Study on Simplified Speed Sensorless Vector Control Systems for Induction Motors

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Chapter 1

Introduction

1.1 Background

The induction motors (IM) are very common and easy to find, because it is low in price and robust. Those are used in industrial applications such as electric train, pump, fan, machine tools, grinders, conveyors and home utilizations. Conventionally, till the last of the twentieth century, the induction motors are operated at a single speed, which is determined by the frequency of main voltage and the number of poles. Typically, they were operated from fixed-frequency sources 50 Hz or 60 Hz in most cases.

To control the IM speed, a variable-frequency source is required, and such kind source did not readily exist. Thus, applications necessitating variable speed were serviced by direct-current motors (DCM). So, controlling the speed of IM is more difficult than controlling the speed of DCM, because the relationship between the motor current and the resulting torque is not linear for IM.

The availability of solid-state power switches, development of power electronics and microelectronics change this scheme hugely. Recently developed solid-state power device such as insulated gate bipolar transistors (IGBT), metal-oxide semiconductor field-effect transistors (MOSFET) are applied to many favorable switches with wide range power ratings and switching frequency that make them suitable candidates for variable-frequency generation. Now the power electronics are capable for supplying the variable voltage/current, or frequency drive to realize variable-speed
performance for IM. About the microelectronics, a digital signal processor (DSP) has been developed. By using C language, control algorithm is programed and realized without adding hardware.

Recently, to reduce the pollution problem in urban area and the availability of fossil fuel, the electric car is getting attention; but because of the limited performance of the battery as an energy storage, it would take considerable time to use pure electric car commonly. Many of electric cars use interior permanent magnet synchronous motor (IPMSM) from the viewpoint of motor efficiency. However, it is interesting that the Tesla Company uses the IM for electric car.

According to the development of pulse width modulation (PWM) inverter, the constant volt per hertz (\(V/f\)) control is applied at first as variable-speed control method for IM. However, the constant \(V/f\) control cannot control torque instantaneously. On the other hand, the vector control (or field oriented control) can control torque instantaneously and control speed of IM quickly. In order to realize vector control, the information of rotor flux angle is indispensable. In order to estimate the rotor flux angle, the angular frequency is computed by adding the rotor speed to the slip angular frequency. The slip angular frequency is obtained from a current model of IM by assuming that the \(q\) – axis flux becomes zero. The rotor flux angle is obtained by integrating the angular frequency. Many researches for rotor resistance identification are reported to compute accurate the slip frequency [10], [16], [20], [21].

By using the rotor flux angle, the three phases currents are transformed to two-axis (\(d – q\)) currents. In general, the \(d\) – axis is selected as the direction of rotor flux vector. Therefore, the \(d\) – axis current is proportional to the magnitude of the rotor flux. The electromagnetic torque is proportional to the product of \(d\) – and \(q\) – axis currents. By controlling the \(d\) –
and $q$ – axis currents quickly, the torque is produced instantaneously without transient. In general, this currents control are achieved by PI control through voltage control by PWM inverter. At steady – state the $d$ and $q$ variables become constant. The $d$ and $q$ currents correspond to the field current and the armature current of DCM respectively.

However, the use of such direct speed sensor induces additional electronics, extra wiring, extra space, frequent maintenance, careful mounting and default probability. The use of speed sensor has problems occurred, mainly in harsh environment. Moreover, the speed sensor is sensitive for electromagnetic noise in hostile environments and has a limited temperature range. To avoid mechanical sensor (speed and position) of IM, several methods called sensorless vector control is used. In order to simplify the hardware, the speed sensorless vector control is studied and applied in practical uses.

In order to improve the performance of induction motor control without speed sensor, many model reference adaptive system (MRAS) based methods are studied. Representative speed estimation is MRAS based method proposed by Schauder [1]. Figure 1.1 shows the two independent simulators are constructed to estimate the component of the rotor flux vector. Hence, voltage model does not involve the quantity of rotor electrical-angular speed, this model is used as a reference model, and current model which does involve rotor electrical-angular speed is used as an adjustable model. The errors between the states of the two models are used to derive a proper adaptation mechanism that generates the estimation of rotor electrical-angular speed for the adjustable model. The terminal currents and voltages are measured and transformed to $\alpha – \beta$ variables. However, the vector control system using MRAS speed estimate is composed independently. Therefore, the system becomes complex. The speed estimation using flux simulator is
used for flux observer-based field-orientation (FOFO) controller [2]. The motor speed is estimated by comparing the output fluxes of two simulators. Its estimated speed is in good agreement with actual speed at high speed, but it has transient error in the lower speed.

The full-order observer is proposed in [3], [5], [22], [26]-[29]. Kubota estimates the $\alpha – \beta$ stator currents and $\alpha – \beta$ rotor fluxes in stationary reference frame [3], [5], [22]. The speed is estimated by using these variables. The vector rotator calculates the stator current command by using information from the flux angle from the observer. The influence of stator resistance variation on the speed estimation and the torque control at low speed region is minimized by identifying it.

The flux observer based sensorless system has an unstable region in the regenerating mode at low speeds. To remedy the problem, Hinkkanen proposed a modified speed-adaptation law [28]. Instead of using only the current estimation error perpendicular to the estimated flux, the parallel component is also exploited in the regenerating mode.

![Figure 1.1 Model reference adaptive system (MRAS) proposed by Schauder [1].]
Design strategy of both observer gains and speed estimation gains for an adaptive full-order observer based sensorless system is necessary issue to assure the stability and the tracking performance. Suwankawin proposed a design of observer gains to achieve the stability over the whole operation especially in the low-speed region, including the regenerating operation [26]. The speed estimation gains are designed by considering the ramp response of the speed estimator. There is still an unstable region of experimental result in plugging region.

Tursini proposed an adaptive speed-sensorless field-oriented control of an induction motor, based on a sliding-mode observer [24]. Using the voltage equation, the observer computes the stator current and the rotor fluxes in $\alpha–\beta$ stationary reference frame. In the case of sliding-mode observer, it is considered that the observer gains are very large. The rotor speed estimation is obtained by using relation with a Lyapunov function. The system performance with different observer gains and the influence of the motor parameters deviations are shown. Sliding-mode full-order observer also are reported in [25], [30].

Tsuji proposed a speed estimation method by using $q$–axis flux [15], [39], [40]. This method is based on a reduce order flux observer and the adaptive control method. The $q$ – axis rotor flux converges to zero for speed estimation. To solve the pure integral problem of flux computation using the voltage model, a flux-observer modified by a current model is applied. The method is constructed by using synchronously rotating reference frame which is on the flux of current model. The fluxes from the current model becomes simple by defining $d–q$ axis, and all fluxes quantities of current model and voltage model are in DC form. By using DC form, the system stability is easily analyzed. Fig.1.2 shows the system proposed by Tsuji. The $q$ – axis
rotor flux is used to estimate the rotor speed by using PI controller. The flux is an output of flux-observer.

Another reduced-order observer-based sensorless method are discussed by Harnefors [31], [32]. By using the flux electromotive forces (EMF) of the voltage model and current model, the reduce-order observer for the rotor flux is considered. The rotor speed is estimated by the EMF and rotor flux. The reduced-order observer is simpler than the full-order observer.

Manipulated design of the observer can improve the stability of the sensorless system even at low speed regenerating operation. However, the configurations of these systems are relatively complicated. It is because the MRAS based methods need state observer and many PI controllers ($d – q$ currents, speed and speed estimation). On the other hand, some simplified speed sensorless vector controls are proposed.

![Flux observer proposed by Tsuji [39]](image)

*Fig. 1.2 Flux observer proposed by Tsuji [39].*
Simplifying the system configuration by removing the current regulators is proposed [33], and the stability is improved by adding a flux-stabilizing controller using derivative of magnetic current [34]. However, these papers have no information about the stability of regenerating mode. A sensorless method using the induced $d$–axis and $q$–axis voltage obtained by a voltage model has been proposed [35], and a similar method is applied to railway vehicle traction [36]. However, the stable region is not clear in these papers. Furthermore, a primary flux control method are proposed in [17], [37] and the stability is improved at regenerating mode [38]. Simple method of stator – flux orientation is proposed in [23]. Some survey papers for IM sensorless control systems are reported in [4], [9], [18]. Parameter estimation of stator and rotor resistances are also important problem for the sensorless systems [6], [11].

In this thesis, a new simplified speed-sensorless vector control method of IM based on $q$–axis rotor flux is proposed [41] – [44]. A flux vector is obtained from voltage model, in which the derivative term is neglected. A flux angle of a current model must be aligned with the flux angle of voltage model. Since the output voltage of $d$–axis PI current controller is used for the flux angle estimation and speed control ($q$–axis voltage control), the system is simplified and stabilized at regenerating mode [41]. In conventional simplified methods, this scheme is not reported. A linear model of the proposed system in the state space equation is obtained to study the system stability by showing root loci. By virtue of the stability analysis, we can design the parameters of controller. The nonlinear simulation and experimental results of the proposed system show stable transient responses in both motoring and regenerating modes [43]. In order to improve the system stability at plugging region, a PI speed controller is studied instead of original I (integral) controller [42]. Furthermore, a simplified sensorless system that
uses a PI $q$– axis current controller and estimates the rotor speed is studied and compared [44].

1.2 Contents of Chapters

This thesis is divided into five chapters with the arrangement:

In chapter 1, the background and the purpose of this research are explained and the contents of chapters are specified.

Chapter 2 describes a space vector representation of induction motor. A three-phase mathematical model of induction motor is transformed to a $d$ – $q$ model by using two-axis theory. This chapter introduces a non-linear model and a linear model of induction motor. The linear model of induction motor is derived by considering small perturbations at a steady state operating point.

In chapter 3, new simplified speed-sensorless vector control methods of IM based on rotor flux linkage are proposed. The two simplified sensorless systems are called as system A and system B. The main difference is the presence (in system A) or the absence (in system B) of $q$ – axis PI current controller. In system A, the angular frequency of rotor flux is estimated to bring $q$–axis flux to zero by using PI controller. The $q$ – axis flux is obtained by the output of $d$ – axis PI current controller with a non-interference control. The rotor speed is computed by subtracting a slip speed from the angular frequency. Flux angle is obtained by integrating the angular frequency. When $q$ – axis flux is larger (smaller) than zero and rotor flux is leading (lagging) than $d$ – axis, the controller must increase (decrease) the value of flux frequency. In system B, the computation of $q$ – axis flux is as same as the system A. The angular frequency of rotor flux is computed by adding the $q$ – axis flux to the speed command. The $q$ – axis flux is also used to control the rotor speed, by adjusting the $q$ – axis voltage. Non-linear models are derived
in both proposed systems A and B. From these non-linear models, linear models of the systems are derived in state space equations. The selection of PI current and speed controllers gains are outlined.

Chapter 4 demonstrates systems stability by showing the root loci obtained by the linear models, the transient responses of simulation results and the stable regions. The performance of both systems A and B are compared by using linear models and non-linear models. Transient responses of linear and non-linear models are computed and compared. Since both responses are almost same around a steady state operating point, the validity of the linear models are confirmed. By using the proposed methods, not only the motoring operation but also the low speed regenerating operation can be stabilized. Quick torque and speed responses of nonlinear models are obtained in both systems A and B. A digital signal processor based PWM inverter fed IM system is equipped and tested. It is confirmed that the experimental results are very close to those of simulation. Therefore, the effectiveness of the proposed methods are also demonstrated experimentally. It is considered that the system B is superior to the system A because its simple structure.

Chapter 5 is the conclusions presented in this thesis.
Chapter 2
Models of Induction Motor

2.1 Space Vector Representation of Induction Motor

For convenience of proposed systems analysis, the models of IM are outlined. In order to analyze the IM, it must be embodied in three-phase mathematical model. From three-phase model, the IM is simplified into two-phase by using the two-axis transformation [12], [13], [14].

To show windings configuration and to calculate inductances, the cross-section of simplified three-phase IM with two-poles is shown in Fig.2.1. The winding configurations are assumed the same to each phases of both the stator and the rotor. The winding of three-phase IM is separated by 120 electrical degrees with respect to each other as shown by the coils \( a-a^* \), \( b-b^* \) and, \( c-c^* \).

Fig.2.2 shows a circuit model of the three-phase IM with two-poles. An equivalent three-phase winding is used in case of short-circuited squirrel-cage rotor. The stator and rotor windings are wye connected. The \( a_s \), \( b_s \) and \( c_s \) mean the axes of stator \( a \), \( b \) and \( c \) phase winding respectively. The angle \( \theta_r \) is an angular displacement of \( a \)-phase rotor winding axis from the axis \( a_s \). By using the rotational angular velocity \( \omega_r \), the angle \( \theta_r \) is expressed as

\[
\theta_r = \int_0^t \omega_r \, dt + \theta_r (0) \quad \text{......................................................... (2.1)}
\]
The stator winding of IM is fundamentally the same as for a synchronous motor. In the below equations, the subscript $s$ denotes variables and parameters associated with the stator circuits, and the subscript $r$ denotes variables and parameters associated with the rotor circuits.

In Fig.2.2 (b), the leakage inductance at stator and rotor windings are $l_s$ and $l_r$ respectively. The mutual inductances $M_{sr}$ has the relation with self-inductances $L_{ss}$ and $L_{rr}$ as follows:

\[ L_{ss} L_{rr} = M_{sr}^2 \]  \hspace{1cm} \text{(2.2)}

Fig. 2.1 Three-phase stator and rotor windings of IM.
(a) Definitions of axes.

(b) Definition of circuit parameters.

