NONLINEAR FLOW AND FRACTAL PROPERTIES OF
ROCK FRACTURE NETWORKS

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ABSTRACT

Permeability is a crucial hydro-mechanical property of rock masses and is important in many areas of geosciences and geoengineering, including water resources management, contaminant pollution control, and petroleum reservoirs. Fracture network modelling techniques that are rapidly developed since 1980s have been adopted as one of the effective methodologies to investigate the permeability of fractured rock masses (i.e., granite and basalt), since fractures play a more significant role in conducting fluid flow and solute transport, comparing with that of the rock matrix. However, there are still many problems that are not solved when calculating the permeability, due to the numerous uncertainties such as fracture geometries, locations, and stress environment in the un-visual underground and the limitation of computing power. This thesis is focused on contributing to two aspects: nonlinear flow and fractal properties of fractured rock masses, both experimentally and numerically.

First, a review study is presented to introduce previous studies on estimating equivalent permeability of discrete fracture networks (DFNs). Mathematical expressions of the equivalent permeability are summarized with the geometric properties, including (i) fracture length distribution, (ii) aperture distribution, (iii) boundary stress, (iv) fractal dimension, (v) dual-porosity models, (vi) anisotropy, and (vii) model size. The hydraulic properties of 3-D fracture networks are also reviewed, and the results are compared with those of 2-D fracture networks. The results of the previous studies show that (1) the equivalent permeability of fracture networks is strongly correlated to the geometric properties of fractured rock masses, e.g., length, aperture, orientation, and connectivity of fractures. Fractal dimension is a useful tool to represent the geometric properties and can be utilized to predict the value of equivalent permeability. (2) the permeability tensor is utilized to represent the anisotropy of the fractured rock masses, and is affected significantly by the scale effects of the DFN models, and (3) the equivalent permeability of the 2-D DFN models is usually underestimated by a factor of 3 to up to 3 orders of magnitude, since the 2-D fracture network is just a cut plane of the 3-D model.

Second, high-precision fluid flow tests and numerical simulations by solving the Navier-Stokes equations are conducted to investigate the nonlinear flow properties of fluid in both intersections and fracture networks. The results of fluid flow at intersections show that with the increment of the hydraulic gradient, the ratio of the flow rate to the hydraulic gradient decreases. When taking account of the fracture
surface roughness with $JRC = 0 \sim 20$, the ratio of the flow rate to the hydraulic gradient would reduce by $0 \sim 26.55\%$. Here, $JRC$ is the abbreviation of joint roughness coefficient. The influences of the intersecting angles on the normalized flow rate and the ratio of the hydraulic aperture to the mechanical aperture can be negligible when the hydraulic gradient is less than $10^{-3}$. However, they would change significantly when the hydraulic gradient is larger than $10^{-2}$ and the influences of the intersecting angles have to be considered. An empirical expression is proposed to calculate the hydraulic gradient and its predictions agree well with the numerical simulation results. For the ratio of mechanical aperture to the radius of the truncated model larger than $10^{-2}$, the ratio of the hydraulic aperture to the mechanical aperture varies significantly and the scale effects have to be considered, while for the ratio of mechanical aperture to the radius of the truncated model less than $10^{-3}$, the scale effect is less significant and can be neglected. The results of fluid flow in fracture networks show that the relationship between hydraulic gradient and flow rate can be well quantified by Forchheimer’s law; when the hydraulic gradient drops below the critical hydraulic gradient, it reduces to the widely used cubic law, by diminishing the nonlinear term. Larger apertures, rougher fracture surfaces, and a greater number of intersections in a DFN would result in the onset of nonlinear flow at a lower critical hydraulic gradient. Mathematical expressions of the critical hydraulic gradient and the coefficients involved in Forchheimer’s law were developed based on multi-variable regressions of simulation results, which can help to choose proper governing equations when solving problems associated with fluid flow in fracture networks.

Finally, the fractal properties of rock fracture networks are characterized and a fractal model is proposed to link the fractal characteristics with the equivalent permeability of the fracture networks. The fractal dimension $D_f$ that represents the tortuosity of the fluid flow and another fractal dimension $D_l$ that represents the geometric distribution of fractures in the networks, are introduced into the model. The results indicate that the equivalent permeability of a fracture network can be significantly influenced by the tortuosity of the fluid flow, the aperture of the fractures and a random number used to generate the fractal length distribution of the fractures in the network. The correlation of fracture number and fracture length agrees well with the results of previous studies, and the calculated fractal dimensions $D_f$ are consistent with their theoretical values, which confirms the reliability of the proposed fractal length distribution and the stochastically generated fracture network models. The optimal hydraulic path can be identified in the longer fractures along the fluid flow direction. Using the proposed fractal model, a mathematical expression between the equivalent
permeability $K$ and the fractal dimension $D_f$ is proposed for models with large values of $D_f$. The differences in the calculated flow volumes between the models that consider and those that do not consider the influence of fluid flow tortuosity are as high as $17.64\% - 19.51\%$, which emphasizes that the effects of tortuosity should not be neglected and should be included in the fractal model to accurately estimate the hydraulic behavior of fracture networks. Based on the new proposed fractal model, a governing equation for fluid flow in fractures that considers the effects of tortuosity and takes into account the out-of-plane geometry of fractures was proposed, and the REV size of DFNs and the effect of random number on equivalent permeability were estimated. The results show that the flow rate in the proposed governing equation for fluid flow in single fractures is proportional to $e^{6-D_T}$, where $D_T$ ranges from 1 to 2. This model fits better with several datasets of in-situ measurements than the cubic law in which the flow rate is proportional to $e^3$. By taking into account the out-of-plane geometry of fractures, the proposed governing equation incorporated the 3-D geometry of opening-mode fractures into a 2-D framework to facilitate efficient solutions for the fluid flow in DFNs. The REV decreases with increasing $D_f$, because the flow paths become more homogeneous as increasing number of fractures in a DFN. The random number utilized to generate the fracture length has larger impacts on the calculated equivalent permeability than those for generating the orientation and center point of fractures.

Keywords: Discrete fracture network; Nonlinear flow; Fractal dimension; Critical hydraulic gradient; Scale effect; Surface roughness; Intersection; Inertial effect; Navier-Stokes equations; Forchheimer’s law; Cubic law; Dead-end fracture; Particle transport; Tortuosity
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1 Introduction

1.1 Background

In recent years, discrete fracture network (DFN) modeling techniques have been extensively applied to investigations of hydro-mechanical and mass transport behaviors of fractured rock systems [Leung and Zimmerman 2012; Lang et al. 2014]. Many studies presumed that fluid flow in each single fracture in DFNs follows the cubic law, which suggests a linear relationship between the flow rate and the pressure drop in the calculations of fluid flow in fracture systems [Long et al. 1982; Jing 2003; Min et al. 2004; Liu et al. 2014]. However, accurate estimations by the cubic law that neglects the inertial effects can only be anticipated for sufficiently low Reynolds numbers (Re), and past studies have revealed that the flow rate could be nonlinearly related to the pressure drop when the applied pressure/flow rate becomes large [Cooke 1973; Holditch and Morse 1976; Gale 1984; Elsworth and Doe 1986; Jung 1989; Kohl et al. 1997; Yeo and Ge 2001; Wen et al. 2006].

Field mapping provides information of geometric characteristics of fractures for establishing DFN models, which could then be used for assessing the fluid flow characteristics numerically. A number of parameters are involved in the description of fracture geometries in rock masses (e.g., length, aperture, and dip angle). It is still a challenging and time consuming task to accurately obtain the geometric information of each single fracture in a field at both the macro-structural (i.e., geometry of the fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [Oliveira and Graca 1987; Hudson and Harrison 1997; Yu and Cheng 2002; Liu et al. 2015]. As an alternative, a number of mathematical expressions have been proposed to represent the characteristics of fracture distributions and to describe fluid flow behavior in single fractures with complex void geometries. Jafari and Babadagli [2008; 2009] have showed that the density and length of fractures are the two most critical parameters for calculating the equivalent fracture network permeability (EFNP). De dreuzy et al. [2001a; 2001b; 2002] have semi-empirically found that the fracture lengths follow a power law distribution, yet it has to heavily account for the length of each fracture to determine the power law exponent. Other studies have shown that the geometric distributions of fractures in fracture networks, especially the length of fractures, exhibit fractal characteristics, which presented a promising approach to quantitatively estimate the characteristics of fracture distributions. [e.g., Vermilye 1995; Hatton 1994; Renshaw 1997; Jiang et al. 2006].
1.1.1 Nonlinear flow in fracture networks

Previous studies have focused on the determination of the critical \(Re\) for the onset of nonlinear flow through single rock fractures, which suggested different ranges from 0.001 to 100 that vary with the surface roughness of fractures and applied normal and/or shear stresses [Witherspoon et al. 1980; Oron and Berkowitz 1998; Zimmerman et al. 1996, 2004; Koyama et al. 2008; Xiong et al. 2011; Radilla et al. 2013; Javadi et al. 2014; Chen et al. 2015]. The surface roughness that contributes to complex void geometries and streamline structures can significantly reduce the critical \(Re\) for the flow regime transition [Parrish 1963; Zimmerman et al. 2004; Konzu and Kueper 2004; Ranjith and Darlington 2007; Javadi et al. 2010; Tzelepis et al. 2015]. Studies on fluid flow through crossed fractures revealed complex nonlinear flow patterns when \(1 < Re < 100\) and different mixing behaviors, bounded by complete mixing and streamline routing within fracture intersections [Stockman et al. 1997; Kosakowski and Berkowitz 1999; Mourzenko et al. 2002].

