Adaptive nulling in thinned planar arrays through Genetic Algorithms

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Abstract: The nulling of interferences or jammers impinging on a planar antenna array is addressed by means of an efficient adaptive control strategy based on a Genetic Algorithm. Each element of the array is uniformly weighted and the nulling feature is yielded by controlling a set of radio-frequency switches that connect or disconnect the elements from the beam forming network. Under the assumption that the direction of arrival of the desired signal is known, the on/off setup of the array elements is optimized to maximize the ratio between the power of the desired signal and the total power collected by the antenna array. Representative results concerned with different interfering scenarios with multiple undesired signals are reported to assess the effectiveness of the approach.

Keywords: adaptive planar arrays, thinned arrays, Genetic Algorithms, pattern nulling

Classification: Electromagnetic theory

References

1 Introduction

In recent years, there has been an enormous proliferation of wireless services and devices along with a still growing request of fast and wideband (e.g., video) communications. To improve the system capacity and the quality of the received signals at the physical layer, the use of smart antennas has been studied [1]. Smart antennas are systems able to reconfigure the radiation characteristics (e.g., pattern, polarization, frequency) according to the conditions of the electromagnetic scenario where they operate for maximizing the power of the desired signal/s with respect to the power of the undesired ones (e.g., interferences or jammers) [2]. Towards this end, pattern nulling strategies aimed at generating deep sidelobes and possibly nulls along the directions of arrival (DoAs) of the undesired interferences have been introduced. As for the minimization of the reconfiguration time versus the effectiveness of the pattern nulls generation, various antenna array architectures and control strategies have been proposed ranging from the use of controllable complex (i.e., amplitude and phase) excitation weights [3] to phase-only nulling techniques [4, 5]. More recently, the availability of enhanced computation facilities has enabled the use of optimization algorithms for reconfiguring the control points of the array elements. Global optimization algorithms based on evolutionary strategies, like the Genetic Algorithms (GAs) [6] or the Differential Evolution (DE) [7, 8], or on swarm intelligence, as for example the Particle Swarm Optimization (PSO) [9, 10] or the Ant Colony Optimization (ACO) [11], have been profitably exploited. However, if a key advantage of such approaches is the possibility to easily take into account unconventional constraints (e.g., the control of only the least significant bits of digital phase shifters [12]), as opposite, their main limitations are the difficulty to deal with large antenna arrays (i.e., high-dimension solution spaces) and the slow convergence to the optimal setting providing the best nulling performance, even though for this latter issue, a suitable memory-enhanced technique has been investigated in [13].
As far as large arrays are concerned, the control of the on/off status has been addressed by equipping each radiating element with a radio-frequency (RF) switch that implement the physical connection or disconnection to the beam-forming network (BFN) \[14\] according to dynamic \[15\] and adaptive \[16\] strategies. More in detail, the dynamic thinning is based on the off-line definition of a finite set of pre-computed on/off switch sequences that are sequentially tried until an increment of the signal-to-noise-plus-interference ratio (SINR) at the receiving system is measured \[15\]. Although easy to implement by means of a simple look-up table, such a strategy does not guarantee the generation of deep nulls along the interference directions. On the contrary, the direct on-time optimization of the SINR has been carried out in \[16\] for linear arrangements.

In this work, the performances of the adaptive thinning proposed in \[16\] is extended to the case of planar array layouts. The remaining of the paper is then organized as follows. The interference suppression problem for 2D arrays is mathematically formulated in Sect. 2, where the adaptive GA-based strategy is briefly recalled, as well. A set of numerical results concerned with noisy scenarios and multiple interferences/jammers impinging from different DoAs is reported (Sect. 3) to assess the effectiveness of the adaptive thinning also for planar arrays. Eventually, some conclusions are drawn (Sect. 4).

2 Mathematical formulation

Let us consider a \(N_x \times N_y\) planar array with elements uniformly distributed in a lattice of cell sides \(d_x\) and \(d_y\) along the two coordinate axes of the xy plane. Each \(mn\)-th \((m = 1, \ldots, N_x, n = 1, \ldots, N_y)\) array element has equal (unitary) amplitude excitation and it can be either turned on \((b_{mn} = 1)\) or off \((b_{mn} = 0)\) by means of a RF switch (Fig. 1). The arising array factor turns out to be given by

\[
AF(u, v) = \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} b_{mn} e^{j[\beta(md_xu+nd_yv)+\varphi_{mn}]}
\]

where \(\{\varphi_{mn}; m = 0, \ldots, N_x - 1, n = 0, \ldots, N_y - 1\}\) is the set of phase weights used for beam steering, \(\beta = \frac{2\pi}{\lambda}\), \(\lambda\) being the free space wavelength, while \(u \triangleq \sin \theta \cos \phi\) and \(v \triangleq \sin \theta \sin \phi\) are the cosine direction angles.