Fig.2.2 Circuit model of three-phase IM with \(d-q\) axis.
The mutual inductance between stator and rotor changes when the rotor moves by the angle $\theta_r$. The rotor $a$-phase current $i_{ra}$ generates flux in the stator $a$-phase winding as shown in Fig 2.3. If $\theta_r = 0$, all flux by $i_{ra}$ passes $L_{ss}$ and the mutual inductance is $M_{sr}$. As the cosine component of the flux by $i_{ra}$ passes $L_{ss}$, the mutual inductance becomes $M_{sr} \cos \theta_r$. The flux linkages in three-phase windings of the stator and the rotor are defined as $\psi_{sa}$, $\psi_{sb}$, $\psi_{sc}$ and $\psi_{ra}$, $\psi_{rb}$, $\psi_{rc}$ respectively. By considering the cosine component of the angle of the winding, we can obtain the flux linkages of $\psi_{sa}$, $\psi_{sb}$, $\psi_{sc}$, $\psi_{ra}$, $\psi_{rb}$, $\psi_{rc}$ as follows:

$$
\begin{bmatrix}
\psi_{sa} \\
\psi_{sb} \\
\psi_{sc}
\end{bmatrix} =
\begin{bmatrix}
l_s + L_{ss} & -L_{ss}/2 & -L_{ss}/2 \\
-L_{ss}/2 & l_s + L_{ss} & -L_{ss}/2 \\
-L_{ss}/2 & -L_{ss}/2 & l_s + L_{ss}
\end{bmatrix}
\begin{bmatrix}
i_{sa} \\
i_{sb} \\
i_{sc}
\end{bmatrix}
+ M_{sr}
\begin{bmatrix}
\cos \theta_r & \cos \left(\theta_r + \frac{2}{3} \pi\right) & \cos \left(\theta_r - \frac{2}{3} \pi\right) \\
\cos \left(\theta_r - \frac{2}{3} \pi\right) & \cos \theta_r & \cos \left(\theta_r + \frac{2}{3} \pi\right) \\
\cos \left(\theta_r + \frac{2}{3} \pi\right) & \cos \left(\theta_r - \frac{2}{3} \pi\right) & \cos \theta_r
\end{bmatrix}
\begin{bmatrix}
i_{ra} \\
i_{rb} \\
i_{rc}
\end{bmatrix}
ê.... (2.3)
\[
\begin{align*}
\begin{bmatrix}
\psi_{ra} \\
\psi_{rb} \\
\psi_{rc}
\end{bmatrix}
&= 
\begin{bmatrix}
L_r + L_{rr} & -L_{rr}/2 & -L_{rr}/2 \\
-L_{rr}/2 & L_r + L_{rr} & -L_{rr}/2 \\
-L_{rr}/2 & -L_{rr}/2 & L_r + L_{rr}
\end{bmatrix}
\begin{bmatrix}
i_{ra} \\
i_{rb} \\
i_{rc}
\end{bmatrix} \\
+ M_{sr} 
&= 
\begin{bmatrix}
\cos \theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\
\cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos \theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\
\cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos \theta_r
\end{bmatrix}
\begin{bmatrix}
i_{sa} \\
i_{sb} \\
i_{sc}
\end{bmatrix}
\end{align*}
\]

By using the flux linkages, the following voltage equations are obtained for stator and rotor windings.

\[
\begin{align*}
\begin{bmatrix}
e_{sa} \\
e_{sb} \\
e_{sc}
\end{bmatrix}
&= 
\begin{bmatrix}
i_{sa} \\
i_{sb} \\
i_{sc}
\end{bmatrix} + p
\begin{bmatrix}
\psi_{sa} \\
\psi_{sb} \\
\psi_{sc}
\end{bmatrix} \\
&= 
\begin{bmatrix}
e_{ra} \\
e_{rb} \\
e_{rc}
\end{bmatrix} = 
\begin{bmatrix}
i_{ra} \\
i_{rb} \\
i_{rc}
\end{bmatrix} + p
\begin{bmatrix}
\psi_{ra} \\
\psi_{rb} \\
\psi_{rc}
\end{bmatrix}
\end{align*}
\]

where, \( p \) means \( d/dt \), \( R_s \) and \( R_r \) is stator resistance and rotor resistance respectively.

All flux by \( i_{ra} \) passes \( L_{ss} \)

Mutual inductance \( M_{sr} \)

\[
\begin{align*}
\cos \theta_r \text{ component of flux} \\
\text{by } i_{ra} \text{ passes } L_{ss}
\end{align*}
\]

Mutual inductance \( M_{sr} \cos \theta_r \) \( 0 \leq \theta_r \leq 2\pi \)

Fig.2.3    Mutual inductance of windings.
In stationary reference frame, by considering cosine components of the angle that is formed between the winding axis and $\alpha - \beta$ axis as shown in Fig.2.4, the $\alpha - \beta$ components $f_{sa}$ and $f_{sb}$ for the stator side are obtained. The $f$ means voltage, current and flux linkage. The space vector is defined as

$$\dot{s} = f_{sa} + j f_{sb}$$

(2.7)

Fig.2.4 $\alpha - \beta$ stationary reference frame.

then,

$$\dot{s} = \sqrt{\frac{2}{3}} \left( f_{sa} + e^{\frac{j 2\pi}{3}} f_{sb} + e^{-\frac{j 2\pi}{3}} f_{sc} \right)$$

(2.8)

For the rotor side, the following equations are defined:

$$
\begin{bmatrix}
    f_{ra} \\
    f_{rb} \\
    f_{rc}
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
    \cos \theta_r & \cos(\theta_r + \frac{2}{3} \pi) & \cos(\theta_r - \frac{2}{3} \pi) \\
    \sin \theta_r & \sin(\theta_r + \frac{2}{3} \pi) & \sin(\theta_r - \frac{2}{3} \pi)
\end{bmatrix}
\begin{bmatrix}
    f_{ra} \\
    f_{rb} \\
    f_{rc}
\end{bmatrix}
$$

(2.9)
By using $f_{r\alpha}$ and $f_{r\beta}$, the space vector is defined as

$$\dot{f}_r = f_{r\alpha} + j f_{r\beta}$$ \hspace{1cm} \text{............................................................... (2.10)}$$

then,

$$\dot{f}_r = \sqrt{\frac{2}{3}} e^{j\theta_r} \left( f_{ra} + e^{j\frac{2}{3}\pi} f_{rb} + e^{-j\frac{2}{3}\pi} f_{rc} \right) \hspace{1cm} \text{......................... (2.11)}$$

The space vector of stator voltage is expressed as

$$\dot{e}_s = \sqrt{\frac{2}{3}} (e_{sa} + e^{j\frac{2}{3}\pi} e_{sb} + e^{-j\frac{2}{3}\pi} e_{sc}) \hspace{1cm} \text{..................................................... (2.12)}$$

From (2.5), we have

$$\dot{e}_s = R_s \dot{i}_s + p \dot{\psi}_s \hspace{1cm} \text{................................................................. (2.13)}$$

The stator flux linkage is computed from (2.3) as

$$\dot{\psi}_s = \sqrt{\frac{2}{3}} (\psi_{sa} + e^{j\frac{2}{3}\pi} \psi_{sb} + e^{-j\frac{2}{3}\pi} \psi_{sc})$$

$$= (i_s + L_{ss}) \dot{i}_s - \frac{L_{ss}}{2} \left\{ i_{sb} + i_{sc} \right\} + \frac{\sqrt{2}}{3} \left\{ i_{sa} + i_{sc} \right\} \right\}$$

$$+ \frac{\sqrt{2}}{3} M_{sr} \cos \theta_r \left\{ i_{ra} + e^{j\frac{2}{3}\pi} i_{rb} + e^{-j\frac{2}{3}\pi} i_{rc} \right\}$$

$$+ \frac{\sqrt{2}}{3} M_{sr} \cos \left( \theta_r + \frac{2}{3}\pi \right) \left\{ i_{rb} + e^{j\frac{2}{3}\pi} i_{rc} + e^{-j\frac{2}{3}\pi} i_{ra} \right\}$$

$$+ \frac{\sqrt{2}}{3} M_{sr} \cos \left( \theta_r - \frac{2}{3}\pi \right) \left\{ i_{rc} + e^{j\frac{2}{3}\pi} i_{ra} + e^{-j\frac{2}{3}\pi} i_{rb} \right\}$$
By using \( i_{sa} + i_{sb} = -i_{sc}, \ i_{sb} + i_{sc} = -i_{sa} \) and \( i_{sc} + i_{sa} = -i_{sb} \), we have

\[
\dot{\psi}_s = \left( l_s + \frac{3}{2} L_{ss} \right) i_s + M_{sr} \left\{ i_r e^{-j\theta_r} \cos\theta_r + i_r e^{-j\theta_r} e^{-j\frac{2}{3}\pi} \cos\left(\theta_r + \frac{2}{3}\pi\right) + i_r e^{-j\theta_r} e^{j\frac{2}{3}\pi} \cos\left(\theta_r - \frac{2}{3}\pi\right) \right\}.
\]

\[
= \left( l_s + \frac{3}{2} L_{ss} \right) i_s + M_{sr} i_r e^{-j\theta_r} \cdot \frac{3}{2} e^{j\theta_r},
\]

\[
= \left( l_s + \frac{3}{2} L_{ss} \right) i_s + \frac{3}{2} M_{sr} i_r \quad \text{.............................} \quad (2.14)
\]

By setting \( L_s = \frac{3}{2} L_{ss} + l_s \) and \( M = \frac{3}{2} M_{sr} \), (2.14) can be written as:

\[
\dot{\psi}_s = L_s \dot{i}_s + M \dot{i}_r \quad \text{........................................} \quad (2.15)
\]

Substituting (2.15) into (2.13), we have

\[
\dot{\psi}_s = (R_s + pL_s) \dot{i}_s + pM \dot{i}_r \quad \text{........................................} \quad (2.16)
\]

The space vector of rotor is expressed as follows:

\[
\dot{\psi}_r = \sqrt{\frac{2}{3}} e^{j\theta_r} \left( \psi_{ra} + e^{j\frac{2}{3}\pi} \psi_{rb} + e^{j\frac{2}{3}\pi} \psi_{rc} \right) \quad \text{.............................} \quad (2.17)
\]

From (2.6), we have

\[
\dot{\psi}_r = R_r \dot{i}_r + \sqrt{\frac{2}{3}} e^{j\theta_r} p \left( \psi_{ra} + e^{j\frac{2}{3}\pi} \psi_{rb} + e^{j\frac{2}{3}\pi} \psi_{rc} \right) \quad \text{...............} \quad (2.18)
\]

hence,

\[
\dot{\psi}_r = R_r \dot{i}_r + e^{j\theta_r} p \left( \dot{\psi}_r e^{-j\theta_r} \right) \quad \text{.............................} \quad (2.19)
\]
The space vector of the rotor flux linkage is computed from (2.4) as

\[
\psi_r = \frac{2}{\sqrt{3}} e^{j\theta_r} \left( \psi_{ra} + e^{\frac{2\pi}{3}} \psi_{rb} + e^{-\frac{2\pi}{3}} \psi_{rc} \right)
\]

\[
= \left( l_r + \frac{3}{2} L_{rr} \right) i_r + \frac{2}{\sqrt{3}} e^{j\theta_r} M_{sr} \left\{ \cos \theta_r \left( i_{sa} + e^{\frac{2\pi}{3}} i_{sb} + e^{-\frac{2\pi}{3}} i_{sc} \right) \\
+ \cos \left( \theta_r - \frac{2\pi}{3} \right) \left( i_{sb} + e^{\frac{2\pi}{3}} i_{sc} + e^{-\frac{2\pi}{3}} i_{sa} \right) \\
+ \cos \left( \theta_r + \frac{2\pi}{3} \right) \left( i_{sc} + e^{\frac{2\pi}{3}} i_{sa} + e^{-\frac{2\pi}{3}} i_{sb} \right) \right\}
\]

................................. (2.20)

therefore,

\[
\psi_r = \left( l_r + \frac{3}{2} L_{rr} \right) i_r + \frac{3}{2} M_{sr} i_s
\]

................................. (2.21)

By setting \( L_r = \frac{3}{2} L_{rr} + l_r \), the rotor flux linkage is expressed as

\[
\psi_r = L_r i_r + M i_s
\]

................................. (2.22)

From (2.19), we have

\[
\dot{\psi}_r = R_r \dot{i}_r + (p - j \omega_r) L_r \dot{i}_r + (p - j \omega_r) M \dot{i}_s
\]

................................. (2.23)

The space vector equation of IM from equations (2.16) and (2.23) is expressed as

\[
\begin{bmatrix}
\dot{\varphi}_s \\
\dot{\varphi}_r 
\end{bmatrix} =
\begin{bmatrix}
R_s + L_s p & M p \\
(p - j \omega_r) M & R_r + (p - j \omega_r) L_r 
\end{bmatrix}
\begin{bmatrix}
i_s \\
i_r 
\end{bmatrix}
\]

................................. (2.24)
The voltages $\dot{e}_s$ and $\dot{e}_r$ in (2.24) are divided into real and imaginary parts to have $\alpha - \beta$ stationary reference model as

\[
\begin{bmatrix}
  e_{s\alpha} \\
e_{s\beta} \\
e_{r\alpha} \\
e_{r\beta}
\end{bmatrix} =
\begin{bmatrix}
  R_s + L_s p & 0 & M_p & 0 \\
  0 & R_s + L_s p & 0 & M_p \\
  M_p & \omega_r M & R_r + L_r p & \omega_r L_r \\
-\omega_r M & M_p & -\omega_r L_r & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
i_{s\alpha} \\
i_{s\beta} \\
i_{r\alpha} \\
i_{r\beta}
\end{bmatrix}
\]

............... (2.25)

### 2.2 $d-q$ Model

We consider a rotating $d-q$ axis shown in Fig.2.5. Where, $\theta$ is the angle between $\alpha$ – axis and $d$ – axis. The $d$ – axis rotates at an arbitrary angular velocity $\omega$. Then, $\theta$ can be expressed by the following equation:

\[
\theta = \int_0^t \omega \, dt + \theta(0) \quad \text{.................................................................} \quad (2.26)
\]

The $\alpha - \beta$ stationary reference frame quantities are transformed into rotating reference frame quantities using $d-q$ transformation as follows:

\[
\dot{j}_{sdq} = e^{-j\theta} \left( f_{s\alpha} + j f_{s\beta} \right) = e^{-j\theta} \dot{j}_s \quad \text{..........................................................} \quad (2.27)
\]

\[
\dot{j}_{rdq} = e^{-j\theta} \dot{j}_r \quad \text{..........................................................} \quad (2.28)
\]
Fig. 2.5  Reference frame transformation from $\alpha - \beta$ to $d - q$.