The \(Re\) was typically incorporated in some criteria for detecting the nonlinear flow through single fractures, which may not apply to DFNs, because flow in each single fracture can have a different \(Re\), and the assessment of the values of localized \(Re\) in DFNs with large amounts of fractures would be a tough task. In contrast, the hydraulic gradient \((J)\), defined as the ratio of hydraulic head difference to DFN side length, is typically a known parameter in many practices on fractured rock masses, such as hydraulic pumping tests with prescribed hydraulic pressures. For a single fracture, the magnitude of \(Re\) is proportional to that of \(J\), and \(J\) is also a dimensionless parameter representing how fast a pressure drops over a given region. Therefore, \(J\) may be a more practical parameter for establishing a criterion for the onset of nonlinear flow in DFNs.

Although the mechanisms, such as the formation of vortices in the positions with dramatic geometric variations at a large \(Re\) that drive the nonlinear flow in rough-surfaced fractures and fracture intersections, have been extensively investigated [e.g., Kosakowski and Berkowitz 1999; Zimmerman et al. 2004; Koyama et al. 2008], the impacts of these micro-phenomena on macro hydraulic properties of DFNs have not been quantitatively estimated. Due to the enormous difficulties of establishing DFN models to consider the roughness of each single fracture and of solving the Navier-Stokes (NS) equations composed of a set of coupled nonlinear partial derivatives of varying orders [Zimmerman and Bodvarsson 1996; Brush and Thomson 2003; Javadi et al. 2010], most previous works presumed that the cubic law was always applicable, disregarding the magnitude of \(J\), such as \(J = 1\) [Long 1982; Zhang et al. 1996; Zhang et
al. 1999; Klimczak et al. 2010; Zhao et al. 2010; Zhao et al. 2011; Liu et al. 2015], \( J = 0.1 \) [Zhang and Sanderson 1996], \( J = 0.001 \) [Cvetkovic et al. 2004; Zhao et al. 2013], and \( J = \) unknown constants [Min et al. 2004; Baghbanan and Jing 2007; 2008; Parashar and Reeves 2012; Reeves et al. 2013; Latham et al. 2013]. It is therefore a crucial issue to determine the critical hydraulic gradient \( (J_c) \) for the onset of nonlinear flow in DFNs, below which the widely used cubic law is sufficiently applicable, and above which some nonlinear governing equations (e.g., Forchheimer’s law) need to be employed.

1.1.2 Fractal properties of fractured rock masses

During the past four decades, many researches have verified that fracture length exhibit fractal properties and they have also tried to link the equivalent permeability of fractured rock masses to the fractal dimension of fracture network backbone. Barton and Larsen [1985], La Pointe [1988], and Barton and Hsieh [1989] found that natural fracture patterns exhibit fractal characteristics based on statistical analysis of natural cracks and fractures. Babadagli et al. [2001] mapped natural fracture patterns of 2-D fracture networks of geothermal reservoirs at different scales. They observed that the fracture networks exhibit scale-invariant properties, however, fractal dimensions might significantly differ when the mass dimensions were measured by different methods. Bagde et al. [2002] calculated the fractal dimensions of blasted fragments and in situ rock blocks using size distribution curves. They concluded that the change of fractal dimension is nominal beyond a uniaxial compressive strength (UCS) value of 20 MPa, while there is a sharp increase in fractal dimension for rock mass rating (RMR) greater than 40. Zhao et al. [2009] found that the random distribution of fractures in a geologic mass agrees well with the fractal law. Their observations and statistics based on the data of three sites demonstrated that fracture distribution of each group, classified by the strike of the strata, still follows the fractal law, although the fractal dimension varies with different strikes to some extent. Zheng and Yu [2012] established a fractal permeability model for gas flow through dual-porosity media by embedding fractal-like tree networks. Their calculation results showed that the porous matrix can be seen as a gas storage medium with negligible contributions to gas flow and the permeability of a dual-porosity medium may primarily be controlled by the fractures. Jafari and Babadagli [2012] analyzed the influences of fracture network characteristics (density, length, orientation, connectivity, and aperture) on permeability using different calculation methods of the fractal dimension. A nonlinear multivariable regression was derived to estimate the equivalent fracture permeability based on five independent
variables. Jafari and Babadagli [2013] later presented the relationship between percolation-fractal properties and permeability of 2-D fracture networks. They found that the fractal dimension of fracture lines obtained using the box counting method yields a more accurate estimation, comparing with the fractal dimensions of intersection point, connectivity index, and scanning lines in X- and Y- directions, for EFNP. Kruhl [2013] reviewed the applications of fractal-geometry techniques in the quantification of complex rock structures considering the scale effect, inhomogeneity, and anisotropy of rock masses. Miao and Yu [2015] derived an analytical expression for permeability of fractured rocks involving fractal dimensions for representing the fracture area, area porosity, fracture density, the maximum fracture length, aperture, fracture azimuth, and fracture dip angle. In most previous 2-D DFN models, the fractures were typically treated as straight lines, without considering their geometric properties in the out-of-plane orientations. The hydro-mechanical properties of single rough rock fractures have been extensively studied experimentally and numerically [e.g., Barton and Choubey 1977; Zimmerman et al. 2004; Xiong et al. 2011], however, fractures in DFN models are still treated typically as parallel-plate models to allow the application of the cubic law, in which the flow rate is proportional to the cube of aperture by assuming a unit value of fracture width that represents the out-of-plane geometry of fractures. Klimczak et al. [2010] has addressed the influences of fracture width and derived a modified cubic law showing that flow rate is proportional to the quantic of the aperture, yet they did not consider the effects of fracture surface roughness. Most of the previous studies [i.e., Barton and Larsen, 1985; Zhao et al., 2009; Jafari and Babadagli, 2012] found that fracture length distribution shows fractal properties by measuring field mapping data, rather than theoretically deriving some expressions for fracture length distribution. Miao and Yu [2015] has stressed the influences of tortuosity on the total equivalent permeability of a fracture network, yet their model assumed that each fracture cuts through the model without considering the orientations and intersections of the fractures, which, to some extent, deviates from the engineering practices. These issues need to be addressed.

1.2 Objective and thesis structure

The objective of this thesis is to investigate the nonlinear flow and fractal properties of rock fracture networks.

Chapter 1 gives a brief introduction of the background, the objective and the structure of this thesis.
Chapter 2 provides a review of estimating the equivalent fracture network permeability (EFNP) of discrete fracture networks (DFNs) by considering the geometric properties of the fractured rock masses. The previous studies of mathematical expressions of the equivalent permeability are summarized with the geometric properties, including (i) fracture length distribution, (ii) aperture distribution, (iii) boundary stress, (iv) fractal dimension, (v) dual-porosity models, (vi) anisotropy, and (vii) model size. The hydraulic properties of 3-D fracture networks are also reviewed, and the results are compared with those of 2-D fracture networks. Finally, the potential future works are addressed.

In Chapter 3, the derivations of the Navier-Stokes equations and its simplification forms, such as Stokes equations, Reynolds equations, and cubic law were presented. According to the geometries of fracture surface and the flow rate, the associated governing equations should be selected.

In Chapter 4, to clearly understand the mechanisms of fluid redistribution and nonlinear flow characteristics, as well as the mathematical descriptions of the hydraulic aperture for each segment connected to the fracture intersections, fluid flow tests are carried out and corresponding numerical simulations by solving the Navier-Stokes equations are performed to exhibit the existence of nonlinear relationship of the flow rate and the pressure drop at fracture intersections. Then, large scale numerical models are established to extensively study the roles of hydraulic gradient, fracture surface roughness, intersecting angle, and scale effect. Finally, an empirical expression is proposed to calculate the hydraulic aperture mathematically.