Under the assumption that the angle of arrival \((u_d, v_d)\) of the desired signal \(f_d\) is known, the binary sequence \(b = \{b_{mn} : m = 0, \ldots, N_x - 1; n = 0, \ldots, N_y - 1\}\) that maximizes the following cost function \[6\]

![Fig. 1. Sketch of the array architecture.](image-url)
$$\Psi(b) = \frac{P(u_d,v_d)}{P_{TOT}}$$  \hspace{1cm} (2)

is the optimal solution since it also optimize the SINR [6] by generating a radiation pattern with nulls along the DoAs of narrow-band interferences, \((u_s,v_s), s = 1, \ldots, S\), \(S\) being the total number of the undesired signals, \(\{f_s; s = 1, \ldots, S\}\), impinging on the planar antenna array. The function (2) is the ratio between the power of the desired signal

$$P(u_d,v_d) = \left| \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} b_m n^\dagger e^{j[\beta(md,u_d+nd,v_d)+\varphi_m]} \right|^2$$  \hspace{1cm} (3)

and the total power collected by the array

$$P_{TOT} = P(u_d,v_d) + \sum_{s=1}^{S} P(u_s,v_s) + P_n$$  \hspace{1cm} (4)

where

$$P(u_s,v_s) = \left| \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} b_m n^\dagger e^{j[\beta(md,u_s+nd,v_s)+\varphi_m]} \right|^2$$

is the power associated to the \(s\)-th interference and \(P_n\) is the background noise power [16].

To determine the optimal on/off configuration of the array elements that maximizes (2), the following binary GA-based technique is used:

(a) Initialization \((k = 0)\) - A set of \(P\) binary-coded individuals, \(\{b_p^k = b_p^0; p = 1, \ldots, P\}\), is randomly generated;

(b) Fitness Evaluation - The value of the cost function (2) is computed for each individual of the initial population: \(\Psi_p^0 = \Psi(b_p^0), p = 1, \ldots, P\);

(c) Population Update \((k \leftarrow k + 1)\) - A new set of \(P\) individuals, \(\{b_p^k; p = 1, \ldots, P\}\), (i.e., the children) is generated by mating the individuals of the population of the previous generation, \(\{b_p^{k-1}; p = 1, \ldots, P\}\), (i.e., the parents) by means of the genetic operators Selection, Crossover, and Mutation [17];

(d) Fitness Evaluation - The value of the cost function (2) is computed for each children: \(\Psi_p^k = \Psi(b_p^k), p = 1, \ldots, P\);

(e) Convergence Check - The iterative evolutionary process is stopped when \(k \geq K_{end}\), \(K_{end}\) being a maximum number of iterations a-priori set by the user on the basis of the timing requested for the antenna reconfiguration. Otherwise, the steps (c)→(d)→(e) are iterated;

(f) Array Reconfiguration \((k = K_{end})\) - At convergence, the RF switches of the array are set to the sequence coded into the best individual of the convergence population, \(b_{opt} = \arg\{\max_{p=1,\ldots,P}(\Psi_p^{K_{end}})\}\).

3 Numerical results

In this section, the GA-based thinning approach for the adaptive nulling in planar arrays is validated by considering different layout sizes as well as interference scenarios characterized by either two or three undesired signals, \(S = \{2,3\}\). In all examples, square arrays \((N_x = N_y)\) have been taken into account with element
spacing equal to $d_x = d_y = \frac{\lambda}{2}$. Moreover, the $DoA$ of $f_d$ has been fixed at broadside [i.e., $(u_d, v_d) = (0, 0)$] and the power of the undesired signals $|f_s|^2$, $s = 1, \ldots, S$ and the noise $P_n$ have been set to 30 dB above and below the desired signal one, respectively. As for the control parameters of the $GA$, they have been chosen according to [16]: $P = \frac{3}{2}$, $p_c = 0.9$ (crossover probability), $p_m = 0.01$ (mutation probability), and $K_{end} = 200$.