Hence, the transformation of reference frame from $\alpha - \beta$ to $d - q$ axis can be expressed as

\[
\begin{bmatrix}
    f_{sd} \\
    f_{sq}
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
    f_{s\alpha} \\
    f_{s\beta}
\end{bmatrix}
\]

........................................     (2.29)

From (2.16) and (2.27), we have

\[
\dot{e}_{sdq} = e^{-j\theta} \dot{e}_s
\]

\[
= e^{-j\theta} \left( (R_s + pL_s)\dot{i}_s + pM \dot{i}_r \right)
\]

\[
= R_s \dot{i}_{sdq} + e^{-j\theta} L_s p \left( e^{j\theta} \dot{i}_{sdq} \right) + e^{-j\theta} M p \left( e^{j\theta} \dot{i}_{rdq} \right)
\]

\[
= R_s \dot{i}_{sdq} + L_s p \dot{i}_{sdq} + j \omega L_s \dot{i}_{sdq} + M \ p \dot{i}_{rdq} + j \omega M \ p \dot{i}_{rdq} \quad \text{......... (2.30)}
\]
From (2.23) and (2.28), we have

\[
\dot{e}_{rdq} = e^{-j\theta} \dot{e}_r
\]
\[
= e^{-j\theta} \left( (p - j \omega_r) M i_s + \left( R_r + (p - j \omega_r) L_r \right) i_r \right)
\]
\[
= e^{-j\theta} R_r \left( e^{j\theta} i_{rdq} \right) + e^{-j\theta} M p \left( e^{j\theta} i_{sdq} \right) - e^{-j\theta} j \omega_r M \left( e^{j\theta} i_{rdq} \right)
\]
\[
+ e^{-j\theta} L_r p \left( e^{j\theta} i_{rdq} \right) - e^{-j\theta} j \omega_r L_r \left( e^{j\theta} i_{rdq} \right)
\]
\[
= R_r i_{rdq} + M p i_{sdq} + j \omega M i_{sdq} - j \omega_r M i_{rdq}
\]
\[
+ L_r p i_{rdq} + j \omega L_r i_{rdq} - j \omega_r L_r i_{rdq} \hspace{1cm} \text{..................................} \hspace{1cm} (2.31)
\]

The voltage equations of (2.30) and (2.31) can be written in matrix form as

\[
\begin{bmatrix}
\dot{e}_{sdq} \\
\dot{e}_{rdq}
\end{bmatrix}
= \begin{bmatrix}
R_s + L_s p + j \omega L_s & Mp + j \omega M \\
Mp + j(\omega - \omega_r) M + R_r + L_r p + j(\omega - \omega_r) L_r
\end{bmatrix}
\begin{bmatrix}
i_{sdq} \\
i_{rdq}
\end{bmatrix} \hspace{1cm} \text{...........} \hspace{1cm} (2.32)
\]

Hence, \( \dot{e}_{sdq} \) and \( \dot{e}_{rdq} \) can be divided into real and imaginary parts as follows:

\[
\begin{bmatrix}
e_{sd} \\
e_{sq} \\
e_{rd} \\
e_{rq}
\end{bmatrix}
= \begin{bmatrix}
R_s + L_s p & -\omega L_s & Mp & -\omega M \\
\omega L_s & R_s + L_s p & \omega M & Mp \\
Mp & -(\omega - \omega_r) M & R_r + L_r p & -(\omega - \omega_r) L_r \\
(\omega - \omega_r) M & Mp & (\omega - \omega_r) L_r & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
i_{rd} \\
i_{rq}
\end{bmatrix} \hspace{1cm} \text{...........} \hspace{1cm} (2.33)
\]

The relation between \( \dot{j}_{sdq} \) and three-phase variables can be expressed as

\[
\dot{j}_{sdq} = e^{-j\theta} \sqrt{\frac{2}{3}} \left( f_{sa} + e^{\frac{2\pi}{3}} f_{sb} + e^{-\frac{2\pi}{3}} f_{sc} \right) \hspace{1cm} \text{..................} \hspace{1cm} (2.34)
\]
Therefore, we have

\[
\begin{bmatrix}
    f_{sd} \\
    f_{sq}
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
    \cos \theta & \cos \left( \theta - \frac{2}{3} \pi \right) & \cos \left( \theta + \frac{2}{3} \pi \right) \\
    -\sin \theta & -\sin \left( \theta - \frac{2}{3} \pi \right) & -\sin \left( \theta + \frac{2}{3} \pi \right)
\end{bmatrix}
\begin{bmatrix}
    f_{sa} \\
    f_{sb} \\
    f_{sc}
\end{bmatrix} \tag{2.35}
\]

The relation between \( \dot{f}_{rdq} \) and three-phase variables can be expressed as

\[
\dot{f}_{rdq} = e^{-j\theta} \frac{2}{\sqrt{3}} \left( e^{j\frac{2}{3} \pi} f_{ra} + e^{-j\frac{2}{3} \pi} f_{rb} \right) \tag{2.36}
\]

Therefore, we have

\[
\begin{bmatrix}
    f_{rd} \\
    f_{rq}
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
    \cos \beta & \cos \left( \beta - \frac{2}{3} \pi \right) & \cos \left( \beta + \frac{2}{3} \pi \right) \\
    -\sin \beta & -\sin \left( \beta - \frac{2}{3} \pi \right) & -\sin \left( \beta + \frac{2}{3} \pi \right)
\end{bmatrix}
\begin{bmatrix}
    f_{ra} \\
    f_{rb} \\
    f_{rc}
\end{bmatrix} \tag{2.37}
\]

From (2.14) and (2.21), the flux linkages are expressed as

\[
\begin{bmatrix}
    \psi_{sd} \\
    \psi_{sq} \\
    \psi_{rd} \\
    \psi_{rq}
\end{bmatrix} = \begin{bmatrix}
    L_s & 0 & M & 0 \\
    0 & L_s & 0 & M \\
    M & 0 & L_r & 0 \\
    0 & M & 0 & L_r
\end{bmatrix}
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{rd} \\
    i_{rq}
\end{bmatrix} \tag{2.38}
\]
Since the rotor is squirrel-cage short-circuited type and the voltage equations are derived from (2.33) and (2.38) by using rotor flux linkage as follows:

\[
\begin{bmatrix}
  e_{sd} \\
e_{sq} \\
 0 \\
 0
\end{bmatrix} =
\begin{bmatrix}
  R_s + \sigma L_s p & -\omega \sigma L_s & \frac{M}{L_p} p & -\frac{\omega M}{L_p} \\
  \omega \sigma L_s & R_s + \sigma L_s p & \frac{\omega M}{L_p} p & \frac{M}{L_p} p \\
  \frac{M}{\tau_r} & 0 & \frac{1}{\tau_r} + p & -(\omega - \omega_r) \\
  0 & \frac{M}{\tau_r} & (\omega - \omega_r) & \frac{1}{\tau_r} + p
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
i_{ra} \\
i_{rb}
\end{bmatrix}
\] .... (2.39)

where,

\[\sigma = 1 - \frac{M^2}{L_s L_r} \quad \text{and} \quad \tau_r = \frac{L_r}{R_r} \quad \text{(rotor open circuit time constant)}.
\]

### 2.3 Electromagnetic Torque

To determine the torque developed in IM, we can calculate the input power as follows:

\[
P_{in} = e_{sa} i_{sa} + e_{sb} i_{sb} + e_{sc} i_{sc} + e_{ra} i_{ra} + e_{rb} i_{rb} + e_{rc} i_{rc}
\]

\[
= [e_{sa} \quad e_{sb} \quad e_{sc}] \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + [e_{ra} \quad e_{rb} \quad e_{rc}] \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix}
\]

\[
= (C_s^T e_s)^T C_s^T i_s + (C_r^T e_r)^T C_r^T i_r
\]

\[
= e_s^T C_s C_s^T i_s + e_r^T C_r C_r^T i_r
\]

\[
= e_s^T i_s + e_r^T i_r
\] ................................................................. (2.40)
where,

$$C_s = \sqrt{2} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}, \quad C_r = \sqrt{2} \begin{bmatrix} \cos \theta_r & \cos \left(\frac{\theta_r + 2\pi}{2}\right) & \cos \left(\frac{\theta_r - 2\pi}{2}\right) \\ \sin \theta_r & \sin \left(\frac{\theta_r + 2\pi}{2}\right) & \sin \left(\frac{\theta_r - 2\pi}{2}\right) \end{bmatrix}$$

$$e_s = \begin{bmatrix} e_s \alpha \\ e_s \beta \end{bmatrix}, \quad i_s = \begin{bmatrix} i_s \alpha \\ i_s \beta \end{bmatrix}, \quad e_r = \begin{bmatrix} e_r \alpha \\ e_r \beta \end{bmatrix}, \quad i_r = \begin{bmatrix} i_r \alpha \\ i_r \beta \end{bmatrix}$$

Therefore,

$$P_{in} = e_s \alpha i_s + e_s \beta i_s + e_r \alpha i_r + e_r \beta i_r$$

............................... (2.41)

By using space vector, $P_{in}$ is expressed as

$$P_{in} = \text{Re} \left( \hat{e}_s \hat{i}_s^* + \hat{e}_r \hat{i}_r^* \right)$$

$$= \text{Re} \left\{ \hat{i}_s^* \left( R_s \hat{i}_s + L_s p \hat{i}_s + M p \hat{i}_r \right) + \hat{i}_r^* \left( M p \hat{i}_s - j \omega_r M \hat{i}_s + R_r \hat{i}_r + L_r p \hat{i}_r - j \omega_r L_r \hat{i}_r \right) \right\}$$

$$= R_s |\hat{i}_s|^2 + R_r |\hat{i}_r|^2 + \frac{1}{2} L_s p |\hat{i}_s|^2 + \frac{1}{2} L_r p |\hat{i}_r|^2$$

$$+ \frac{1}{2} M p \left( i_s^* i_r + i_s i_r^* \right) + \text{Re} \left( -j \omega_r M \hat{i}_s \hat{i}_r^* \right)$$

............... (2.42)

The each term of (2.42) is considered as

$R_s |\hat{i}_s|^2$: primary copper loss

$R_r |\hat{i}_r|^2$: secondary copper loss

$\frac{1}{2} L_s p |\hat{i}_s|^2 + \frac{1}{2} L_r p |\hat{i}_r|^2$: reactive power of self-inductance
\[ \frac{1}{2} M p (i_s^* i_r + i_s i_r^*) : \text{reactive power of mutual inductance} \]

\[ \text{Re}(j \omega_r M i_s^* i_r^*) : \text{mechanical power} \]

Hence, the electromagnetic torque \( \tau_e \) is given by

\[
\tau_e = \frac{\text{Re}\left\{ j \omega_r M (i_s^* i_r^*) \right\}}{2p} \]

\[ = \frac{P}{2} M \text{Im}(i_s^* i_r^*) \]

\[
\tau_e = \frac{P}{2} M (i_{sq} i_{rd} - i_{sd} i_{rq}) = \frac{P}{2} M \left( i_{sq} \psi_{rd} - i_{sd} \psi_{rq} \right) \]

\[
\text{where, } P : \text{number of poles.} \]

Therefore,

\[
\tau_e = \frac{P}{2} M (i_{sq} i_{rd} - i_{sd} i_{rq}) = \frac{P}{2} M \left( i_{sq} \psi_{rd} - i_{sd} \psi_{rq} \right) \]

\[
\text{......} \quad (2.43) \]

\[ 2.4 \quad \text{Non–linear State Equation} \]

In order to compute the transient responses, we derive a non-linear state equation from (2.39). The non-linear state equation is expressed as

\[
p i_{sd} = \frac{e_{sd}}{\sigma L_s} - \left( \frac{R_s}{\sigma L_s} + \frac{M^2}{\sigma L_s L_r \tau_r} \right) i_{sd} + \omega i_{sq} + \frac{M \psi_{rd}}{\sigma L_s L_r \tau_r} + \frac{\omega_r M \psi_{rq}}{\sigma L_s L_r \tau_r} \]

\[
p i_{sq} = \frac{e_{sq}}{\sigma L_s} - \omega i_{sd} - \left( \frac{R_s}{\sigma L_s} + \frac{M^2}{\sigma L_s L_r \tau_r} \right) i_{sq} - \frac{\omega_r M \psi_{rd}}{\sigma L_s L_r \tau_r} + \frac{M \psi_{rq}}{\sigma L_s L_r \tau_r} \]

\[
p \psi_{rd} = \frac{M}{\tau_r} i_{sd} - \frac{1}{\tau_r} \psi_{rd} + (\omega - \omega_r) \psi_{rq} \]

\[
p \psi_{rq} = \frac{M}{\tau_r} i_{sq} - (\omega - \omega_r) \psi_{rd} - \frac{1}{\tau_r} \psi_{rq} \]

\[
\text{......} \quad (2.44) \]

\[
\text{......} \quad (2.45) \]

\[
\text{......} \quad (2.46) \]

\[
\text{......} \quad (2.47) \]

\[
\text{......} \quad (2.48) \]
The mechanical equation of motion is expressed as

\[ p\omega_r = \frac{P}{2J}(\tau_e - T_L) = \frac{M}{4J L_r} \left(i_{sq} \psi_{rd} - i_{sd} \psi_{rq}\right) - \frac{P}{2J} T_L \] ............ (2.49)

where, \( J \): inertia of the rotor plus load, \( T_L \): load torque.