Chapter 5 deals with nonlinear fluid flow behavior in rock fracture networks. A series of DFNs with different apertures, roughness, and numbers of intersections are established. Based on multi-variable regressions of the simulation results, mathematical expressions of the critical hydraulic gradient, \( J_c \), and the coefficients involved in Forchheimer’s law (\( A \) and \( B \)) are established. These expressions are then applied to another series of DFNs with well-known geometric characteristics of fractures to verify their validity by comparing the predicted results with the fluid flow simulation results, and their nonlinear flow behaviors are analyzed and discussed.

Chapter 6 focuses on the nonlinear flow properties and approaches for investigating solute transport in the DFNs that contain hundreds/thousands of fracture segments and intersections. In this Chapter, the fluid flow and solute transport processes within single fracture intersections, and the effects of intersection and dead-end of fractures on nonlinear flow and particle transport in 2-D DFNs were investigated both experimentally and numerically.
In Chapter 7, a fractal model is established to assess the equivalent permeability of 2-D rock fracture networks. The fractal dimension $D_T$ and the fractal dimension $D_f$ are used in the model to represent the effects of the tortuosity of fluid flow in the fractures (micro-scale) and the geometric characteristics of the fracture distributions (macro-scale), respectively. Fluid flow is simulated in the generated fracture network models, and the relation between the fractal dimension and the equivalent permeability is analyzed.

In Chapter 8, a multiple fractal model that considers the fractal properties of both porous matrices and fracture networks is proposed for the permeability of dual-porosity media embedded with randomly distributed fractures. In this model, the aperture distribution is verified to follow the fractal scaling law, and the porous matrix is assumed to comprise a bundle of tortuous capillaries that also follow the fractal scaling law. Analytical expressions for fractal aperture distribution, total flow rate, total equivalent permeability, and dimensionless permeability are established, where the dimensionless permeability is defined as the ratio of permeability of the porous matrices to that of the fracture networks.

Chapter 9 summarizes the major conclusions and provides some discussions of this thesis.

References:


Min, K. B., L. Jing, O. Stephansson (2004), Determining the equivalent permeability tensor for fractured rock masses using a stochastic REV approach: method and application to the field data


2 Mathematical expressions for characterizing equivalent permeability of rock fracture networks: A review

2.1 Introduction

Permeability is a crucial hydro-mechanical property of rock masses and plays an important role in applications of many projects in geosciences and geoengineering, such as CO₂ sequestration, enhanced oil recovery, geothermal energy development, and risk assessment of water inrush in karst tunnels and coal mines [Li et al., 2014a; 2014b; 2014c; Lei et al., 2014; 2015a]. The permeability of a fracture network is governed by the inherent properties of fractures themselves (i.e., surface roughness, aperture, orientation, density, persistence, infilling, and intersection) and the applied mechanical and hydraulic pressures (i.e., boundary stress and hydraulic pressure). Because the permeability of fracture networks is usually much greater than that of the rock matrix in the fractured rock masses (i.e., granite and basalt) [Louis 1969; Murphy et al., 2004], the discrete fracture network (DFN) approach that neglects fluid flow in rock matrix has gained much attention in recent explorations [Long et al, 1982; Jing, 2003].

Previous review works have focused on the coupled hydro-mechanical behaviors of rock masses. Rutqvist and Stephansson [2003] reviewed the studies on hydro-mechanical (HM) couplings of rocks since 1960s. The HM interaction is one of the main reasons of a series of events, such as landslides, dam failure, and injection-induced earthquakes. Analysis on laboratory and field measurements indicated that stress can significantly affect the permeability of rock masses at shallow depth and in areas of low in-situ permeability. In highly permeable fractured rock masses, fluid flows mainly in clusters of connected fractures. Berkowit [2002] analyzed the measurements, conceptual pictures and mathematical models of fluid flow and transport phenomena in fractured systems, focusing on the following four important issues: (i) geometric distributions of fracture networks, (ii) water flow, (iii) solute transport and reactive models, and (iv) two-phase flow and transport. Jing [2003] reviewed the techniques, advances, problems, and future developments in numerical modeling for rock mechanics. The special nature of rock masses was discussed when modeling the inherent characteristics of discontinuousness, anisotropy, inhomogeneity, and inelasticity of rock masses. Couplings between hydraulic, mechanical and thermal
processes were characterized for different models. Jiang et al. [2008] reviewed and analyzed the previous studies on the fluid flow characteristics of rock fractures, including the relationship between mechanical aperture and hydraulic aperture, the coupled shear-flow properties of fractures, and the visualization techniques for measuring fluid flow and solute transport in fractures [Koyama et al., 2008a]. Ma [2015] presented a review on the permeability evolution model for fractured porous media. Different models and functions to calculate the permeability of fracture networks were summarized, including the permeability evolution models based on porosity, the permeability functions based on the stress and damage concepts, equivalent channel models, pore network models, and so on. Tsang et al. [2015] reviewed the studies on hydrologic issues associated with nuclear waste repositories in three rock types (fractured crystalline rock, unsaturated tuff, and clay-rich formations) over the last 35 years. These studies were focused on flow and transport in saturated fractured rocks from a depth of 500 m to the shallow subsurface, flow through unsaturated zone from the shallow subsurface to the repository depth, and hydrologic issues in clay, for the three different rocks, respectively. They stated that although much work has been done on hydrology, the future works would be focused on the coupled thermo-hydro-mechanical (THM) processes.

To date, there are still no generic mathematical expressions to calculate the equivalent permeability of fracture networks, because the equivalent permeability is influenced by both the geometric properties of fracture networks and the surrounding mechanical and hydraulic environments, primarily including the following important parameters: (i) fracture length distribution, (ii) aperture distribution, (iii) fracture surface roughness, (iv) fracture dead-end, (v) number of intersections, (vi) hydraulic gradient, (vii) boundary stress, (viii) anisotropy, and (ix) scale (see Fig. 2-1). Previous works were focused on one or several of these parameters by neglecting others. To systematically understand the relationship of equivalent permeability with these parameters, this work attempts to review the mathematical expressions for characterizing equivalent permeability of 2-D rock fracture networks reported during the last 35 years. Recent developments of 3-D fracture networks are briefly reviewed, and the potential future works are addressed. Vast studies have been reported in literature regarding to the estimation of permeability of fractured rock masses. In this review, we only attempt to summarize the well-known and/or well-accepted mathematical expressions for most parameters. When few works and mathematical expressions have been reported on a parameter (e.g., number of intersections and hydraulic gradient), unsolved problems along with recent developments and studies
done by the authors are presented. Since great success has been achieved by applying fractal theories to characterize the geometric properties of fracture networks, mathematical models based on fractal theories are highlighted. This work aims to provide a reference for engineers and researchers especially the beginners to achieve a quick access to the existing mathematical models related to estimations of the permeability of fractured rock masses.

Fig. 2-1 Schematic view of the geometric parameters involved in a fracture network, including: (i) fracture length, (ii) aperture, (iii) surface roughness, (iv) dead-end, (v) number of intersections, (vi) hydraulic gradient, (vii) boundary stress, (viii) anisotropy, and (ix) scale.

\[ J = \frac{(P_1 - P_2)l}{\rho g L_0} \]
2.2 2-D fracture networks

Natural fractures can vary over several orders of magnitude from pore scale to kilometer scale [Bonnet et al., 2001]. It is a challenging task to perfectly represent the real 3-D fracture networks in numerical models, predominantly due to the difficulty of measuring and characterizing fractures inside of the rock masses [De Dreuzy et al., 2004]. The insufficient information of the fractures obtained from outcrops or intersecting well-bores is usually utilized to generate 2-D DFNs using the Monte Carlo method [Parashar and Reeves, 2012]. The equivalent permeability of a DFN, which can be calculated by using the classical Darcy’s law for fluid flow in fracture networks [Long et al., 1982; Baghbanan and Jing, 2007], is commonly utilized as a practical parameter to characterize the conductivity of fractured rock masses [Long and Witherspoon, 1985; Hestir and Long, 1990]. Fluid flow through single fractures is assumed to obey the cubic law, in which the flow rate is linearly proportional to the pressure drop when the Reynolds number is sufficiently low [Min and Jing, 2003; Javadi et al., 2010; Adler et al., 2013; Liu et al., 2016]. By applying constant hydraulic pressure drops on the two opposite boundaries of a DFN and by assuming that the other boundaries are impermeable, the equivalent permeability of a DFN can be calculated based on the hydraulic pressure - flow rate relationship when the fluid flow reaches the steady state [Zimmerman and Bodvarsson, 1996b; Zhao et al., 2014a]. Here, the equivalent permeability could be back-calculated according to the Darcy’s law, as follows [Min and Jing, 2003; Liu et al., 2015a]:

\[ K = \frac{\mu Q}{A \rho g J} \]  \hspace{1cm} (2-1)

where \( K \) is the equivalent permeability of a fracture network, \( Q \) is the macroscopic flow rate, \( \rho \) is the fluid density, \( A \) is the cross-sectional area, \( g \) is the gravitational acceleration, and \( J \) is the hydraulic gradient.