The first example (Test Case 1) deals with an array of $N_x \times N_y = 4 \times 4 = 16$ elements and $S = 2$ jammers with $DoAs$ $(u_1, v_1) = (0.03, 0.97)$ and $(u_2, v_2) = (-0.57, -0.48)$. Since the $GA$ is a stochastic global optimizer, $R = 100$ simulations with different initial populations have been run for having a statistical relevance of the analysis outcomes. The arising statistics (mean, standard deviation, minimum, and maximum) of the $SINR$ and of the null depths, $P(u_s, v_s)$, $s = \{1, 2\}$, are listed in Table I. Moreover, the behavior of the average $SINR$ in correspondence with the best individuals of each population versus the $GA$ generations, $\text{avg}_{\text{SINR}}^k \triangleq \frac{1}{R} \sum_{r=1}^{R} \text{SINR}^k_r$, $k = 1, \ldots, K_{end}$, is shown in Fig. 2 along with the deviation boundaries defined as $\text{avg}_{\text{SINR}}^k \pm \text{std} - \text{dev}_{\text{SINR}}^k$, being $\text{std} - \text{dev}_{\text{SINR}}^k \triangleq \frac{\sqrt{\sum_{r=1}^{R} (\text{SINR}^k_r - \text{avg}_{\text{SINR}}^k)^2}}{R}$, $k = 1, \ldots, K_{end}$ the standard deviation.

From Table I, it turns out that the average $SINR$ performance is only few decibels above zero ($\text{avg}_{\text{SINR}}^{K_{end}} = 2.6$ dB) with a standard deviation more than three times larger ($\frac{\text{std} - \text{dev}_{\text{SINR}}^{K_{end}}}{\text{avg}_{\text{SINR}}^{K_{end}}} \sim 3$).

**Table I.** Test Case 1 - [$N_x = N_y = 4$, $d_x = d_y = \frac{\lambda}{2}$, $S = 2$] - $SINR$ and null depths $P(u_s, v_s)$, $s = \{1, 2\}$ statistics.

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<tr>
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</thead>
<tbody>
<tr>
<td>$SINR$</td>
<td>2.6</td>
<td>6.8</td>
<td>-6.8</td>
<td>24.1</td>
</tr>
<tr>
<td>$P(u_1, v_1)$</td>
<td>-45.4</td>
<td>15.8</td>
<td>-82.1</td>
<td>-23.9</td>
</tr>
<tr>
<td>$P(u_2, v_2)$</td>
<td>-34.6</td>
<td>9.0</td>
<td>-60.2</td>
<td>-24.5</td>
</tr>
</tbody>
</table>

**Fig. 2.** Test Case 1 - [$N_x = N_y = 4$, $d_x = d_y = \frac{\lambda}{2}$, $S = 2$] - Statistics of the achieved $SINR$ through the optimization process over 100 $GA$-runs.
The optimal on/off configuration retrieved by the GA among the \( R \) runs and the corresponding power pattern are reported in Fig. 3(a) and Fig. 3(b), respectively. Moreover, the evolution of the ‘fitness’ (i.e., cost function value) of the best individual, \( \Psi_{\text{opt}} \), and the corresponding \( \text{SINR} \) values versus the iteration index, \( k = 1, \ldots, K_{\text{end}} \), are shown in Fig. 4(a), while Fig. 4(b) gives the behavior of null depths along the \( S = 2 \) interferer directions. As it can be observed [Fig. 4(b)], nulls lower than \(-55 \text{ dB}\) with respect to the main lobe peak have been already generated starting from iteration \( k = 4 \) along the directions of the two interferences [Fig. 4(b)] that is 10 dB and 20 dB better than the average (Table I) for the interference \( s = 1 \) and \( s = 2 \), respectively. On the other hand, it is worth noticing that even though the \( \text{SINR} \) at convergence is equal to 24.1 dB [Fig. 4(a) - Table I], only half elements are active [\( N_{\text{on}} = 8 \) - Fig. 3(a)] and thus the aperture efficiency turns out quite limited, \( \eta = \frac{N_{\text{on}}}{N_x N_y} = 0.50 \).