The above equations are described by a non-linear state equation

\[ p x_s = f(x_s, u_s, T_L) \] ......................................................... (2.50)

where, \( x_s \) is state vector and \( u_s \) is input vector of IM.

\[ x_s = \begin{bmatrix} i_{sd}, i_{sq}, \psi_{rd}, \psi_{rq}, \omega_r \end{bmatrix}^T \] ........................................ (2.51)

\[ u_s = \begin{bmatrix} e_{sd}, e_{sq}, \omega \end{bmatrix}^T \] ..................................................... (2.52)

2.5 Linear State Equation

An essential problem connected to the modeling of the induction machine is the non-linearity of the equations that describe its operation. This non-linearity is caused by the voltage equations and the electromagnetic torque relation as well, due to the products between the state variables. When a control system is designed, it is very convenient to linearize the machine equations [7], [8].

The stability analysis of non-linear system is difficult in general. So, we derive a linear model of IM by considering small perturbation at a steady state operating point which is obtained by setting \( p = 0 \). The linear model of IM is derived as follows from (2.50):

\[ p\Delta x_s = A_s \Delta x_s + B_s \Delta u_s + B L \Delta T_L \] ........................................ (2.53)
where,

\[ \Delta x_s = \begin{bmatrix} \Delta i_{sd}, \Delta i_{sq}, \Delta \psi_{rd}, \Delta \psi_{rq}, \Delta \omega \end{bmatrix}^T \]

\[ \Delta u_s = \begin{bmatrix} \Delta e_{sd}, \Delta e_{sq}, \Delta \omega \end{bmatrix}^T \]

\[
A_s = \begin{bmatrix}
-a_1 & \omega & \frac{M}{\sigma L_s L_r \tau_r} & \frac{M \omega_r}{\sigma L_s L_r} & \frac{M \psi_{rq}}{\sigma L_s L_r} \\
-\omega & -a_1 & - \frac{M \omega_r}{\sigma L_s L_r} & \frac{M}{\sigma L_s L_r \tau_r} & - \frac{M \psi_{rd}}{\sigma L_s L_r} \\
\frac{M}{\tau_r} & 0 & - \frac{1}{\tau_r} & (\omega - \omega_r) & - \psi_{rq} \\
0 & \frac{M}{\tau_r} & -(\omega - \omega_r) & - \frac{1}{\tau_r} & \psi_{rd} \\
-a_2 \psi_{rq} & a_2 \psi_{rd} & a_2 i_{sq} & -a_2 i_{sd} & 0
\end{bmatrix}
\]

\[ a_1 = \frac{R_s}{\sigma L_s} + \frac{M^2}{\sigma L_s L_r \tau_r}, \quad a_2 = \frac{MP^2}{4JL_r} \]

\[
B_s = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & i_{sq} \\
0 & \frac{1}{\sigma L_s} & -i_{sd} \\
0 & 0 & \psi_{rq} \\
0 & 0 & -\psi_{rd} \\
0 & 0 & 0
\end{bmatrix}, \quad B_L = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\frac{P}{2J}
\end{bmatrix}
\]

(2.55)
Chapter 3

Speed Sensorless Vector Control Systems

3.1 Proposed System A

A speed sensorless vector control system in which the rotor speed of IM is estimated explicitly is proposed and analyzed to investigate the system stability. We call the sensorless system “system A”.

3.1.1 Block Diagram of System A

In order to simplify the controller and to stabilize the system at low speed regenerating operations, the speed sensorless vector control system A is proposed as shown in Fig.3.1. As described in (2.39), the $d-q$ rotating reference frame equations of IM are modified as

Voltage model:

\[ e_{sd}^* = (R_s^* + \sigma L_s p) i_{sd}^* - \omega^* \sigma L_s i_{sq} + \frac{M}{L_r} p \psi_{rd}^* - \frac{M}{L_r} \omega^* \psi_{rq}^* \]  
\[ \text{............... (3.1)} \]

\[ e_{sq}^* = \omega^* \sigma L_s i_{sd}^* + (R_s^* + \sigma L_s p) i_{sq} + \frac{M}{L_r} \omega^* \psi_{rd}^* + \frac{M}{L_r} p \psi_{rq}^* \]  
\[ \text{............... (3.2)} \]
Current model

\[
0 = -\frac{M}{\tau_r^*} i_{sd}^* + \left(\frac{1}{\tau_r^*} + p\right)\psi_{rd}^* - (\omega^* - \hat{\omega}_r)\psi_{rq}^* \quad \text{.......................... (3.3)}
\]

\[
0 = -\frac{M}{\tau_r^*} i_{sq}^* + (\omega^* - \hat{\omega}_r)\psi_{rd}^* + \left(\frac{1}{\tau_r^*} + p\right)\psi_{rq}^* \quad \text{.......................... (3.4)}
\]

Where, \( \tau_r^* = L_r/R_r^* \), \( \hat{\omega}_r \): estimated rotor speed.

The synchronously rotating angular frequency \( \omega^* \) of \( d-q \) reference frame is defined to satisfy that the rotor flux of the current model \( \psi_{rq}^* \) is zero.

Therefore,

\[
\psi_{rq}^* = 0 \quad \text{......................................................... (3.5)}
\]
This means that the $d - q$ axis is selected to synchronize the direction of the current model rotor flux. By assuming that the $d - axis current reference $i_{sd}^*$ is constant, from (3.3) we have

$$\psi_{rd}^* = Mi_{sd}^*$$ ................................................................. (3.6)

By substituting (3.5) and (3.6) into (3.4) we have

$$\omega^* - \hat{\omega}_r = \frac{i_{sq}}{\tau_r i_{sd}^*}$$ ................................................................. (3.7)

Therefore, the estimated slip speed $\omega_e$ is defined as

$$\omega_e = \frac{i_{sq}}{\tau_r i_{sd}^*}$$ ................................................................. (3.8)

The assumption of constant $i_{sd}^*$ causes that the $d - axis flux becomes constant described in (3.6). Therefore, we make the following assumption:

$$p\psi_{rd}^* = 0$$ ................................................................. (3.9)

By using (3.9), from (3.1) we have

$$e_{sd}^* = R_s i_{sd}^* - \omega^* \sigma L_s i_{sq} - \frac{\omega^* M}{L_r} \psi_{rq}^*$$ ................................................................. (3.10)

In (3.10), the induced voltage $e_{d}^*$ is defined as

$$e_{d}^* = -\frac{\omega^* M}{L_r} \psi_{rq}^*$$ ................................................................. (3.11)

In the proposed system, the induced voltage $e_{d}^*$ is computed by using the output voltage of $d - axis PI current controller. We can estimate the phase of rotor flux $\theta^*$ by changing $\omega^*$ to satisfy

$$\psi_{rq}^* = 0$$ ................................................................. (3.12)
Fig. 3.2 shows the rotor flux vector of current model $\psi^*_r$ and that of voltage model $\psi^*_r$. The $\psi^*_r$ is on the $d$-axis. When $\psi^*_{rq} > 0$, we must increase $\omega^*$ and we must decrease $\omega^*$ when $\psi^*_{rq} < 0$ to satisfy (3.11).

Because the $\psi^*_r$ is proportional to $e^*_d$ in (3.11), the angular speed $\omega^*$ is estimated by using PI control as

$$\omega^* = -\left(\frac{K_{\omega}}{T_{\omega s}} + K_{\omega}\right)e^*_d$$  \hspace{1cm} (3.13)

The sign of $K_{\omega}$ must be changed according to the sign of $\omega^*$ as

$$sign(\omega^*) = 1 \text{ if } \omega^* > 0 \text{ and } -1 \text{ if } \omega^* < 0.$$  \hspace{1cm} (3.14)

The angle of the rotor flux is computed by integrating the angular speed as follows:

$$\theta^* = \frac{\omega^*}{s}$$ \hspace{1cm} (3.15)

Fig. 3.2 The rotor flux vectors and $d-q$ reference frame.
The rotor speed is estimated by using (3.7). The speed PI control and \( q \)–axis PI current control are composed like conventional system as shown in Fig.3.1.

By assuming \( \psi_{rq}^* = 0 \) and \( \psi_{rd}^* = M i_{sd}^* \), we have following equation from (3.2):

\[
e_{sq}^* = \omega^* \sigma L_s i_{sd}^* + (R_s^* + \sigma L_s p)i_{sq}^* + \frac{M^2}{L_r} \omega^* i_{sd}^*
\]

\[
= \omega^* L_s i_{sd}^* + (R_s^* + \sigma L_s p)i_{sq}^*
\]

............................................ (3.16)

The 2\( \phi \)/3\( \phi \) transformation of Fig.3.1 is performed as

\[
\begin{bmatrix}
i_{sd}^* \\
i_{sq}^*
\end{bmatrix} = \frac{2}{\sqrt{3}} 
\begin{bmatrix}
\cos \theta^* & \cos \left( \theta^* - \frac{2}{3} \pi \right) & \cos \left( \theta^* + \frac{2}{3} \pi \right) \\
-\sin \theta^* & -\sin \left( \theta^* - \frac{2}{3} \pi \right) & -\sin \left( \theta^* + \frac{2}{3} \pi \right)
\end{bmatrix} 
\begin{bmatrix}
i_{sa}^* \\
i_{sb}^* \\
i_{sc}^*
\end{bmatrix}
\]

...... (3.17)

\[
\begin{bmatrix}
e_{sa}^* \\
e_{sb}^* \\
e_{sc}^*
\end{bmatrix} = \frac{2}{\sqrt{3}} 
\begin{bmatrix}
\cos \theta^* & -\sin \\
\cos \left( \theta^* - \frac{2}{3} \pi \right) & -\sin \left( \theta^* - \frac{2}{3} \pi \right) \\
\cos \left( \theta^* + \frac{2}{3} \pi \right) & -\sin \left( \theta^* + \frac{2}{3} \pi \right)
\end{bmatrix} 
\begin{bmatrix}
e_{sd}^* \\
e_{sq}^*
\end{bmatrix}
\]

............... (3.18)

3.1.2 Description of System

The following assumptions are set in analysis.

(1) \( i_{sd}^* \) is constant.

(2) Voltage control is performed ideally and the following equation is valid:

\[
e_{sa}^* = e_{sa}, \ e_{sb}^* = e_{sb}, \ e_{sc}^* = e_{sc}
\]

............................................ (3.19)
In order to analyze an IM, the $d-q$ reference frame that rotates synchronously with the flux angle $\theta^*$ is taken as $\theta$ of (2.26). Thus, $\omega = \omega^*$. Therefore, the $d-q$ transformation is expressed as

$$
\begin{bmatrix}
    f_{sd} \\
    f_{sq}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    \cos \theta^* & \cos \left(\theta^* - \frac{2}{3} \pi\right) & \cos \left(\theta^* + \frac{2}{3} \pi\right) \\
    -\sin \theta^* & -\sin \left(\theta^* - \frac{2}{3} \pi\right) & -\sin \left(\theta^* + \frac{2}{3} \pi\right)
\end{bmatrix} \begin{bmatrix}
    f_{sa} \\
    f_{sb} \\
    f_{sc}
\end{bmatrix}.
$$

(3.20)

From (3.18), (3.19) and (3.20) we have

$$
e_{sd}^* = e_{sd}, e_{sq}^* = e_{sq} \hspace{1cm} \text{................................. (3.21)}
$$

The PI $d-$ axis controller is expressed as

$$
e_d^* = K_p \left( i_{sd}^* - i_{sd} \right) + e_{cd} \hspace{1cm} \text{................................. (3.22)}
$$

where,

$$
e_{cd} = \frac{K_c}{s} \left( i_{sd}^* - i_{sd} \right) \hspace{1cm} \text{................................. (3.23)}
$$

Then the derivative of $e_{cd}$ is expressed as

$$
p e_{cd} = K_i \left( i_{sd}^* - i_{sd} \right) \hspace{1cm} \text{................................. (3.24)}
$$

The PI angular speed estimator is expressed as

$$
\omega^* = e_{\omega} - K_{\omega} e_d^* \hspace{1cm} \text{................................. (3.25)}
$$

where,

$$
e_{\omega} = \frac{K_{\omega}}{T_{\omega} s} e_d^* \hspace{1cm} \text{................................. (3.26)}
$$
Then, the derivative of $e_\omega$ is expressed as

$$pe_\omega = -\frac{K_{e_\omega}}{T_{e_\omega}} e^*_d$$

.......................................................................................... (3.27)

By using (3.22), we have

$$pe_\omega = -\frac{K_{e_\omega}}{T_{e_\omega}} \left( K_p \left( i^*_i - i_i \right) + e_{cd} \right)$$

.......................................................................................... (3.28)

From (3.22) and (3.25)

$$\omega^* = e_\omega - K_{e_\omega}K_p^i \hat{i}^*_i + K_{e_\omega}K_p i_i - K_{e_\omega}e_{cd}$$

.......................................................................................... (3.29)

The output variable of PI speed controller $i_{sq}^*$ is expressed as

$$i_{sq}^* = \left( K_{ps} + \frac{K_{is}}{s} \right) \left( \omega^*_r - \hat{\omega}_r \right)$$

$$= K_{ps} \left( \omega^*_r - \omega^* + \frac{i_{sq}^*}{\tau_r i^*_{sd}} \right) + \omega_{cd}$$

.......................................................................................... (3.30)

where,

$$\omega_{cd} = \frac{K_{is}}{s} \left( \omega^*_r - \omega^* + \frac{i_{sq}^*}{\tau_r i^*_{sd}} \right)$$

.......................................................................................... (3.31)

By using $\omega^*$ in (3.29), we have

$$p\omega_{cd} = K_{is} \left( \omega^*_r + K_{e_\omega}K_p^i \hat{i}^*_i - K_{e_\omega}K_p i_i - K_{e_\omega}e_{cd} - e_{cd} + \frac{i_{sq}^*}{\tau_r i^*_{sd}} \right)$$

................................................................. (3.32)

The $d$–axis voltage is expressed as

$$e^*_{sd} = K_p \left( \hat{i}^*_{i_d} - i_i \right) + e_{cd} - \omega^* \sigma L_s i_{sq} + R_s i_{sd}^*$$

................................................................. (3.33)

The $q$–axis voltage is expressed as

$$e^*_{sq} = \left( K_p + \frac{K_i}{s} \right) \left( i^*_{sq} - i_{sq} \right) + \omega^* L_s i_{sd}^*$$

$$= K_p \left( i_{sq}^* - i_{sq} \right) + e_{cq} + \omega^* L_s i_{sd}^*$$

................................................................. (3.34)
where,
\[ e_{cq} = \frac{K}{s} \left( i_{sq}^* - i_{sq} \right) \] ............................................................... (3.35)

Then, the derivative of \( e_{cq} \) is expressed as
\[ pe_{cq} = K_i \left( i_{sq}^* - i_{sq} \right) \] ............................................................... (3.36)

3.1.3 Steady State Analysis

In this system, we can choose any value of speed command \( \omega_r^* \) and magnetizing current command \( i_{sd}^* \). Load torque \( T_L \) is an arbitrary input that depends on any load connected to motor. When we set the \( \omega_r^* , i_{sd}^* , T_L \), other all quantities can be determined. It has three degrees of freedom. In order to simplify the procedure of computation, the slip speed is given instead of the load torque.