2.2.1 Permeability and fracture length distribution

The length distribution of fractures is one of the most important parameters to calculate the equivalent permeability of a rock fracture network. In some early works, due to the lack of in-situ measurement of fracture length, fractures were typically assumed to have constant lengths [Englman et al., 1983; Robinson, 1983; 1984; Balberg et al., 1984; 1991; Gueguen and Dienes, 1989]. Actually, in natural fractured rock masses, the fracture length has very broad distributions and is observed to follow power
law, exponential, and lognormal types of functions [Chelidze and Guguen, 1990; Chang and Yortsos, 1990; Sahimi, 1993; Watanabe and Takahashi, 1995a; 1995b; Andrade et al., 2009; Torabi and Berg, 2011; Kolyukhin and Torabi, 2012; 2013]. Among them, the power law distribution has been most widely utilized [Segall and Pollard, 1983; Gudmunsson, 1987; Villemin and Sunwoo, 1987; Childs et al., 1990; Sornette et al., 1993; Davy, 1993; Bour and Davy, 1997; Bogdanov et al., 2007; Reeves et al., 2013], with a typical form of

\[ n(l) = \alpha_l l^{-\alpha_1} \]  

(2-2)

where \( n(l)dl \) is the number of fractures having a length in the range of \([l, l + dl]\), \( \alpha_l \) is the proportionality coefficient for fracture number - length relationship, and \( \alpha_1 \) is the power law exponent for fracture number - length relationship, varying generally between 1 and 3.5 due to the influences of stress history, linkage of faults, sampling bias, and size of dataset [Dverstorp and Anderson, 1989; Tsang et al., 1996; Liu et al., 2015a], and etc. De Dreuzy et al. [2001a, 2001b] reported that a fracture network is comprised of many small fractures and the classical percolation model is applicable when \( \alpha_1 \geq 3 \); whereas for \( \alpha_1 \leq 2 \), the largest fractures make up the DFN model and govern the main flow paths. Between the two bounds, i.e., \( 2 < \alpha_1 < 3 \), relatively uniform fluid flow occurs in all of the fractures.

Based on power law distribution, Bour and Davy [1997] conducted a theoretical and numerical study on the connectivity of fracture networks, and proposed a criterion at which fractures are always connected, i.e., at least one flow path exists in a model. Then, mathematical expressions of equivalent permeability of DFNs were derived, written as:

\[ K(p) \sim (p - p_c)^v \]  

(2-3)

\[ K(L) \sim L^{-\psi/v} \]  

(2-4)

where \( p \) is the probability that any site or bond belongs to the infinite cluster, \( p_c \) is the percolation threshold, \( L \) is the size of the fracture system, and \( \psi \) and \( v \) are two coefficients depending on the Euclidean dimension of the system [Stauffer and Aharony, 1992].

De Dreuzy et al. [2001a] subsequently linked the number of fractures to the fracture center density within a truncated fracture length range and proposed a power law length distribution as:
\[ n(l) = d_C L^2 \cdot (a_1 - 1) \cdot \frac{l_{\text{max}}^a}{l_{\text{min}}^{a_1}} \quad \text{for} \quad l \in [l_{\text{min}}, l_{\text{max}}] \]  
\hspace{1cm} (2-5)

where \( l_{\text{min}} \) and \( l_{\text{max}} \) are the minimum and the maximum fracture length, and \( d_C \) is the fracture center density (number of fracture centers per unit area).

Calculation results showed that the permeability depends on the values of \( a_1 \) and \( p \) as shown in Table 2-1. According to Eq. (2-5), when \( a_1 \) is infinitely large, the fracture network is made up of fractures with a constant length of \( l_{\text{min}} \), and the percolation theory applies. On the contrary, when \( a_1 \) is smaller than 2, the largest fractures dominate the main flow paths of the fracture networks [Klimczak et al., 2010].

<table>
<thead>
<tr>
<th>( 1 &lt; a_1 &lt; 2 )</th>
<th>( 2 &lt; a_1 &lt; 3 )</th>
<th>( a_1 &gt; 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p &lt; p_c )</td>
<td>( K \sim \frac{p}{L} \cdot T_f )</td>
<td>( K \sim \frac{p}{L} \cdot T_f )</td>
</tr>
<tr>
<td>( p \sim p_c )</td>
<td>( K \sim \frac{p}{L} \cdot \xi \cdot T_f )</td>
<td>( K \sim \frac{p}{L} \cdot \xi \cdot T_f )</td>
</tr>
<tr>
<td>( p &gt; p_c )</td>
<td>( K \sim \frac{p - p^*}{L} \cdot \xi \cdot T_f )</td>
<td>( K \sim \frac{p - p^*}{L} \cdot \xi \cdot T_f )</td>
</tr>
</tbody>
</table>

where \( p^* \) is the generalized expression of a percolation, which is contributed by the fracture tips that do not conduct fluid flow, \( T_f \) is the fracture transmissivity, \( \xi \) is a characteristic scale, \( \mu/\nu \) is a constant that equals to 0.9826 [Grassberger, 1999], \( C \) is a dimensional function depending only on the geometric parameters of the model, and \( p_c \) is a characteristic exponent.

Leung and Zimmerman [2012] generated DFNs with fracture lengths following the power law distribution, and mathematically related the fracture network permeability to the complex fracture geometric properties (Eq. (2-6)). Their method is able to estimate the permeability of fracture networks that span over 10 orders of magnitude.

\[ K = K_0 B \sqrt{1 + 2 \xi} \frac{N \bar{I}}{2L} \]  
\hspace{1cm} (2-6)
where $K_0$ is the permeability of a fracture network consisting of an orthogonal pair of fractures, each passing straight through the system, $B$ is a dimensionless constant of proportionality that has a value of 9.26E-02, $N$ is the total number of fractures, $\overline{l}$ is the arithmetic mean of all the lengths of fractures, and $\zeta$ is the overall connectivity of a fracture network.

Since Mandelbrot et al. [1982] introduced the concept of fractal dimension to represent the size distribution of the islands on the surface of the earth, fractal dimension has been rapidly utilized in many scientific aspects in geology and hydrology [e.g., Yu and Li, 2001; Jiang et al., 2006b; Wu and Yu, 2007; Yun et al., 2008]. Researchers found that the distribution of pores in porous matrix has fractal properties [Yu and Cheng, 2002; Yu et al., 2005; Yun et al., 2009; Cai et al., 2010; Miao et al., 2014], and other researchers verified that the fracture distribution in fractured rock masses also exhibits fractal characteristics at both the macro-structural (i.e., geometry of fracture networks) [Berkowitz and Hadad, 1997; Babadagli, 2001; Bagde et al., 2002; Jiang et al., 2006a; Kruhl, 2013; Miao et al., 2015a; Liu et al., 2015a] and micro-structural (i.e., geometry of single fractures) [Odling, 1994; Kulatilake et al., 1995; Babadagli and Develi, 2003; Jiang et al., 2006b; Askari and Ahmadi, 2007; Li and Huang, 2015; Babadagli et al., 2015] levels.

The results of a site characterization at the Sellafield area, Cumbria, England, conducted by the United Kingdom Nirex Limited [Nirex, 1997; Andersson and Knight, 2000], showed that the fracture length follows a fractal scaling law. The cumulative probability density function of fracture length was derived using the concept of fractal dimension as follows [Min et al., 2004; Baghbanan and Jing, 2008; Jing et al., 2009; Zhao et al., 2010b]:

\[
(l - l_{min}) - R(l_{max} - l_{min}) = \frac{1}{D_f}
\]

(2-7)

where $l$ is the fracture length, $D_f$ is the fractal dimension of the fracture backbone, and $R$ is the uniformly distributed random number in the range $[0, 1]$.

Jafari and Babadagli [2008; 2009; 2011] found that the equivalent fracture network permeability (EFNP) is strongly correlated to the fractal properties of 2-D fracture networks, based on the in-situ measurements of outcrops of geothermal reservoirs in southwestern Turkey. They [Jafari and Babadagli, 2012; 2013] mathematically established two formulations to calculate the EFNP as follows.

\[
\ln(K) = 3.724\exp(0.556X_1) + 1.265\ln(X_2) + 0.990\ln(X_3) + 11.62, \ R^2 = 0.9568
\]

(2-8)
where $X_1$ is the connectivity index, $X_2$ is the box-counting dimension of fracture lines, and $X_3$ is the hydraulic conductivity.

$$K = 88.411 (\rho' - \rho_c)^{0.4602}, \quad R^2 = 0.8494 \quad (2-9)$$

where $(\rho' - \rho_c)$ is the dimensionless percolation threshold, in which $\rho'$ is the dimensionless density. Studies of Barton and Larson [1985] and Nolte et al. [1989] revealed that $\rho_c = 3.6$. The validity of these expressions has been verified by applying them to different fracture patterns that span a wide range of length values.