In the second example (Test Case 2), the same interference configuration of the previous example has been taken into account, but with an array of 64 elements (i.e., \( N_x = N_y = 8 \)). As expected, the larger number of elements (i.e.,
degrees of freedom of the synthesis problem) enables higher reception performances (avg $f_{SINR}$ end $opt$ $g_j$ $8/C_2^8 \approx 14.5$ dB vs. avg $f_{SINR}$ end $opt$ $g_j$ $4/C_2^4 \approx 2.6$ dB) and an enhanced nulling effectiveness ($std - dev[f_{SINR}$ end $opt$ $g_j$ $8/C_2^8 \approx 2.6$ dB vs. $std - dev[f_{SINR}$ end $opt$ $g_j$ $4/C_2^4 \approx 8.6$ dB) as confirmed by the values of the statistics shown in Fig. 5 and Table II. As for the best solution among the $R$ runs, the on/off status of the array elements and the corresponding power pattern are given in Fig. 6(a) and Fig. 6(b), respectively. Although the optimal $SINR$ is slightly below that of the previous example ($max[f_{SINR}$ end $opt$ $g_j$ $8/C_2^8 \approx 22.4$ dB vs. $max[f_{SINR}$ end $opt$ $g_j$ $4/C_2^4 \approx 24.1$ dB), there are now only 20 elements off out of 64 with a resulting aperture efficiency of $\eta \approx 0.73$, namely 23% higher than that in the Test Case 1.
In the last representative example (Test Case 3), $S = 3$ jamming signals with $\text{DoA}(u_1, v_1) = (0.03, 0.97)$, $(u_2, v_2) = (-0.57, -0.48)$, and $(u_3, v_3) = (0.44, 0.37)$ have been considered with an array of $N_x = N_y = 12$ elements. Despite the presence of $S = 3$ undesired signals, the presence of 144 radiating elements still guaranteed good reception performances ($\text{avg}\{\text{SINR}_{\text{endopt}}\} = 9.5 \text{ dB and } \text{std-dev}\{\text{SINR}_{\text{endopt}}\} = 1.9 \text{ dB}$-Table III) with a minimum value of the SINR equal to $\text{min}\{\text{SINR}_{\text{endopt}}\} = 5.4 \text{ dB}$, that is more than 12 dB above the minimum of the two previous examples ($\text{min}\{\text{SINR}_{\text{endopt}}\}|_{4 \times 4} = -6.8 \text{ dB and } \text{min}\{\text{SINR}_{\text{endopt}}\}|_{4 \times 4} = -8.5 \text{ dB}$). For the sake of completeness, the statistics of the SINR throughout the GA generations (Fig. 7) and the graphs for the best solution (Fig. 8 and Fig. 9) are given, as well.

![Fig. 7. Test Case 3 - $[N_x = N_y = 12, d_x = d_y = \frac{\lambda}{4}, S = 3]$ - Statistics of the achieved SINR through the optimization process over 100 GA-runs.](image)

<table>
<thead>
<tr>
<th>$\text{SINR} \text{ [dB]}$</th>
<th>$\text{avg} {\cdot}$</th>
<th>$\text{std-dev} {\cdot}$</th>
<th>$\text{min} {\cdot}$</th>
<th>$\text{max} {\cdot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(u_1, v_1)$ [dB]</td>
<td>-46.0</td>
<td>5.6</td>
<td>-64.3</td>
<td>-36.3</td>
</tr>
<tr>
<td>$P(u_2, v_2)$ [dB]</td>
<td>-47.1</td>
<td>6.4</td>
<td>-73.0</td>
<td>-39.2</td>
</tr>
<tr>
<td>$P(u_3, v_3)$ [dB]</td>
<td>-46.8</td>
<td>6.1</td>
<td>-67.4</td>
<td>-38.2</td>
</tr>
</tbody>
</table>

Table III. Test Case 3 - $[N_x = N_y = 12, d_x = d_y = \frac{\lambda}{4}, S = 3]$ - SINR and null depths $P(u_s, v_s), s = \{1, 2, 3\}$ statistics.

![Fig. 8. Test Case 3 - $[N_x = N_y = 12, d_x = d_y = \frac{\lambda}{4}, S = 3]$ - Plot of the (a) on-off configuration and (b) power pattern for the optimal solution obtained by means of the adaptive GA-based method.](image)
Concerning the computation time for re-adapting the system, the average CPU-time on a standard laptop machine (i.e., 2 GHz PC with 1 GB of RAM) amounts to 0.03 [sec], 0.11 [sec], and 0.25 [sec] when \( N = 16 \), \( N = 64 \), and \( N = 144 \), respectively.

4 Conclusions

The pattern nulling performance of reconfigurable planar arrays has been assessed. By controlling the on/off status of the array elements with a binary GA, the SINR has been maximized to generate deep sidelobes or nulls along the arbitrary and unknown DoAs of the interferences or jammers impinging on the antenna. Representative numerical results have shown the existence of a trade-off between the nulling capability and the number of control points, namely the number of array elements, as well as the computation costs for re-configuring the RF switches at their optimal values.

Fig. 9. Test Case 3 - \([N_x = N_y = 12, d_x = d_y = \frac{\lambda}{2}, S = 3]\) - Behavior of the (a) fitness of the best individual of the population and (b) corresponding null depths along the interference DoAs through the optimization process.

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