Actually, angular frequency \( \omega^* \), rotor speed \( \omega_r \) are constant in steady state condition, and then the differential equation of the system becomes linear. If the system is linear, it is similar to DC circuit. Then, we can set the differential operator \( p = 0 \) in steady state analysis because there is no change of state quantity.

The following equations is obtained by letting \( p = 0 \) in steady state condition:
\[ i_{sd}^* = i_{sd} , i_{sq}^* = i_{sq} , e_d^* = 0 , e_{cd} = 0 , \omega_r^* = \hat{\omega}_r \] ........................ (3.37)
From (2.39), the induction motor is expressed as

\[ e_{sd}^* = R_s i_{sd} - \sigma L_s i_{sq} - \frac{\omega^* M}{L_r} \psi_{rq} \] ........................................ (3.38)

\[ e_{sq}^* = \omega^* L_s i_{sd} + R_s i_{sq} + \frac{\omega^* M}{L_r} \psi_{rd} \] ........................................ (3.39)

\[ 0 = -\frac{M}{\tau_r} i_{sd} + \frac{1}{\tau_r} \psi_{rd} - (\omega^* - \omega_r) \psi_{rq} \] ........................................ (3.40)

\[ 0 = -\frac{M}{\tau_r} i_{sq} + (\omega^* - \omega_r) \psi_{rd} + \frac{1}{\tau_r} \psi_{rq} \] ........................................ (3.41)

\[ \tau_e = T_L \] ............................................................................ (3.42)

From (3.33) and (3.37), we have

\[ e_{sd}^* = R^* i_{sd}^* - \sigma L_s i_{sq} \] ........................................ (3.43)

If we assume \( R_s = R^*_s \), from (3.38) and (3.43), we have

\[ \psi_{rq} = 0 \] ............................................................................ (3.44)

From (3.40) and (3.44)

\[ \psi_{rd} = M i_{sd} \] ............................................................................ (3.45)

From (3.41) and (3.44),

\[ \omega^* - \omega_r = \frac{M i_{sq}}{\tau_r \psi_{rd}} = \frac{i_{sq}}{\tau_r i_{sd}^*} \] ........................................ (3.46)

36
By assuming \( R_r = R_r^* \) and using (3.7), we have

\[
\omega_r = \hat{\omega}_r
\]

(3.47)

Therefore,

\[
\omega_r = \omega_r^*
\]

(3.48)

By comparing (3.34) and (3.39), we have

\[
e_{cq} = R_s i_{sq}
\]

(3.49)

A steady-state solution is calculated in the following procedures by setting \( N_r^*, \) \( N_{sl}^* \), and \( i_{sd}^* \) as given values.

[1] Electrical angular speed command \( \omega_r^* \) is calculated by

\[
\omega_r^* = \frac{P}{2} 2\pi \frac{N_r^*}{60}
\]

(3.50)

[2] Similarly, electrical slip angular frequency \( \omega_{sl} \) is calculated by

\[
\omega_{sl} = \omega^* - \omega_r = \frac{P}{2} 2\pi \frac{N_{sl}}{60}
\]

(3.51)

[3] \( \psi_{rq} = 0 \) is calculated by referred to (3.12)

[4] \( \psi_{rd} = M i_{sd} \)

[5] \( i_{sq} \) is calculated by

\[
\hat{i}_{sq} = \frac{\omega_{sl} T_r}{M} \psi_{rd}
\]

(3.52)
[6] Slip speed $\omega_s$ is equal to $\omega_{sl}$ and rotor speed $\omega_r$ is equal to $\omega^*$.

[7] $\omega^*$ is calculated by

$$\omega^* = \omega_r + \omega_{sl} \hspace{200pt} \text{.................................................................................. (3.53)}$$

[8] $e_{sd}^* = R_s i_{sd} - \omega^* \sigma L_s i_{sq}$

[9] $e_{sq}^* = \omega^* L_s i_{sd} + R_s i_{sq}$

[10] $e_{cq} = R_s i_{sq}$

[11] $\omega_{cd} = i_{sq}^*$

[12] $e_\omega = \omega^*$

[13] $\tau_e = \frac{P M^2}{2 L_r} i_{sd} i_{sq}$

The non-linear state equation of induction motor in the rotating reference frame that rotates synchronously with $\theta^*$ is obtained from (2.45) to (2.49). In these equations, $e_{sd}$, $e_{sq}$ and $\omega$ must be replaced with $e_{sd}^*$, $e_{sq}^*$ and $\omega^*$ respectively by using (3.21).
3.1.4 Linear Model

From (2.53) the linear model of IM of Fig.3.1 is expressed by

$$p \Delta x_s = A_s \Delta x_s + B_s \Delta u_s + B_L \Delta T_L$$ ................................................. (3.54)

where,

$$\Delta x_s = [\Delta i_{sd}, \Delta i_{sq}, \Delta \psi_{rd}, \Delta \psi_{rq}, \Delta \omega_r]^T$$

$$\Delta u_s = [\Delta e_{sd}^*, \Delta e_{sq}^*, \Delta \omega_r^*]^T$$

In these equations, $\Delta e_{sd}, \Delta e_{sq}, \Delta \omega$ and $\omega$ were replaced with $\Delta e_{sd}^*, \Delta e_{sq}^*, \Delta \omega_r^*$ and $\omega^*$ respectively.

By considering small perturbation at a steady state operating point, the following equations are obtained from equation (3.24), (3.28), (3.32) and (3.36).

$$p \Delta e_{cd} = -K_i \Delta i_{sd}$$ ................................................................. (3.55)

$$p \Delta e_{\omega} = \frac{K_{\omega}}{T_{\omega}} K_p \Delta i_{sd} - \frac{K_{\omega}}{T_{\omega}} \Delta e_{cd}$$ ................................................................. (3.56)

$$p \Delta \omega_{cd} = K_{is} \left( \Delta \omega_r^* - K_{\omega} K_p \Delta i_{sd} + K_{\omega} \Delta e_{cd} - \Delta e_{\omega} + \frac{\Delta i_{sq}}{\tau_{r i_{sd}}} \right) \ldots (3.57)$$

$$p \Delta e_{cq} = K_i \Delta i_{sq} - K_i \Delta i_{sq}$$ ................................................. (3.58)

By using (3.30), (3.58) becomes

$$p \Delta e_{cq} = -K_i K_{ps} K_{\omega} K_p \Delta i_{sd} + K_i \left( \frac{K_{ps}}{\tau_{r i_{sd}}} - 1 \right) \Delta i_{sq}$$

$$+ K_i K_{ps} K_{\omega} \Delta e_{cd} - K_i K_{ps} \Delta e_{\omega} + K_i \Delta \omega_{cd} + K_i K_{ps} \Delta \omega_r^* \ldots (3.59)$$
These equations can be expressed in a matrix form as follows:

\[
P \begin{bmatrix}
\Delta e_{cd} \\
\Delta e_{d} \\
\Delta \omega_{cd} \\
\Delta \omega_{cq}
\end{bmatrix} = \begin{bmatrix}
-K_i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-K_{is}K_{o}K_p & 0 & 0 & 0 \\
-K_iK_{ps}K_{o}K_p & K_i\left(\frac{K_p}{T_{r}i_{sd}}-1\right) & 0 & 0
\end{bmatrix} \begin{bmatrix}
\Delta i_{sd} \\
\Delta i_{sq} \\
\Delta \psi_{cd} \\
\Delta \psi_{rq} \\
\Delta \omega_r
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & 0 & 0 & 0 \\
-K_{o}/T_{o} & 0 & 0 & 0 \\
K_{is}K_{o} & -K_{is} & 0 & 0 \\
K_{i}K_{ps}K_{o} & -K_{i}K_{ps} & K_i & 0
\end{bmatrix} \begin{bmatrix}
\Delta e_{cd} \\
\Delta e_{d} \\
\Delta \omega_{cd} \\
\Delta \omega_{cq}
\end{bmatrix}
\]

Simply is expressed as

\[
p \Delta z = A_z \Delta x + A_{\Delta z} \Delta z + B_r \Delta r \quad \ldots \ldots \quad (3.60)
\]

where,

\[
\Delta z = \begin{bmatrix}
\Delta e_{cd} \\
\Delta e_{d} \\
\Delta \omega_{cd} \\
\Delta \omega_{cq}
\end{bmatrix}^T, \quad \Delta r = \begin{bmatrix}
\Delta \omega_r^*
\end{bmatrix}
\]

From the equations (3.29), (3.33) and (3.34) we have

\[
\Delta \omega^* = K_{o}K_{p}\Delta i_{sd} - K_{o}\Delta e_{cd} + \Delta e_{d} \quad \ldots \ldots \quad (3.61)
\]

\[
\Delta e_{sd}^* = -K_{p}\Delta i_{sd} + \Delta e_{cd} - \omega^*\sigma L_r\Delta i_{sq} - \Delta \omega^*\sigma L_i \Delta i_{sq}
\]

\[
= -K_{p}\left(K_{o}\sigma L_{i_{sq}} + 1\right)\Delta i_{sd} - \omega^*\sigma L_s i_{sq} + \left(K_{o}\sigma L_{i_{sq}} + 1\right)\Delta e_{cd} - \sigma L_{i_{sq}} \Delta e_{d} \quad \ldots \ldots \quad (3.62)
\]
\[
\Delta e^*_sq = K_p \Delta i^*_sq - K_p \Delta i_{sq} + \Delta e_{cq} + L_s i^*_{sd}\Delta \omega^*
\]

\[
= K_p K_{ps} \Delta \omega^*_r + K_p \Delta \omega_{cd} + \left( \frac{K_{ps}}{\tau_{r}^* i_{sd}} - 1 \right) K_p \Delta i_{sq}
\]

\[
+ \Delta e_{cq} + \left( L_s i^*_{sd} - K_p K_{ps} \right) K_o K_p \Delta i_{sd}
\]

\[
+ \left( K_p K_{ps} - L_s i^*_{sd} \right) K_o \Delta e_{cd} + \left( L_s i^*_{sd} - K_p K_{ps} \right) \Delta e_o \quad ...........
\]

(3.63)

We can write a matrix form of \( \Delta u_s \) as follows:

\[
\Delta u_s = F_x \Delta x_s + F_z \Delta z + F_r \Delta r \quad \text{...........................................} \quad (3.64)
\]

\[
\begin{bmatrix}
\Delta e^*_{sd} \\
\Delta e^*_{sq} \\
\Delta \omega^*
\end{bmatrix} = \begin{bmatrix}
-K_p (K_o \sigma L_s i_{sq} + 1) & -\sigma L_s & 0 & 0 & 0 \\
K_p (L_s i^*_{sd} - K_p K_{ps}) & K_p \left( \frac{K_{ps}}{\tau_{r}^* i_{sd}} - 1 \right) & 0 & 0 & 0 \\
K_p K_o & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\Delta i_{sd} \\
\Delta i_{sq} \\
\Delta \psi_{sd} \\
\Delta \psi_{rq} \\
\Delta \omega^*_r
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
K_o \sigma L_s i_{sq} + 1 & -\sigma L_s i_{sq} & 0 & 0 & 0 \\
0 & -K_o & 0 & 0 & 0 \\
L_s i^*_{sd} - K_p K_{ps} & 1 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\Delta e_{cd} \\
\Delta e_{eo} \\
\Delta \omega^*_{cd} \\
\Delta e_{cq} \\
0
\end{bmatrix}
\]

From equation (3.54), (3.61) and (3.64) the linear model of proposed system A can be derived as follows:

\[
p \begin{bmatrix}
\Delta x_s \\
\Delta z
\end{bmatrix} = \begin{bmatrix}
A_x + B_s F_x & B_s F_z \\
A_z & A_z
\end{bmatrix} \begin{bmatrix}
\Delta x_s \\
\Delta z
\end{bmatrix} + \begin{bmatrix}
B_s F_r \\
B_r
\end{bmatrix} \begin{bmatrix}
\Delta r
\end{bmatrix} + \begin{bmatrix}
B_L \\
0
\end{bmatrix} \begin{bmatrix}
\Delta T_L
\end{bmatrix} \quad ...........
\]

(3.65)

Simply we can express as

\[
p \Delta x = A \Delta x_s + B \Delta r + B_L \Delta T_L \quad \quad \text{...........................................} \quad (3.66)
\]
Where,

\[
A = \begin{bmatrix}
-a_1 - \frac{K_p}{\sigma L_s} & 0 & \frac{1}{\tau_r \sigma L_s L_r} \\
-\omega^* + K_y K_p \left( \frac{L_{s}^2 i_{sd} - K_p K_{ps}}{\sigma L_s} - i_{sd} \right) & -a_1 + \frac{K_p}{\sigma L_s} \left( \frac{K_{ps}}{\tau_r i_{sd}} - 1 \right) & -M \omega_r \frac{1}{\sigma L_s L_r} \\
\frac{1}{\tau_r} M + K_y K_p \psi_{rq} & 0 & -\frac{1}{\tau_r} \\
-K_p K_y \psi_{rd} & \frac{1}{\tau_r} M & \omega_r - \omega^* \\
-a_p \psi_{rq} & a_p \psi_{rd} & a_p \psi_{rq} \\
-K_i & 0 & 0 \\
-K_i K_p \psi_{rd} & K_i & 0 \\
K_i K_p \psi_{rd} K_p & -K_{is} & \left( \frac{K_{is}}{\tau_r i_{rd}} - 1 \right) & 0
\end{bmatrix}
\]
The output equation is expressed as

$$\Delta y = C \Delta x$$

$$\Delta y = C \Delta x$$

(3.67)

Where,

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
3.2 Proposed System B

In the proposed system B, $q$–axis current controller is removed. Therefore, the system B is simpler than the system A.