Liu et al. [2015a] derived a fractal model to estimate the equivalent permeability of randomly distributed rock fracture networks, in which the fracture length follows a fractal distribution (Eq. (2-10)) and the fluid flow is governed by Eq. (2-11).

$$l_i = \frac{l_{\min}}{(1 - R_i)^{2/D_f}} = \left( \frac{l_{\min}}{l_{\max}} \right) \frac{l_{\max}}{(1 - R_i)^{2/D_f}} \quad (2-10)$$

where $l_i$ is the length of the $i$th fracture.

$$q(i) = \frac{c_i^{2/D_f} \Delta P_i}{12 \mu l_{\min}^{2/D_f}} (1 - R_i)^{2D_f/D_f} \quad (2-11)$$

Where $q(i)$ is the fluid volume through the $i$th fracture, $\Delta P_i$ is the local hydraulic pressure difference applied between the tips of the fracture, $\mu$ is the viscosity of fluid, and $D_f$ is the fractal dimension of the nonlinear streamline of fluid flow. By performing numerical simulations on the DFNs generated with 10 sets of random numbers, the average equivalent permeability was found to be correlated with the fractal dimension, $D_f$, as:

$$K = 2E - 13 \exp(0.9784D_f) \quad (2-12)$$

Assumptions were made that the fracture center points and orientations are randomly and uniformly distributed and the apertures are correlated to the trace lengths of fractures. Their results showed that the equivalent permeability varies by approximately 5 orders of magnitude when $D_f = 1.3$, and the range of variation decreases to less than 2 orders of magnitude when $D_f = 1.8$, due to the influences of random numbers utilized to generate fracture lengths, orientations, and locations.

Miao et al. [2015a] developed an analytical fractal model to link the fracture network permeability to the fractal dimension of fracture length distribution, fracture area porosity, fracture azimuth, fracture dip angle, and the ratio of the maximum to the minimum length of fractures, written as:
\[
K = \frac{\alpha^3 d_c}{128} \frac{1 - D_f}{4 - D_f} \left[ \frac{1 - \cos^2 \alpha' \sin^2 \theta}{1 - \left( \frac{\rho}{\phi} \right)^{1 - \delta_f}} \right]^{1/3}
\]

(2-13)

where \( \alpha' \) is the fracture azimuth, \( \theta \) is the fracture dip angle, \( \phi \) is the fracture porosity, and \( \alpha_2 \) is the proportionality coefficient in the fractal model. The validity of the proposed model was verified by comparing to the predictions of available numerical simulation results.

Natural fractured rock masses are constituted by rock matrix and discontinuities [Cai et al., 2014], and the former one is a kind of porous media. A number of dual-porosity models have been established to quantify the permeability of porous media [Bibby, 1981; Elsworth and Bai, 1992; Vogel et al., 2000; Li and Yu, 2013]. Xu et al. [2008] proposed a dual-domain model and analyzed the radial flow in the heterogeneous porous media embedded with constructal tree networks. The analytical expressions for seepage velocity, local and global permeability, and pressure drop were derived, and the transport properties of the optimal branching structure were discussed. Wang and Yu [2011] established a fractal model to estimate the starting pressure gradient for Bingham fluids in porous media embedded with fractal-like tree networks. They concluded that with the increasing matrix porosity, diameter ratio, and fractal dimension for mother diameters, the starting pressure gradient would decrease, where the mother diameter is defined as the original largest diameter of a fractal-like tree network. Zheng and Yu [2012] established a fractal permeability model for gas flow through dual-porosity media by embedding fractal-like tree networks. Their calculation results showed that the porous matrix can be seen as a gas storage medium with negligible contributions to gas flow and the permeability of a dual-porosity medium may primarily be controlled by the fractures. Malley et al. [2015] presented a generic framework for estimating the volume available in fractures and undamaged shale matrix during hydraulic fracturing. The amount of water stored in fractures was calculated by accounting the volume of fractures based on in-situ measurements, and the amount of water stored in the undamaged shale matrix was estimated by using a two-phase model at the pore-scale and a single-phase model at the continuum scale. An example showed that the water storage is 915 m\(^3\) in fractures and 10600 m\(^3\) in matrix with a ratio of approximate 1/12. Miao et al. [2015b] analytically investigated the permeability of dual-porosity media embedded with random fractures, which widely exist in water and oil reservoirs (Eq. (2-14)). The porous matrix of the media was assumed to consist of a bundle of tortuous capillaries, the sizes of which follow the fractal scaling law. Their results showed that the ratio of the maximum pore/capillary diameter of a porous matrix
to the maximum fracture aperture of a fracture network plays a significant role in calculating the permeability of a dual-porosity medium.

\[ K_t = K_m + K \]  
\[ K_m = \frac{\pi}{128} l_0^{-D_{TP}} \frac{D_p}{A} \left( D_{TP} - D_{TP} \right) \lambda_{\text{max}}^{4 + D_p} \]  
\[ K = \frac{\beta}{128A} D_f \left( 1 - \cos^2 \alpha' \sin^2 \theta \right) l_0^4 \lambda_{\text{max}}^{4 - D_f} \]

where \( K_t \) is the permeability of the dual-porosity medium, \( K_m \) is the matrix permeability, \( l_0 \) is the straight length of a capillary, \( D_{TP} \) is the fractal dimension for tortuosity of tortuous capillaries, \( D_p \) is the fractal dimension for the size distribution of capillaries/pores, \( \lambda_{\text{max}} \) is the maximum capillary/pore diameter.

Other works based on the measurements and simulations of the length of natural fractures, such as Long et al. [1982], Cacas et al. [1990], Guo et al. [2015a; 2015b], suggested that the fracture length is lognormally distributed. To date, there are still no mathematical expressions, if any, to calculate the equivalent permeability when the rock fracture length is lognormally distributed.

Open questions for Section 2.2.1

- What is the minimum required site scale/volume to obtain enough number of fractures, in order to establish a regression function for the length distribution of a fracture network (i.e., the power law length distribution)? Is it equal to the representative elementary volume (REV) that leads to constant permeability for models with a larger dimension than REV?
- What is the physical meaning that the fracture length typically follows power law distributions? Which character decides the exponent of power law?
- Many methods have been proposed to calculate the fractal dimension of an object, for example, the box-counting method [Cheng, 1997], the compass-walking method [Maerz et al., 1990], the \( h-L \) method [Li and Huang, 2015]. How to determine the most suitable method for a specific fracture network?
- Some fracture length distributions, for example, the normal distribution, do not show fractal characters. How to determine whether the fractal theory applies to a fracture distribution?
- How to design an experimental system that allows us to precisely measure the permeability of rock matrix and fractures simultaneously?
- How to incorporate the effect of stress into the correlation between the aperture and...
the length of fractures since the aperture changes dramatically with the changing stresses acting on a DFN.

2.2.2 Permeability and aperture distribution

Previous studies have shown that the magnitude of hydraulic aperture of single fractures, which could be back-calculated from the pressure - flow rate relationship based on the cubic law, is significantly affected by the surface roughness of fractures (e.g., joint roughness coefficient: JRC) [Olsson and Barton 2001], the Reynolds number (Re) [Zimmerman et al. 2004; Xiong et al. 2011], the contact area ratio and contact shapes [Zimmerman and Bodvarsson 1996a; Li et al. 2008], the shear displacement [Koyama et al. 2008b; Javadi et al. 2014], the hydraulic gradient [Roy Guha and Singh, 2015], and etc. Under different stress environments, cracks in rocks would propagate following three modes: opening mode, sliding model, and tearing mode, and therefore, the aperture of each single fracture is usually different [Irwin, 1957; Sih, 1962; Paris and Sih, 1965; Pollard and Segall, 1987]. Observations on aperture distributions obtained from field mapping results have shown that the fracture aperture is correlated with the fracture length [Hatton et al., 1994; Renshaw and Park, 1997; Zimmerman and Main, 2004]. For sliding-mode or tearing-mode fractures, the maximum shearing displacement is generally correlated to the length as [Bonnet et al., 2001]:

\[ \delta_{\text{max}} = a_3 l^{a_2} \]  

(2-15)

where \( \delta_{\text{max}} \) is the maximum shear displacement, \( a_2 \) is the power law exponent for shear displacement - length relationship, and \( a_3 \) is the proportionality coefficient for shear displacement - length relationship, which is related to the mechanical properties of surrounding rocks. A linear relation \( (a_2 = 1) \) was confirmed based on the linear elastic fracture mechanics (LEFM) theory [Pollard and Segall, 1987], the plane strain model given by Cowie and Scholz [1992a], and the in-situ measurements [Cowie and Scholz, 1992a, 1992b; Scholz et al., 1993; Dawers et al., 1993; Scholz, 2002; Kim and Sanderson, 2005; Schultz et al., 2008]. Variation of \( a_2 \) depends on many factors, such as stress history, lithology, growth mechanism of faults, and etc., therefore, different values of \( a_2 \) have been reported varying from 0.5 to 2 [Walsh and Watterson, 1988; Gillespie et al., 1992; Fossen and Hesthammer, 1997].

