
</tr>
<tr>
<td>Tateno</td>
<td>0.115</td>
<td>12</td>
<td>9</td>
<td>19</td>
<td>41</td>
</tr>
<tr>
<td>Tokyo</td>
<td>0.161</td>
<td>4</td>
<td>7</td>
<td>19</td>
<td>30</td>
</tr>
<tr>
<td>Wajima</td>
<td>0.131</td>
<td>6</td>
<td>19</td>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>Torishima</td>
<td>0.074</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Shionomisaki</td>
<td>0.095</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>Yonago</td>
<td>0.112</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>Fukuoka</td>
<td>0.131</td>
<td>7</td>
<td>8</td>
<td>11</td>
<td>26</td>
</tr>
<tr>
<td>Kagoshima</td>
<td>0.102</td>
<td>15</td>
<td>6</td>
<td>9</td>
<td>31</td>
</tr>
</tbody>
</table>

3-3 The relation between $\beta$ and population

It is easy to consider that $\beta$ becomes greater with increase in population, because $\beta$ depends not only on natural aerosols such as small water droplets, natural dusts and sea salt nuclei but on those of artificial, i.e. particles made from combustion products, car exhausts and so on. It is an interest problem to investigate the relation between $\beta$ and population, but the administrative districts which include the stations are different in area according to their respective ranks. In order to have equal area for each station in Japan, the circle with a radius of 30 kilometers was adopted. The population shown in
Prime Characteristics of Turbidity Coefficient as Estimated from Direct Solar Radiation Measurements

Fig. 7, therefore, is the number of inhabitants who live within the circle with the station in its center. In addition, the annual mean value of $\beta$ was corrected so as to get rid of the difference of latitude. This correction was performed with use of Fig. 22 in the previous paper (1968) under the assumption

$$p_0 = 5 \times 10^4$$

that all the stations were situated at 37°N. Therefore, the value of $\beta$ is slightly different from the true annual mean value of $\beta$. The relation between $\beta$ and population was fairly well verified as seen from the figure. As to Wajima where shows too large value, it is probable that there might be some instrumental errors. The author had no precise data of population in other countries, but it is easy to regard that Leningrad is so large city that the population within the circle would not so greatly vary, even if the suberbun population is not added. In this respect, Leningrad with latitude correction was plotted in the same figure. This also fits well.

3-4 The relation between precipitable water and surface pressure of water vapor

It is not long since the aerological measurements were established, and the aerological network is not so dense as others. The relation between precipitable water and surface pressure is important for these reasons and has been used instead of direct soundings. It is Hann’s formula that has been frequently utilized for this meaning. According to his measurements, the precipitable
Fig. 8 Water vapor amount $w$ plotted against the surface pressure of water vapor $e_s$ for Akita, Tokyo and Fukuoka.

Water in cm or gram, $w$, is given by

$$w = k \cdot e_s$$

(7)

where $k$ is the coefficient of constant, $e_s$ is the surface water vapor pressure. In case that $e_s$ is given by the unit of mb, $k$ comes to be 1.7 and this coefficient is independent of weather conditions. However, as Kitaoka and Matuoka (1948) mentioned in their paper, the value of $k$ seems to be too large in cases of Japan. Fig. 8 shows some of Japanese cases, but they were all in fair weather conditions. Table 2 also shows the value of $k$ estimated graphically for 10 stations in Japan. From this table the coefficient of Hann's formula must be 1.4 for Japan on the average. Another question is whether the relation can be expressed as simple as a linear equation without a term of constant. This relation seems to be kept in higher latitude, but to be discrepant in lower
Table 2  The coefficients of Hann's formula for 10 stations in Japan

<table>
<thead>
<tr>
<th>Stations</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nemuro</td>
<td>0.150</td>
</tr>
<tr>
<td>Sapporo</td>
<td>0.145</td>
</tr>
<tr>
<td>Akita</td>
<td>0.145</td>
</tr>
<tr>
<td>Sendai</td>
<td>0.145</td>
</tr>
<tr>
<td>Tateno</td>
<td>0.145</td>
</tr>
<tr>
<td>Wajima</td>
<td>0.135</td>
</tr>
<tr>
<td>Shionomisaki</td>
<td>0.150</td>
</tr>
<tr>
<td>Yonago</td>
<td>0.145</td>
</tr>
<tr>
<td>Fukuoka</td>
<td>0.150</td>
</tr>
<tr>
<td>Kagoshima</td>
<td>0.125</td>
</tr>
<tr>
<td>Mean</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Fig. 9  Same as Fig. 8 but for Yakutsk, Heiligendamm and Dresden.
latitude. Fig. 9 shows the cases of Yakutsk, Dresden and Odessa. They can be expressed as a linear equation passing the origin, but the values of $k$ are different one another, they are 0.19, 0.16 and 0.17 respectively. On the other hand, Fig. 10, the cases of New Delhi and Taipei, differ from above. They cannot be expressed as such a simple equation any longer and a more complicated equation must be necessary. As seen from Fig. 8, Fukuoka shows an attribute of the latter case slightly, while Tokyo and Akita are belonging to the former case. Therefore, the Hann's formula is essentially available for the region over 40° in latitude, and the value of $k$ is not same over the world. From the figures and table presented here it shows an increasing tendency with latitude. The formula is subject to systematic errors of 30 per cent or more as seen form the figures, therefore, if the errors of the value of $k$ is less than 10 per cent, it may be out of question. As the values of precipitable water were estimated in fair weather only, some of uncertainty might be leaved in these conclusion.
Prime Characteristics of Turbidity Coefficient as Estimated from Direct Solar Radiation Measurements

Acknowledgments

The author wishes to express his sincere thanks to Prof. G. Yamamoto and Mr. M. Tanaka of the Geophysical Institute, Tohoku University for their valuable guidances and discussions. He is deeply indebted to Prof. T. Sato for his encouragements throughout this work. Thanks are also due to the members of the Section of Maritimemeteorology, Nagasaki Marine Observatory for their kindnesses in offering many useful information.

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