3.2.1 Block Diagram of System B

Figure 3.3 shows the block diagram of proposed system B. The output voltage $e_d^*$ of $d$–axis PI current controller is used in both the flux angle estimation and the speed control ($q$–axis voltage control).

![Block diagram of system B](image_url)
The $e^*_d$ is proportional to $\psi^r_{rq}$ as described in (3.11). By using $e^*_d$, the angular frequency $\omega^*$ is modified as follows:

$$\omega^* = \omega^*_r + \omega^*_e - K_{\omega} e^*_d$$

................................................................. (3.68)

The term $\omega^*_r + \omega^*_e$ is considered as feed-forward value of the angular frequency. The gain $K_{\omega}$ sign must be changed according to $\omega^*$ as

$$K_{\omega} = \text{sign}(\omega^*)|K_{\omega}|$$

................................................................. (3.69)

The estimated rotor speed $\hat{\omega}_r$ is defined as

$$\hat{\omega}_r = \omega^* - \omega_e$$

................................................................. (3.70)

where,

$$\omega_e = \frac{i_{sq}^*}{\tau^*_r i_{sd}^*}$$

From (3.68) and (3.70), we have

$$K_{\omega} e^*_d = \omega^*_r - \hat{\omega}_r$$

................................................................. (3.71)

Therefore, the speed control can be accomplished by using $K_{\omega} e^*_d$. Since the torque current $i_{sq}$ is controlled by the $q$–axis voltage $e^*_{sq}$, we compute the $e^*_{sq}$ as

$$e^*_{sq} = L_i i_{sd}^* \left( \omega^*_r + \omega^*_e + \omega^*_c \right)$$

................................................................. (3.72)

where,

$$\omega^*_c = \left( K_{pc} + \frac{K_{ic}}{s} \right) e^*_d$$

................................................................. (3.73)

Other equations of Fig.3.3 are the same as those of Fig. 3.1.
3.2.2 Description of System

The proposed system B shown in Fig.3.3 is described. The \( d \) – axis PI current controller is described by (3.22) – (3.24). The output of PI speed controller \( \omega_c \) is computed by

\[
\omega_c = K_{ao}K_{pc}e_d^* + \omega_{ci}
\]

................................. (3.74)

where,

\[
\omega_{ci} = \frac{K_{ao}K_{ic}}{s}e_d^*
\]

................................. (3.75)

The derivative of \( \omega_{ci} \) is expressed as

\[
p\omega_{ci} = K_{ao}K_{ic}e_d^*
\]

................................. (3.76)

The angular frequency is expressed as

\[
\omega^* = \omega_r^* + \frac{i_{sq}^*}{\tau_r} - K_p \left( K_p \left( i_{sd}^* - i_{sd} \right) + e_{cd} \right)
\]

....................... (3.77)

The variable \( e_{cd} \) is defined in (3.23).

The \( d \) – axis voltage is calculated by (3.33). The \( q \) – axis voltage is written as

\[
e_{sq}^* = L_s i_{sd}^* \left( \omega_r^* + \frac{i_{sq}}{\tau_r} + K_{ao}K_{pc} \left( K_p \left( i_{sd}^* - i_{sd} \right) + e_{cd} \right) + \omega_{ci} \right)
\]

........ (3.78)

In order to analyze the IM of Fig.3.3, the \( d-q \) axis that rotates synchronously with the flux angle \( \theta^* \) is defined. If the ideal voltage control of (3.19) is assumed, the actual \( d-q \) voltage are equal to those references as described in (3.21). Therefore, the equation of IM is described by (2.39). By replacing the variables \( e_{sd}, e_{sq}, \) and \( \omega \) with \( e_{sd}^*, e_{sq}^*, \) and \( \omega^* \) respectively.
3.2.3 Steady State Analysis

At steady state condition, the derivative operator \( p \) is set to zero. By the integral controllers of \( d \) – axis current and speed, we have

\[
i_{sd} = i_{sd}^* \quad \text{................................. (3.79)}
\]

\[
e_d^* = 0 \quad \text{................................. (3.80)}
\]

Therefore, \( e_{cd} = 0 \) from (3.22).

The actual IM equations at steady state are described in (3.38) – (3.42). By assuming \( R_s = R_s^* \), the \( q \) – axis flux \( \psi_{rq} \) becomes zero as the same of system A. Therefore, (3.46) is valid in this case too. From (3.80), we have

\[
\omega^* = \omega_r^* + \frac{i_{sq}^*}{r_s i_{sd}^*} \quad \text{................................. (3.81)}
\]

By assuming \( R_r = R_r^* \), the following equation is obtained from (3.46) and (3.81):

\[
\omega_r^* = \omega_r \quad \text{From (3.78) and (3.81), we have}
\]

\[
e_{sq}^* = \omega^* L_s i_{sd}^* + \omega_{cl} L_s i_{sd}^* \quad \text{................................. (3.82)}
\]

By comparing (3.39), (3.45) and (3.82)

\[
\omega_{cl} = \frac{R_s i_{sq}^*}{L_s i_{sd}^*} \quad \text{................................. (3.83)}
\]

Steady-state values are calculated by giving \( N_r^* \), \( N_{sl} \) and \( i_{sd}^* \) as known values, similar to the case of system A.
3.2.4 Linear Model

The linear state equation of the IM is described by (3.54). From (3.55), (3.76), (3.77) for a small signal perturbation, the following equations are obtained:

\[ p \Delta e_{cd} = -K_j i_{sd} \]

\[ p \Delta \omega_{ci} = K_e K_i \left( \Delta e_{cd} - K_p \Delta i_{sd} \right) \]

\[ \Delta \omega^* = \Delta \omega_r^* + \frac{\Delta i_{sq}}{\tau_{i_{sd}}^*} + K_e \left( K_p \Delta i_{sd} - \Delta e_{cd} \right) \]

The \( d \) – axis voltage \( e_{sd}^* \) in (3.33) is linearized as

\[ \Delta e_{sd} = -K_p \Delta i_{sd} + \Delta e_{cd} - \omega^* \sigma L_s \Delta i_{sq} - \Delta \omega^* \sigma L_s i_{sq} \]

\[ = -\left( \sigma L_s i_{sq} K_o + 1 \right) K_p \Delta i_{sd} - \left( \omega^* + \frac{i_{sq}}{\tau_{i_{sd}}^*} \right) \sigma L_s \Delta i_{sq} \]

\[ + \left( K_o \sigma L_s i_{sq} + 1 \right) \Delta e_{cd} - \sigma L_s i_{sq} \Delta \omega_r^* \]

\[ \text{................................. (3.84)} \]

The \( q \) – axis voltage \( e_{sq}^* \) in (3.78) is linearized as

\[ \Delta e_{sq}^* = L_s i_{sd}^* \left( \Delta \omega_r^* + \frac{\Delta i_{sq}}{\tau_{i_{sd}}^*} + K_e K_p \left( \Delta e_{cd} - K_p \Delta i_{sd} \right) + \Delta \omega_{ci} \right) \]

\[ \text{............. (3.85)} \]
The state equation of the controller is expressed as

$$p\Delta z = A_x \Delta x_s + A_z \Delta z + B_r \Delta r$$ ........................................ (3.86)

$$p \begin{bmatrix} \Delta e_{cd} \\ \Delta \omega_{ci} \end{bmatrix} = \begin{bmatrix} -K_i & 0 & 0 & 0 \\ -K_{\omega} K_{ic} K_p & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{sd} \\ \Delta i_{sq} \\ \Delta \psi_{rd} \\ \Delta \psi_{rq} \\ \Delta \omega_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta e_{cd} \\ \Delta \omega_{ci} \end{bmatrix}$$

$$\Delta u_s = F_x \Delta x_s + F_z \Delta z + F_r \Delta r$$ ............................................... (3.87)

$$\begin{bmatrix} \Delta e_{*sd} \\ \Delta e_{*sq} \\ \Delta \omega_{*ci} \end{bmatrix} = \begin{bmatrix} -\left( K_{\omega} \sigma L_i s_{sq} + 1 \right) K_p & \left( \omega^* - \frac{i_{sq}}{\tau_{i_{sd}}^*} \right) \sigma L_s & 0 \\ -L_s i_{sd}^* K_{\omega} K_{pc} K_p & L_s \tau_{i_{sd}}^* & 0 \\ K_{\omega} K_p & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{sd} \\ \Delta i_{sq} \\ \Delta \psi_{rd} \\ \Delta \psi_{rq} \\ \Delta \omega_r \end{bmatrix}$$

The linear model of whole system is obtained as follows:

$$p \begin{bmatrix} \Delta x_s \\ \Delta z \end{bmatrix} = \begin{bmatrix} A_x + B_s F_x & B_s F_z \\ A_z \end{bmatrix} \begin{bmatrix} \Delta x_s \\ \Delta z \end{bmatrix} + \begin{bmatrix} B_r F_r \\ B_r \end{bmatrix} \Delta r + \begin{bmatrix} B_L \\ 0 \end{bmatrix} \Delta T_L$$

Simply,

$$p \Delta x = A \Delta x + B_r \Delta r + B_L \Delta T_L$$ ........................................ (3.88)
where,

\[
A = \begin{bmatrix}
-a_1 - \frac{K_p}{\sigma L_s} & 0 \\
-\omega^* + i_{sd} K_\omega K_p \left( -\frac{K_{pc}}{\sigma} - 1 \right) & -a_1 + \frac{1}{\sigma \tau^*_r} - \frac{i_{sd}}{\tau^*_r i_{sd}} \\
\frac{M}{\tau_r} + K_\omega K_p \psi_{rq} & \frac{\psi_{rq}}{\tau^*_r i_{sd}} \\
-K_\omega K_p \psi_{rd} & \frac{M}{\tau_r} - \frac{\psi_{rd}}{\tau^*_r i_{sd}} \\
-a_2 \psi_{rq} & a_2 \psi_{rd} \\
-K_i & 0 \\
-K_{sd} K_{ic} K_p & 0
\end{bmatrix}
\]
The output equation is expressed as
\[ \Delta y = C \Delta x \] ................................. \hspace{3cm} (3.89)
where,
\[ C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \]

### 3.3 Gain Selection of Controller

#### 3.3.1 Current Controller

Figure 3.4 shows a block diagram of the PI current control system of IM. The IM is modeled as a series circuit of the resistance $R_{sr}$ and leakage inductance $\sigma L_s$. The disturbance means a counter electromotive force. The resistance $R_{sr}$ is expressed as
\[ R_{sr} = R_s + \left( \frac{M}{L_p} \right)^2 R_r \] ................................. \hspace{3cm} (3.90)
Assuming that there are no disturbances, the closed-loop transfer function of the current control becomes the following equations:

\[
\frac{I}{I^*} = \frac{K_p(T_s + 1)}{(\sigma L_s s + R_{sr})T_i s + K_p(T_i s + 1)}
\]

The \( T_i \) is usually designed as [45]

\[
T_i = \frac{\sigma L_s}{R_{sr}}
\]  \hspace{1cm} \text{(3.91)}

The current transfer function becomes as

\[
\frac{I}{I^*} = \frac{K_p}{\sigma L_s s + K_p} = \frac{1}{T_{eq} s + 1}
\]  \hspace{1cm} \text{(3.92)}

where,

\[
T_{eq} = \frac{\sigma L_s}{K_p}
\]

Cut-off frequency \( \omega_c \) of this transfer function is expressed by the following equation:

\[
\omega_c = \frac{K_p}{R_{sr} T_i} = \frac{1}{T_{eq}}
\]  \hspace{1cm} \text{(3.93)}

Fig.3.4 Block diagram of the current control system.
By setting the cut-off frequency \( \omega_c \), the proportional gain \( K_p \) is determined by (3.93).