For opening-mode fractures, previous studies showed a linear relationship between the maximum opening displacement \( (\epsilon_{\text{max}}) \) and the fracture length \( (l) \) [Vermilye and Scholz, 1995]. Considering two sets of dikes and one set of veins in models, Olson
[2003] and Klimczak et al. [2010] found a sub-linear, square root power-law distribution as:

\[ e_{\text{max}} = \alpha_4 l^{0.5} \]  

(2-16)

where \( e_{\text{max}} \) is the maximum opening displacement, \( \alpha_4 \) is the proportionality coefficient for the opening displacement - length relationship. Here, \( e_{\text{max}} \) is correlated with the hydraulic aperture (\( e \)) as [Olson 2003]:

\[ e = \frac{\pi}{4} e_{\text{max}} \]  

(2-17)

where \( e \) is calculated by averaging the displacements.

The square root relationship (Eq. (2-16)) between \( e_{\text{max}} \) and \( l \) agrees well with the field observations [Walmann et al., 1996; Schultz et al., 2008]. Other relationships have also been reported according to the differences of topological attributes and geo-mechanical constraints [Hatton et al., 1994; Renshaw and Park, 1997; Lei et al., 2015a]. In-situ hydraulic tests showed that the fracture aperture (or the permeability per fracture) follows lognormal distributions [Dversorp and Andersson, 1989; Cacas et al., 1990; Li and Zhang, 2010], which were utilized to accommodate the results of field experiments and theoretical models [Long and Billaux, 1987; Charlaix et al., 1987; Margolin et al., 1998]. By assuming that the aperture in fracture networks follows a lognormal distribution, De Dreuzy et al. [2001b; 2002] studied the hydraulic properties of 2-D random fracture networks, and established a mathematical expression for network permeability as:

\[
K(b, a_1, p, L/l_{\text{min}}) = K(b = 0, a_1, p, L/l_{\text{min}}) \cdot \exp \left[ \frac{\omega (a_1, p, L/l_{\text{min}}) b^2}{2} \right]
\]  

(2-18)

where, \( b \) is the second moment of the lognormal permeability distribution and \( b = 0 \) indicates that all of the fractures have the same constant aperture, and \( \omega \) is the upper and lower bounds of the models and is restricted in the range [-1, 1]. The second term \( \exp((\omega/2)b^2) \) is also a kind of effective permeability [Kirkpatrick, 1971].

The lognormal type distribution of fracture aperture exhibits a long tail with larger aperture values, which would deviate significantly for different random numbers and could robustly change the permeability of fracture networks [Baghbanan and Jing, 2007]. Truncation thresholds are usually utilized to decrease the sampling bias of the in-situ measurements [Hudson and Priest, 1983; Priest and Hudson, 1976; 1981; Einstein and Baecher, 1983; Dershowitz and Einstein, 1988]. By assuming a truncated distribution function of apertures that vary from 1 to 200 μm, Baghbanan and Jing [2007] derived an expression to link the aperture to the fracture length by introducing
the concept of fractal dimension. The aperture of each fracture can be calculated as follows:

\[ g'(e) = g'(e_a) + \left( \frac{e^{D_f} - e^{D_a}}{l_{\text{max}} - l_{\text{max}}} \right) \left[ g'(e_b) - g'(e_a) \right] \]  

(2-19)

where, \( g' \) is the error function, \( e_a \) and \( e_b \) are the lower and upper aperture limits, respectively. They found that the variation of permeability is 1 order of magnitude larger when using Eq. (2-19), comparing with the computation using a constant mean value of aperture (65μm). Baghbanan and Jing [2008] and Liu et al. [2014a] later used this function to extensively calculate the directivities of equivalent permeability taking into account the effects of stresses and the lognormal distributions of fracture length, respectively. Baghbanan and Jing [2008] found that the permeability decreases when applying smaller stress ratios (horizontal/vertical stress) at the model boundaries. Liu et al. [2014a] reported that when the fracture aperture is lognormally distributed, the directivity of permeability is negligible. When fracture aperture is correlated with length as in Eq. (2-19), the directivity of permeability is remarkable, resulting in heterogeneous hydraulic characteristics of rock masses.

Open questions for Section 2.2.2:

- How to accurately measure the magnitude of aperture of each single fracture in fractured rock masses? Flow test can help obtain the hydraulic aperture of single rock fractures, however, precise measurement on fractured rock masses with a large number of fractures of different sizes is still a challenging task.
- The infillings that commonly exist in fractures are seldom considered when measuring the mechanical/hydraulic apertures. How much do the infillings impact on the magnitude of hydraulic aperture and consequently influence the permeability of a fracture network?
- The aperture of fractures measured from outcrops should be different from those inside of a rock mass where different fractures may be subjected to different stresses. How to quantify their differences?
- Does the fracture aperture distribution in a fracture network exhibit fractal characteristics? If it is, how to effectively establish mathematical expressions for aperture distribution with the involvement of fractal dimensions based on limited in-situ measurement data?
2.2.3 Permeability and fracture surface roughness

Natural rock fractures typically have rough-walled surfaces, and the surface roughness significantly influences the magnitude of hydraulic aperture, which is an essential parameter for calculating the volumetric flow rate and equivalent permeability. A particle would travel a longer distance along a tortuous path through a rough-walled fracture than through a smooth-walled fracture [Liu et al., 2015a], resulting in a reduction of the overall fracture permeability [Boull et al., 2006]. The surface roughness of natural rock fractures has been widely investigated [e.g., Neuzil and Tracy, 1981; Brown and Scholz, 1985; Moreno et al., 1988; Dubuc et al., 1989; Schmittbuhil et al., 1995; Develi and Babadagli, 1988; Drazer and Koplik, 2000; Crandall et al., 2010; Rasouli and Hosseinian, 2011], and different approaches have been proposed to characterize them, such as roughness coefficient ($Z_2$) [Myers, 1962], JRC [Barton, 1973], and fractal dimension [Babadagli et al. 2015]. The values of $Z_2$ and JRC are correlated with each other as [Myers, 1962; Tse and Cruden, 1979]:

$$Z_2 = \left[ \frac{1}{M} \sum (z_{i+1} - z_i)^2 \right]^{1/2}$$

(2-20)

$$\text{JRC} = 32.2 + 32.47 \log Z_2$$

(2-21)

where $x_i$ and $z_i$ represent the coordinates of the fracture surface profile, and $M$ is the number of sampling points along the length (i.e., $x$ coordinate) of a fracture. Fractal dimension of a single fracture surface can be calculated using different methods, such as divider, box-counting, variogram, spectral, and roughness-length [Zhao et al., 2014a].

Previous studies on the fracture surface roughness have focused on establishing mathematical expressions for the relation between hydraulic aperture and roughness parameters, based on a great amount of flow tests and numerical simulations on single rock fractures [Lomize, 1951; Louis, 1969; Patir and Cheng, 1978; Walsh, 1981; Quadros, 1982; Hakami, 1985; Renshaw, 1995; Waite et al., 1999; Rasouli and Hosseinian, 2011]. Zimmerman and Bodvarsson [1996] established a function that relates the hydraulic aperture ($e$) to the mechanical aperture ($E$), contact ratio ($C_r$), and the standard deviation of mean mechanical aperture ($\sigma_{apert}$) (see Eq. (2-22)). Olsson and Barton [2001] conducted coupled shear-flow tests on single fractures and found that $e$ is also influenced by the mobilized value of JRC (JRC$_{mrb}$), the shear displacement ($u_s$), and the peak shear displacement ($u_{sp}$) (see Eq. (2-23)). In addition to the standard deviation of mean mechanical aperture ($\sigma_{apert}$), Xiong et al. [2011] considered the effects
of the standard deviation of local slope of fracture surface ($\sigma_{\text{slope}}$), and the Reynolds number ($Re$) (see Eq. (2-24)).

$$e^3 \approx E^3 \left[1 - 1.5 \frac{\sigma_{\text{apert}}^2}{E^2} + \cdots \right] (1 - 2C_r)$$  \hspace{1cm} (2-22)

$$e = E^{1/2} JRC_{\text{mob}} \left( u_s \geq u_{sp} \right)$$  \hspace{1cm} (2-23)

$$e^3 = E^3 \left(1 - 1.0 \frac{\sigma_{\text{apert}}}{E} \left(1 - \frac{\sigma_{\text{apert}}}{E} \right) \sqrt{\frac{\sigma_{\text{slope}}}{10}} \sqrt{Re} \right)$$  \hspace{1cm} (2-24)

A more extensive summary of the relation between the mechanical and hydraulic apertures is presented by Zhao and Li [2015]. The influences of fracture surface roughness on the relationships between mechanical aperture and hydraulic aperture is correlated with the magnitude of mechanical aperture. Cornet et al. [2003] based on in-situ experiments at a scale of approximate 1 m, found that when the mechanical aperture is larger than 15 μm, the effect of surface roughness is eliminated and the hydraulic aperture is equal to the mechanical aperture. Although a large number of studies have focused on the influences of surface roughness on the flow behavior of single fractures, works that incorporated the effects of surface roughness into DFN modelling are still few. Zhao et al. [2014a] established a number of DFNs consisted of parallel-walled fractures with the hydraulic aperture of each fracture being correlated with the surface roughness, based on the work of Li and Jiang [2013], who proposed a regression function to estimate the hydraulic aperture of rough fractures. Their results showed that considering surface roughness ($Z_2 = 0.5$) would result in a decrease of the macroscopic flow rate by 47% and 44% for two kinds of fracture networks, respectively. However, surface roughness has negligible influence on the general patterns of fluid flow and solute migration. Liu et al. [2015a] established DFNs with parallel-walled fractures, in which the tortuous fracture length of each fracture was modified on the basis of a function proposed by Yu and Cheng [2002] as:

$$l_t = e^{1-D_T} l^{D_T}$$  \hspace{1cm} (2-25)

where, $l_t$ is the tortuous length of a fracture. $l_t$ increases with the increasing $D_T$ in the range of $[1, 1.02]$, corresponding to $JRC = [0, 20]$ [Kulatilake et al., 1995].

Their results demonstrated that the equivalent permeability is exponentially correlated with $D_T$ influenced by the trace length ratio, as:
\[ K = \exp(-7.5*D_T - 4.0), \text{ when } l_{min} / l_{max} = 0.5E-04 \]  
\[ K = \exp(-7.5*D_T - 4.6), \text{ when } l_{min} / l_{max} = 1.0E-04 \]  
\[ K = \exp(-7.5*D_T - 5.2), \text{ when } l_{min} / l_{max} = 2.0E-04 \]

Based on numerical simulations on a series of DFNs generated with different sets of random numbers, they found that the maximum permeability discrepancy between the model that considers the effect of surface roughness with the smooth parallel-plate model can be as high as 19.51% when \( D_T = 1.02 \) corresponding to JRC = 20. Liu et al. [2015b] subsequently developed an original code to generate DFN models with 2-D rough-walled fracture surfaces, to characterize the effects of surface roughness on hydraulic properties of DFNs through numerical simulations by solving the Navier-Stokes (NS) equations. One example of calculation result showed that when fluid flow is in the linear regime, fractures with surface roughness of \( Z_2 = [0.35, 0.5] \) would reduce the permeability by 27.1%, compared with the parallel-plate fracture model. Notable is that the numerical simulations are limited to simple DFNs that only include several/tens of fractures, because it is still a time-consuming work to solve the NS equations composed of a set of coupled nonlinear partial derivatives of varying orders in DFNs in the models with a large number of fractures [Zimmerman and Bodvarsson 1996; Brush and Thomson 2003; Javadi et al. 2010].

**Open questions for Section 2.2.3:**

- To date, surface roughness of single fractures could be precisely measured in laboratory experiments via different mechanical or optical methods. How can we obtain the roughness of fractures located inside of fractured rock masses?
- When the flow rate/Reynolds number is sufficiently high, eddies would firstly form in the locations with dramatic geometric variations of aperture induced by surface roughness, thereafter reducing the permeability of fracture networks. How to quantify the role of fracture surface roughness in the transition of fluid flow from Darcy’s flow to non-Darcy’s flow?
- During the in-situ high-pressure packer test, fracture dilation and hydraulic fracturing would occur [Chen et al. 2015a]. What is the role of surface roughness in these phenomena and how to incorporate its effects into DFN models?
- Why does fracture surface display self-affine fractal properties? Can we use a single fractal dimension to describe the roughness of fractures at different scales?
2.2.4 Permeability and fracture dead-end

Most of previous studies assumed that the dead-end of fractures has negligible effects on macroscopic flow behavior in fracture networks, therefore the dead-ends were commonly deleted from the DFNs [Zhao et al., 2011; Bidgoli et al., 2013; Liu et al., 2013]. However, it was also found that although the circulating flow structures in dead-end (or local flow cell) do not contribute to the overall fluid flow, they may significantly affect the mass transport behavior of fracture networks [Taneda, 1979; Shikaze et al., 1998]. Lever et al. [1985] reported that considering dead-end-pore diffusion is successful in explaining most deviations between Fickian theory and experiment. Park et al. [2003] established a series of 3-D DFN models containing a single horizontal fracture and various numbers and sizes of small fractures, and investigated the effects of local flow cell on flow and transport behaviors, where the local flow cell is called “dead-end” in 2-D models. The results suggested that the local flow cell has little effect on the overall fluid flow, however, it can actively contribute to the long breakthrough tailing and retardation of solutions through DFNs.

To stress the effects of fracture dead-end on fluid flow and particle transport, Liu et al. [2015c] particularly established two DFN models with dead-ends and without dead-ends, and performed numerical simulations by solving the NS equations. Hydraulic pressures were applied on the opposing faces (left side and right side), with the other two faces (upper side and bottom side) impermeable. The hydraulic gradient, defined as the ratio of hydraulic head difference of the two permeable faces to the flow length of the model, varied from $10^{-7}$ to $10^{-3}$, experiencing 5 orders of variation. The particles were injected at the left side and collected at the right side, and the travel time of each particle through the model was recorded. The results showed that fracture dead-end has negligible influences on fluid flow with a relative difference of flow rate between the two models less than 1.5%. In contrast, the relative difference of travel time of particles ranges from 5% to 35% at different hydraulic gradients, emphasizing the importance of including the dead-ends in a model when particle transport is encountered.

Open questions for Section 2.2.4:

✧ Different solute transport behaviors could be observed at fracture dead-ends bounded between complete mixing and streamline routing at different Peclet numbers. How do the geometric properties of dead-ends (i.e., length, aperture, and
roughness) impact on the solute transport behavior?

✧ Does the distribution of dead-ends in a DFN follow any mathematical functions?
✧ Once a solute or a nuclide enters into a dead-end, it may be trapped in the segment of dead-end, and may reenter the flow system influenced by the variations of flow rate and concentration. How to simulate this phenomenon and estimate its impacts on macro solute transport behavior in numerical models?

2.2.5 Permeability and number of intersections

Fluid redistribution at fracture intersections can significantly change the hydraulic properties and the mass transport behavior of fractured rock masses, especially for fluid flow in DFNs comprising a large number of intersections (see Fig. 2-1). When the flow rate/Reynolds number is high, inertial effects would be induced at the intersections, which would subsequently reduce the permeability of DFNs [Liu et al., 2016]. Previous studies have mainly focused on the experiments and numerical simulations on fluid flow and mass transport through single fracture intersections. Wilson and Witherspoon [1976] conducted a series of laboratory experiments to determine the magnitude of laminar flow interference effects at fracture intersections. Their results indicated that head loss at the intersection is equivalent to a length of about five pipe diameters with flow at a Reynolds number of 100, and that the inertial effects at intersections are negligibly small when flow is in the laminar flow region. Kosakowski and Berkowitz [1999] numerically calculated flow patterns through intersecting parallel plates with realistic intersection geometries, by solving the NS equations. They found that the nonlinear inertial effects become important for $Re = 1 \sim 100$. Park et al. [2001] based on single fracture intersections and fracture networks, studied the complete mixing and streamline routing, which are significantly dependent on the flow behaviors at the intersections. It was found that only less than 5% of the total number of intersections have un-negligible influences on the choice of mixing assumptions. Johnson et al. [2006] characterized fluid flow and mixing in rough-walled fracture intersections, both experimentally and numerically. They reported that flow channelization through rough-walled intersection fractures significantly enhances the physical mixing compared with the parallel-plate model. Zafarani et al. [2013] proposed an efficient time domain approach for simulating Peclet number – dependent flow and transport through fracture intersections. The breakthrough curves predicted by the proposed approach are in good agreement with those with the benchmark simulations. The new algorithm is 2 ~ 3 orders of magnitude faster than traditional simulations by solving the Stokes equations.
Liu et al. [2014b] conducted flow tests on an artificial crossed fracture model. They directly observed the formation of eddies at the intersection when \( \text{Re} \) was large, where the cubic law was no longer applicable. The inertial effects were negligible when \( \text{Re} < 1 \), and played a robust role when \( \text{Re} > 100 \).