### 3.3.2 Speed Controller

The block diagram of speed control system is shown in Fig.3.5. When the vector control is ideal, the torque can be controlled as

\[
\tau_e = \frac{PM^2}{2L_r} i_{sd}^* i_{sq} = K_T i_{sq} \quad \text{...........................................} \quad (3.94)
\]

The loop transfer function \( G_0 \) of Fig. 3.5 is

\[
G_0 = \left( K_{ps} + \frac{K_{is}}{s} \right) \frac{1}{1 + T_{eq}s} \frac{PK_T}{2J_s} \quad \text{...........................................} \quad (3.95)
\]

The Bode diagram of (3.95) is shown in Fig.3.6. The angular frequency \( \omega_{pi} \) is expressed as

\[
\omega_{pi} = K_{is} / K_{ps} \quad \text{...........................................} \quad (3.96)
\]

**Fig.3.5** Block diagram of the speed control system.
In order to have sufficient phase margin, the following equation should be satisfied [45]:

\[ \omega_{pi} \leq \frac{\omega_{sc}}{5} \] 

(3.97)

Under the condition (3.97), the crossover frequency \( \omega_{sc} \) is obtained as

\[ \omega_{sc} = \frac{PK_T K_{ps}}{2J} \] 

(3.98)

If we set the \( \omega_{pi} \) and \( \omega_{sc} \), the gains \( K_{ps} \) and \( K_{is} \) is determined by (3.98) and (3.96).
4.1 Experimental System

4.1.1 Microcomputer control system

The IM control system by using a digital signal processor (DSP) is shown in Fig.4.1. The power circuit is composed by rectifier, smoothing capacitor, and IGBT inverter. A DC machine is used as a load through a torque sensor for the induction machine. Torque sensor is used for detecting the torsion of the shaft. The motor currents and DC link voltage are detected to the DSP through an analog to digital (A/D) converter.

PWM gate signal generator is connected to IGBT inverter. These signals are carried by an optical fiber cable and are not affected by noise. Resistor is connected to the braking circuit and the regenerative energy is consumed as heat. When the IM operates as a generator, IGBT brake circuit is turned on. The dangerous voltage rise can damage the rectifier circuit diode and the smoothing capacitor, because its energy is not returned to the power supply.
The interrupt signal INT1 of the DSP is sent from the PWM gate signal generator to synchronize the period of PWM. The host computer has a segment that can be connected to the DSP, so it can transfer control program, or display information on the screen from DSP. The PWM inverter and DSP control circuit is made by My Way Co., Ltd. Block diagram of DSP 320C32 is shown in Fig.4.2.
Fig. 4.2 Block diagram of DSP TMS320C32.

The experimental program of the DSP is written by using C language. The developed program is divided into the main routine, INT1 interrupt routine and the timer0 interrupt routine. Fig. 4.3 shows the flow chart of the sensorless vector control B.

First, the main program starts. After the permission of interrupt, the INT1 and Timer0 interrupts can be accepted to DSP and the main program enters into infinite loop to communicate with the host computer. For example, the waveform data is sent to the host computer such as speed and current, when INT1 signal enters the DSP from the gate signal generator, the main program is stopped. Timer0 interrupt program is accepted to DSP after the INT1 program ended. The INT1 has dominant priority. The main program priority is the lowest, and it is executed when two interrupt programs are ended. The INT1 program runs every 200 $\mu s$. 
Fig. 4.3 Flow chart of sensorless vector control B program.
The experimental programs are changed to machine language by using C compiler and sent to RAM of DSP. This process is called “downloading”. Process and communication program commands are made by My Way Co. Ltd. It is necessary for both the DSP and the PC.

### 4.1.2 Parameters of System

The IM specifications used in the experimental system are shown in Table 4.1.

The leakage inductance $\sigma L_s$ is computed as

$$\sigma L_s = \left(1 - \frac{M^2}{L_s L_r}\right) L_s = \left(1 - \frac{0.11^2}{0.115^2}\right) \times 0.115 = 0.00978 \text{H}$$

Resistance of stator and rotor of (3.90) is computed as

$$R_{sr} = R_s + \left(\frac{M}{L_r}\right)^2 R_r = 1.54 + \left(\frac{0.11}{0.115}\right)^2 \times 0.787 = 2.26 \Omega$$

Integration time $T_i$ is computed from (3.91) as

$$T_i = \frac{\sigma L_s}{R_{sr}} = \frac{0.00978}{2.26} = 0.00433$$

By choosing that the cut-off frequency of (3.93) is 1500 rad/s, the current PI gains are computed as follows:

$$K_p = R_{sr} T_i \omega_c = \sigma L_s \omega_c = 0.00978 \times 1500 = 14.7$$

$$K_i = \frac{K_p}{T_i} = \frac{14.7}{0.00433} = 3395$$
Table 4.1 Parameters of the three-phase IM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of poles $P$ [poles]</td>
<td>4</td>
</tr>
<tr>
<td>Rated Output [kW]</td>
<td>1.5</td>
</tr>
<tr>
<td>Rated Torque [N-m]</td>
<td>8.43</td>
</tr>
<tr>
<td>Speed [min$^{-1}$]</td>
<td>1700</td>
</tr>
<tr>
<td>Rated Line Voltage [V]</td>
<td>200</td>
</tr>
<tr>
<td>Rated Current [A]</td>
<td>6.4</td>
</tr>
<tr>
<td>Excitation Current $i_{sd}^*$ [A]</td>
<td>4.2</td>
</tr>
<tr>
<td>Moment Inertia $J$ [kg$\cdot$m$^2$]</td>
<td>0.0126</td>
</tr>
<tr>
<td>Primary Resistance Nominal Value $R_s$ [Ω]</td>
<td>1.54</td>
</tr>
<tr>
<td>Secondary Resistance Nominal Value $R_r$ [Ω]</td>
<td>0.787</td>
</tr>
<tr>
<td>Iron Loss Resistance $R_m$ [Ω]</td>
<td>391</td>
</tr>
<tr>
<td>Primary Self-inductance $L_s$ [H]</td>
<td>0.115</td>
</tr>
<tr>
<td>Secondary Self-Inductance $L_r$ [H]</td>
<td>0.115</td>
</tr>
<tr>
<td>Mutual Inductance $M$ [H]</td>
<td>0.11</td>
</tr>
<tr>
<td>Motor Manufacturer</td>
<td>Mitsubishi Electric Corporation</td>
</tr>
</tbody>
</table>

For the design of PI speed control system, torque constant is computed as

$$K_T = \frac{PM^2}{2L_r} \frac{i_{sd}^*}{2 \times 0.115} \times 4.2 = 0.8838$$

The PI speed proportional gain (3.98) is computed by

$$K_{ps} = \frac{2J\omega_{sc}}{PK_T} = \frac{2 \times 0.0126 \times \omega_{sc}}{4 \times 0.8838} = 0.00713 \times \omega_{sc}$$
When the cut-off frequency of PI speed control system $\omega_{sc}$ is set to 20, then

$$K_{ps} = 0.0713 \times 20 = 0.1426$$

The angular frequency $\omega_{pi}$ of PI speed controller is set to 4 rad/s.

The integral gain $K_{is}$ of PI speed controller is computed as

$$K_{is} = \omega_{pi}K_{ps} = 4 \times 0.1426 = 0.5704$$

The integration time $T_{is}$ of PI speed controller is computed as

$$T_{is} = \frac{K_{ps}}{K_{is}} = \frac{0.1426}{0.5704} = 0.25$$

Table 4.2 shows the control parameters of system A.

<table>
<thead>
<tr>
<th>Control Parameters of System A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current PI Proportional Gain $K_p$</td>
</tr>
<tr>
<td>Current PI Integral Gain $K_i$</td>
</tr>
<tr>
<td>Current PI Integration Time $T_i$</td>
</tr>
<tr>
<td>Current PI Cut-off Frequency $\omega_c$</td>
</tr>
<tr>
<td>Speed PI Proportional Gain $K_{ps}$</td>
</tr>
<tr>
<td>Speed PI Integral Gain $K_{is}$</td>
</tr>
<tr>
<td>Speed PI Integration Time $T_{is}$</td>
</tr>
<tr>
<td>Speed PI Cut-off Frequency $\omega_{sc}$</td>
</tr>
<tr>
<td>PI Proportional Gain $</td>
</tr>
<tr>
<td>PI Integral Gain $</td>
</tr>
</tbody>
</table>
In system B, the PI current control gain and integration time are the same with system A, and other gains are derived from the analysis of system stability. Control parameters of system B are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Current PI Proportional Gain $K_p$</th>
<th>14.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current PI Integral Gain $K_i$</td>
<td>3395</td>
</tr>
<tr>
<td>Current PI Integration Time $T_i$</td>
<td>0.00433</td>
</tr>
<tr>
<td>Current PI Cut-off Frequency $\omega_c$</td>
<td>1500</td>
</tr>
<tr>
<td>PI Proportional Gain $K_{pc}$</td>
<td>1.0</td>
</tr>
<tr>
<td>PI Integral Gain $K_{ic}$</td>
<td>20.0</td>
</tr>
<tr>
<td>Gain $\mid K_\omega \mid$</td>
<td>5.0</td>
</tr>
</tbody>
</table>

### 4.1.3 Experimental IM – DCM System

Figure 4.4 shows the experimental DCM circuit for the forward rotation of IM. In the motoring operation of IM, the direction of rotation, the rotating magnetic field direction and the produced torque $T_e$ are in the same direction. The load torque $T_L$ has equal magnitude to the $T_e$ in the opposite direction at the steady state operation.

If the terminals X and Y are directly connected, the current flows through the bulb in the direction of $I_a$ in motoring operation. The armature current $I_a$ becomes maximum when all the light bulbs which are connected parallels are switched on. The DCM voltage generated is proportional to the velocity. Therefore, this voltage becomes smaller during low-speed operation; the armature current $I_a$ does not increased even if all the light bulbs are turned on. To solve this problem, we use a diode bridge which is connected
between the terminals X and Y. This polarity can be changed depending on whether the DCM operation as motoring or regenerating. By increasing the voltage of the variable transformer connected to the diode bridge, DC voltage of the diode bridge and armature current $I_a$ are increased.

The load torque $T_L$ delivered by DCM is proportional to $I_a$. In this way, it is possible to perform experiment with different load torque. The maximum torque in this experiment is 8.0 N-m. The speed of IM does not change even by changing the load torque $T_L$.

![Experimental DCM circuit for motoring and regenerating operations of IM.](image)

Fig. 4.4 Experimental DCM circuit for motoring and regenerating operations of IM.

$$T_L = K I_f I_a$$

The Terminal Voltage of DCM

$$V_{AH} = K' I_f N_r$$
By considering the climbing slopes, the IM is under resistance from the gravity, that is the load torque as shown in Fig. 4.5. This situation is called motoring operation. When the motor rotates higher than the synchronous speed, it generates a braking force. This condition is called regenerating operation. In this process, the mechanical energy is converted to electrical energy (so-called operation as an induction generator). There is also another circumstance, when the direction of synchronous speed is opposite to the rotor rotation. This situation is called plugging. In this case, the direction of motor rotation follows the load torque and the direction of the torque produced by the motor follows the synchronous speed.

In regenerating operation and forward rotation, the DCM must be operated as a motor. For this reason, in the diode bridge, X and Y are connected to (–) and (+) respectively. The armature current \( I_a \) becomes opposite in regenerating operation, the load torque \( T_L \) generated by DCM is also reversed. However, the speed of IM is does not change if the speed command of IM control system is constant. Therefore, torque current \( i_{sq} \) and \( T_e \) change automatically to keep the constant velocity until \( T_e \) is equal to \( T_L \) the same value in the opposite direction. As a result, the direction of \( T_e \) is reversed with respect to the rotation direction \( N_r \). Mechanical energy goes into the IM; and it is operated as a generator. During this time, DCM provides mechanical energy. This energy is used by the resistance of IM and inverter, but a lot of energy is used to charge the DC capacitor. Then, the capacitor voltage increases rapidly. The energy is not sent to the three-phase power supply side because of the diode bridge, but there is a brake circuit in order to keep a constant capacitor voltage by turning on and off the dynamic brake (DB) switch. Fig. 4.6 shows the experimental IM connected to the DC motor as a load and a torque detector in between.
\[N_0\] synchronous speed
\[N_r\] rotor speed
\[T_e\] produced torque
\[T_L\] load torque

Fig. 4.5 Operating regions of IM.

Fig. 4.6 IM with DCM as a load and a torque detector in between.
4.2 System Stability

Figure 4.7 shows the root trajectories that are computed by the linear model of proposed system B. The speed command $N_r^*$ is $700 \text{ min}^{-1}$, and slip speed $N_{sl}$ are changed from $-80 \text{ min}^{-1}$ to $80 \text{ min}^{-1}$ as parameter of load. The different integral gain $K_{ic}$ such as 100.0, 50.0, 33.3, 25.0 and 20.0 are selected. If the integral gain $K_{ic}$ is very large, the system becomes unstable. However, if $K_{ic}$ is selected smaller than 100.0, the system is stable at both motoring and regenerating operation.

![Root trajectories with parameters of slip speed $N_{sl}$ and integral gain $K_{ic}$ (system B).](image)

*Fig.4.7 Root trajectories with parameters of slip speed $N_{sl}$ and integral gain $K_{ic}$ (system B).*
Figure 4.8 shows the root trajectories when the gain $|K_\omega|$ and the integral gain $K_{ic}$ are changed. The speed command $N_r^*$ is $700 \text{ min}^{-1}$, and slip speed $N_{sl}$ is $35.0 \text{ min}^{-1}$. It is observed that the system becomes stable by choosing larger value of $|K_\omega|$ and smaller value of $K_{ic}$.
To see the movement of roots by changing the gain $|K_\omega|$ and the slip speed $N_{sl}$ is shown in Fig.4.9. The gain $|K_\omega|$ is changed to 0.1, 1.0, 3.0, and 10.0 and the slip speed $N_{sl}$ is changed from $-80 \text{ min}^{-1}$ to $80 \text{ min}^{-1}$. When $|K_\omega|$ is larger than 3.0, the system becomes stable at motoring and regenerating operation. When $|K_\omega|$ is small the system is unstable at high load in both motoring and regenerating operations.

![Fig.4.9 Root trajectories with parameters slip speed $N_{sl}$ and gain $|K_\omega|$ (system B).](image)

Fig.4.9 Root trajectories with parameters slip speed $N_{sl}$ and gain $|K_\omega|$ (system B).
In Fig.4.10, the root loci is shown when $N_r^* = 1500 \text{ min}^{-1}$, $K_{ic} = 20.0$, $|K_{ic}| = 3.0$, for the change of slip speed $N_{sl}$ from $-80 \text{ min}^{-1}$ to $80 \text{ min}^{-1}$. The system is stable in all operating points. By comparing Fig.4.9 and Fig.4.10 the system works more stable at higher speed in motoring operations, because the real part of the root is smaller.

Figures 4.11 and 4.12 show the root trajectories computed by the linear models of the systems A and B in the case of $N_r^* = 50 \text{ min}^{-1}$ and $N_r^* = 100 \text{ min}^{-1}$ respectively. The slip speed is changed from $-80 \text{ min}^{-1}$ to $80 \text{ min}^{-1}$ as parameter of load by the increment $4 \text{ min}^{-1}$. The system is stable at low speed regenerating operation. However, the system is unstable at plugging region as shown in Fig.4.11 (a) and (b) by the poles on real axis in both systems.

Fig.4.10 Root trajectory at $1500 \text{ min}^{-1}$ (system B).
Fig. 4.11  Root trajectories with parameter $N_{sl}$ at speed $N_r^* = 50 \text{ min}^{-1}$ (systems A and B).
Fig. 4.12 Root trajectories with parameter $N_{sl}$ at speed $N_r^* = 100 \text{ min}^{-1}$ (systems A and B).
Figures 4.13 shows the unstable operating region of system A with parameters; $|K_{p\omega}| = 20.0$, $|K_{i\omega}| = 20.0$, and $\omega_{sc} = 20.0$. The $\omega_{sc}$ is cut-off angular frequency of the speed control. These gains are selected to have wide stable operating points.

![Fig.4.13 Unstable region of system A.](image)

Fig.4.13 Unstable region of system A.
In Fig.4.14 shows the unstable operating region when speed command and slip speed are changed with parameters $|\omega_0| = 5.0$, $K_{ic} = 20.0$, and $K_{pc} = 0.0$ for system B. By comparing the results of Fig.4.13 and 4.14, it is observed that the unstable plugging region is almost same.

![Diagram of unstable region](image)

Fig.4.14 Unstable region with parameter $K_{pc} = 0.0$ for system B.
The increasing gain value of speed control proportional gain $K_{pc}$ can improve the stability region at low speed of motoring and plugging operation as shown in Fig. 4.15.

![Diagram showing unstable region with parameter $K_{pc} = 1.0$ of system B.](image)

Fig. 4.15 Unstable region with parameter $K_{pc} = 1.0$ of system B.
4.3 Simulation Results

Figure 4.16 shows the simulation results of non-linear and linear models of system A. The simulation results of linear and non-linear are almost same. Therefore, the modeling of the system is appropriate. The parameters used in this simulations are $|K_{po}| = 20.0$, $|K_{io}| = 20.0$ and $\omega_{sc} = 20.0$ for system A. In the case of system B, the result is shown in Fig. 4.17. For system B, the parameters are: $|K_{ao}| = 5.0$, $K_{ic} = 20.0$, and $K_{pc} = 1$. The load torque $T_L = 4.0$ N-m is selected.

In Figs.4.18, 4.20 and 4.22, the transient responses of system A and B are shown for the step change of speed command at motoring operation. These figures are correspondent to low-speed, medium-speed and high-speed operations. On the other hand, the transient responses at regenerating operations are shown in Figs 4.19, 4.21 and 4.23 for low-speed, medium-speed and high-speed operations respectively.

In system A, the estimated speed $\hat{\omega}_r$ is shown to compare with rotor speed, while in system B flux frequency $\omega^*$ is shown. Both systems A and B are stable in both regenerating and motoring operations.
Fig. 4.16   Transient responses in motoring operation of system A.
Fig.4.17 Transient responses in motoring operation of system B.
Fig. 4.18 Transient response for the step change 50 min\(^{-1}\) → 150 min\(^{-1}\) → 50 min\(^{-1}\) and \(T_L = 4.0\) N-m with selected parameters.
Fig. 4.19 Transient response for the step change $50 \text{ min}^{-1} \to 150 \text{ min}^{-1} \to 50 \text{ min}^{-1}$ and $T_L = -4.0 \text{ N-m}$ with selected parameters.
Fig. 4.20  Transient response for the step change $500 \text{ min}^{-1} \rightarrow 600 \text{ min}^{-1} \rightarrow 500 \text{ min}^{-1}$ and $T_L = 4.0 \text{ N-m}$ with selected parameters.
Fig. 4.21 Transient response for the step change 500 min$^{-1} \to 600$ min$^{-1} \to 500$ min$^{-1}$ and $T_L = -4.0$ N-m with selected parameters.
Fig. 4.22  Transient response for the step change 1000 min$^{-1}$ → 1100 min$^{-1}$ → 1000 min$^{-1}$ and $T_L = 4.0$ N-m with selected parameters.
Fig. 4.23 Transient response for the step change 1000 min$^{-1}$ → 1100 min$^{-1}$ → 1000 min$^{-1}$ and $T_L = -4.0$ N-m with selected parameters.
4.4 Experimental Results

All experimental results are shown in the case of system B. Figures 4.24, 4.25 and 4.26 shows the simulation and the experimental results for low, medium and high speed respectively. \( N_r \) is actual motor speed \( (\omega_r) \) and \( N^* \) is synchronous speed \( (\omega^*) \). The control parameters are set as \( |K_\omega| = 5.0 \), \( K_{ic} = 20.0 \) and \( K_{pc} = 0.0 \). The load torque \( T_L \) is set to 4.0 N-m (half of rated torque). From these comparisons, the experimental results are very close to those of simulation. Therefore, the validity of proposed method is confirmed, except for high frequency ripples. The high frequency ripples of \( N^* \) and \( e_d^* \) are caused by PWM voltage control in experimental system. However, since the flux angle \( \theta^* \) is obtained by integrating \( \omega^* \), the actual rotor speed has little ripples.
Fig. 4.24  Transient response for the step change $50 \text{ min}^{-1} \rightarrow 150 \text{ min}^{-1} \rightarrow 50 \text{ min}^{-1}$ at $T_L = 4.0 \text{ N-m}$. 
Fig. 4.25 Transient for the step change $500 \text{ min}^{-1} \rightarrow 600 \text{ min}^{-1} \rightarrow 500 \text{ min}^{-1}$ at $T_L = 4.0 \text{ N-m}$. 

(a) Simulation

(b) Experiment
Fig. 4.26 Transient response for the step change $1000 \text{ min}^{-1} \rightarrow 1100 \text{ min}^{-1} \rightarrow 1000 \text{ min}^{-1}$ at $T_L = 4.0 \text{N-m}$.
Figures 4.27, 4.28 and 4.29 shows the simulation and the experimental result in regenerating operation. Same change of speed command are tested under the load torque $T_L = -4.0 \text{N-m}$ for low, medium and high speed respectively. In any cases, quick responses of $N_r$ are obtained. The high frequency ripples of $N^*$ in Figs. 4.27, 4.28 and 4.29 are smaller than those in Figs. 4.24, 4.25 and 4.26. The reason for this difference can be considered that the amplitude modulation ratio of regenerating operation is smaller if the voltage control has allowance under limited DC-bus voltage; the distortion of stator currents is reduced. The larger we choose the gain $|K_\omega|$, the larger high frequency ripples of $N^*$ are induced.
Fig. 4.27 Transient response for the step change 50 min\(^{-1}\) → 150 min\(^{-1}\) → 50 min\(^{-1}\) at \(T_L = -4.0\) N-m.
Fig. 4.28 Transient for the step change $500 \text{ min}^{-1} \rightarrow 600 \text{ min}^{-1} \rightarrow 500 \text{ min}^{-1}$ at $T_L = -4.0 \text{ N-m}$. 
Fig. 4.29 Transient response for the step change $1000 \text{ min}^{-1} \rightarrow 1100 \text{ min}^{-1} \rightarrow 1000 \text{ min}^{-1}$ at $T_L = -4.0 \text{ N-m}$. 
Figure 4.30 shows the transient responses of experiment for the step change of speed command from 100 min\(^{-1}\) to \(-100\) min\(^{-1}\) and back to 100 min\(^{-1}\). In this figure, the motor is driven between the operating points of motoring and regenerating through the unstable plugging region. It is observed that the system is operated stably. The control parameter \(K_{pc} = 1.0\) is set for the following experiments.

Figure 4.31 shows the experimental results when the speed command is changed from 100 min\(^{-1}\) to 5 min\(^{-1}\). In these case, the system can be operated stably. On the other hand, in Fig. 4.32, the speed command is changed from 100 min\(^{-1}\) to \(-25\) min\(^{-1}\). The system becomes unstable at plugging region and is operated using speed sensor for the protection after \(t = 0.98\) s. This experimental result can prove the unstable region of Fig.4.15.

Fig.4.30 Transient response for the step change 100 min\(^{-1}\) \(\rightarrow\) \(-100\) min\(^{-1}\) \(\rightarrow\) 100 min\(^{-1}\) at \(T_L = 4.0\) N-m.
Fig. 4.31 Transient response for the step change $100 \text{ min}^{-1} \rightarrow 5 \text{ min}^{-1}$ at $T_L = 5.0 \text{N-m}$.

Fig. 4.32 Transient response for the step change $100 \text{ min}^{-1} \rightarrow -25 \text{ min}^{-1}$ at $T_L = 5.0 \text{N-m}$. 
Moreover, Figs. 4.33 and 4.34 show the experimental results for the step change of the speed command $N_r^*$ with speed changed $50 \text{ min}^{-1} \rightarrow 150 \text{ min}^{-1} \rightarrow 50 \text{ min}^{-1}$ corresponding to Fig. 4.18 (b) and 4.19 (b) in motoring and regenerating operation respectively. These figures can compare the responses of actual rotor speed $N_r$ for control parameter $K_{pc} = 1.0$.

Figure 4.35 shows the transient response at no-load condition ($T_L \approx 0$), with speed changed $50 \text{ min}^{-1} \rightarrow 150 \text{ min}^{-1} \rightarrow 50 \text{ min}^{-1}$ corresponding to Figs. 4.33 and 4.34. The speed responses give almost the same characteristics for each load torque. However, the $i_{sq}$ differs from each parameter of $T_L$.

![Graph](image)

**Fig. 4.33** Transient response for the step change $50 \text{ min}^{-1} \rightarrow 150 \text{ min}^{-1} \rightarrow 50 \text{ min}^{-1}$ at $T_L = 4.0 \text{ N-m.}$
Fig.4.34  Transient response for the step change
50 min\(^{-1}\) → 150 min\(^{-1}\) → 50 min\(^{-1}\) at \(T_L = -4.0\) N-m.

Fig.4.35  Transient response for the step change
50 min\(^{-1}\) → 150 min\(^{-1}\) → 50 min\(^{-1}\) at \(T_L \approx 0\) N-m.
Chapter 5
Conclusion

Study on simplified speed sensorless vector control system for induction motors is described in this thesis. The results obtained from this study are summarized as follows:

1. New simplified speed sensorless vector control methods of IM based on rotor flux linkage are proposed. The two simplified sensorless systems are called as system A and system B. In both methods, the $q$ – axis flux is obtained by the output voltage of $d$ – axis PI current controller with a non-interference control.

2. In system A, the angular frequency of rotor flux is estimated to bring $q$–axis flux to zero by using PI controller. The flux angle is obtained by integrating the angular frequency. The rotor speed is computed by subtracting a slip speed from the angular frequency.

3. In system B, the computation of $q$–axis flux is as same as the system A. The angular frequency of rotor flux is computed by subtracting the $q$–axis flux from the speed command adding with a slip speed. The $q$–axis flux is also used to control the rotor speed by adjusting the $q$–axis flux. The flux angle is obtained by integrating the angular frequency.
In both systems A and B, the $q$–axis flux is obtained from voltage model, in which the derivative term of the $d$–axis flux is assumed zero because of constant magnetizing current command. A flux angle of a current model is aligned with the flux angle of voltage model by changing the angular frequency of current model. Since the output voltage of $d$–axis PI current controller is used for the flux angle estimation and speed control ($q$–axis voltage control), the system is simplified and stabilized at regenerating mode.

By choosing a reference frame which rotates synchronously with the computed flux angle, non-linear models are derived in both proposed systems A and B. From these non-linear models, linear models of the systems are derived in state space equations by considering a small perturbation around a steady-state operating point.

The performance of both systems A and B are compared by using linear models and non-linear models. Transient responses of linear models and non-linear models are computed and compared. Since both responses are almost same around a steady state operating point, the validity of the linear models are confirmed. By using the linear models, the system stability is studied by showing root loci.

In plugging mode of both systems, most of the poles of the root-loci are located on the positive real axis, it means the system becomes unstable. In motoring and regenerating operations of systems A and B, all poles are located in the left half plane. Therefore, the system is stable in both operations.

By virtue of the stability analysis, we can design the parameters of controller in both systems.
A digital signal processor (DSP) based PWM inverter fed IM system is equipped and tested. It is confirmed that the experimental results are very close to those of non-linear simulation except for high frequency ripples. Therefore, the effectiveness of the proposed methods are also demonstrated experimentally.

The nonlinear simulation and experimental results of the proposed system show stable and quick transient responses in both motoring and regenerating modes.

By using PI speed control ($q$ – axis voltage control), the unstable region is improved comparing with the case of I control in system B.

It is considered that the system B is superior to the system A because its simple structure in the case of speed control applications.
References


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