Although fluid flow characterizations at single fracture intersections has attracted much attention during the past several decades, when implementing fluid flow simulation in DFNs, the influence of fracture intersection on the macroscopic flow is usually neglected, due to the difficulties of establishing DFNs that consider the precise geometry of fracture intersections and of solving the complex NS equations. To solve this problem, Liu et al. [2016] utilized their originally developed code to generate a series of DFNs with well-known geometric distributions of fractures, and investigated the roles of fracture surface roughness, number of intersections, and hydraulic aperture in the hydraulic properties of DFNs. They found that the obtained \( Q-J \) relationship can be well depicted by the following Forchheimer’s law-based function:

\[
J = A'Q + B'Q^2
\]

(2-27)

where \( A' \) and \( B' \) are two coefficients related to fracture geometry. Based on multi-variable regressions, mathematical expressions of \( A' \) and \( B' \) were established as follows:

\[
A' = (\lambda' E_M)^{-0.5} \cdot \exp \left\{ 310.5(\lambda' E_M)^{-0.008} - 0.001 \text{JRC} - 0.7 N_i^{0.25} - 303 \right\} \tag{2-28}
\]

\[
B' = (\lambda' E_M)^{-2} \cdot \exp \left\{ 719(\lambda' E_M)^{-0.002} - 0.01 \text{JRC} - 5 N_i^{0.15} - 700.4 \right\} \tag{2-29}
\]

where, \( E_M \) is the mean mechanical aperture of fractures in DFNs, \( N_i \) is the number of fracture intersections, \( \lambda' \) is a coefficient that quantifies the reduction of \( E_M \) due to the variation of fracture apertures in a DFN, with an unit of mm\(^{-1}\). With the increment of \( N_i \), both \( A' \) and \( B' \) decrease, leading to the decrease in \( J \) at a constant \( Q \) according to Eq. (2-27). In such a condition, the equivalent permeability of DFNs will increase according to its definition (Eq. (2-1)).

**Open questions for Section 2.2.5:**

\[\blacklozenge\] The geometry of a single fracture intersection has significant influences on the flow patterns within the intersection. How to characterize the geometry of each intersection in DFNs, and how to estimate its effects on the macro hydraulic properties of DFNs?
Increasing the number of intersections would increase the connectivity of a fracture network, and consequently increase the equivalent permeability. How to mathematically develop the relationship between equivalent permeability with the number of intersections?

How to precisely count the number of intersections in a studied natural rock mass?

2.2.6 Permeability and hydraulic gradient

Although the nonlinear flow has been studied for more than four decades within single fractures [Zimmerman et al., 2004; Zeng and Grigg, 2006; Adler et al., 2013; Zhang and Nemcik, 2013; Zhang et al., 2013; Javadi et al., 2010; 2014; Chen et al., 2015b; Zhou et al., 2015] and single fracture intersections [Wilson and Witherspoon, 1976; Kosakowski and Berkowitz, 1999; Johnson and Brown, 2001; Johnson et al., 2006], the nonlinear flow in DFNs consisted of hundreds/thousands of fracture segments and intersections has not received extensive studies, due to the enormous difficulties of establishing complex DFN models and of solving the NS equations [Zimmerman and Bodvarsson, 1996; Brush and Thomson, 2003; Javadi et al., 2010]. The previous works of calculating the equivalent permeability of fracture networks commonly presumed that fluid flow follows the cubic law by applying constant hydraulic gradients ($J$) on the opposite boundaries, such as $J = 1$ [Long et al., 1982; Zhang et al. 1996; Zhang et al. 1999; Klimczak et al. 2010; Zhao et al. 2010; Zhao et al. 2011; Liu et al. 2015a], $J = 0.1$ [Zhang and Sanderson 1996], $J = 0.001$ [Cvetkovic et al. 2004; Zhao et al. 2013], and $J = \text{unknown constants}$ [Min et al. 2004; Baghbanan and Jing 2007; 2008; Parashar and Reeves 2012; Reeves et al. 2013; Latham et al. 2013], which, however, deviates from the in-situ pumping test results where increasing the hydraulic gradient would decrease the permeability of rock masses when the hydraulic gradient is sufficiently large [Gale, 1985; Kohl et al., 1997; Chen et al., 2015a; 2015c].

Liu et al. [2016] established a series of DFN models as described in Section 2.5, and studied the effects of hydraulic gradient on the equivalent permeability of rock fracture networks. They found that with the increment of hydraulic gradient, fluid flow experiences variation of three stages: (i) linear regime where the equivalent permeability holds constant values, (ii) weak inertial regime where the equivalent permeability varies slightly, and (iii) strong inertial regime where the equivalent permeability changes significantly. To characterize the onset of nonlinear flow, they defined a critical hydraulic gradient ($J_c$), below which the equivalent permeability holds constant values and the cubic law is applicable, and above which the equivalent
permeability varies with the increment of hydraulic gradient and the nonlinear governing equations, such as Forchheimer’s law, should be employed. Based on numerical simulations on fluid flow through the established DFNs, a regression function to calculate $J_c$ was proposed as:

$$J_c = (\lambda' E_M)^{0.5} \cdot \exp\left\{300.8(\lambda' E_M)^{4031} - 0.03 JRC - 0.3 N_i^{0.5} - 303.3\right\}$$ (2-30)

The physical meanings of $\lambda'$, $E_M$, JRC, and $N_i$, are identical to those in Eqs. (2-28) and (2-29). Among these three parameters ($E_M$, $N_i$, and JRC), $J_c$ is most sensitive to $E_M$, followed by $N_i$ and JRC. Simulation results suggested values of $J_c = 10^{-5}$ and $10^{-4}$ for two DFN models with different geometric properties, respectively [Liu et al., 2015b; 2015c].

*Open questions for Section 2.2.6:*

- Inertial effects would decrease the equivalent permeability at large hydraulic gradient, while fracture dilation and hydraulic fracturing would give rise to the equivalent permeability. How to distinguish their competitive roles in high-pressure pumping tests and how to model them in numerical simulations?
- How to characterize nonlinear fluid flow in large-scale DFNs with hundreds/thousands of rough-walled fractures when addressing the influence of hydraulic gradient?
- What is the inherent correlation between hydraulic gradient and Reynolds number? The latter one is commonly adopted in the detection of nonlinear flow.

2.2.7 Permeability and boundary stress

Natural fractured rock masses are subjected to shear and normal stresses from the surrounding rocks, which would consequently change their permeability [Esaki et al., 1999; Olsson and Barton, 2001; Hans and Boulon, 2003; Auradou et al., 2005; Roman et al., 2012; Zhao, 2013; Indraratna et al., 2015]. Several early experimental studies conducted by Gangi [1978], Tsang and Witherspoon [1981] and Barton et al. [1985] have verified this phenomenon. Cappa et al. [2005] based on field experiments of fluid flow through a fracture network comprised of low permeable bed planes and high permeable faults, found that the hydromechanical properties of a single fracture is robustly affected by boundary conditions. They studied the relationship between the permeability and the effective confining pressure in the range of 0 ~ 50 MPa. The
results showed that with the increment of effective confining pressure, the permeability decreases by approximately 1 - 3 orders of magnitude for Pierre shale (Neuzil, 1986), Westerly granite (Brace et al., 1968), and MWX tight sand gas (Kilmer et al., 1987). In numerical simulations, the action of in-situ stresses are usually reproduced by applying stresses on the boundaries of DFN models. Zhang and Sanderson [1996] studied the effect of orientation of the maximum applied stress on the 2-D permeability tensor of natural fracture networks. They concluded that the differential stress has a significant effect on both the magnitude and direction of permeability. The permeability decreases with the increment of the major horizontal stress at a fixed minor horizontal stress. Chen et al. [2007] derived an equivalent elastic-perfectly plastic constitutive model with non-associated flow rule and mobilized dilatancy to characterize the strain-dependent hydraulic conductivity for fractured rock masses, which is written as:

\[
K = \sum_j k_j (\delta - n_j \otimes n_j) = \sum_j k_{0j} \left(1 + \frac{s_j}{b_{0j}} \Delta \varepsilon_{ij}\right)^3 (\delta - n_j \otimes n_j)
\]  \hspace{1cm} (2-31)

where \(k_j\) is the equivalent hydraulic conductivity of the \(j\)th set of fractures under loading, \(\Delta \varepsilon_{ij}\) is the increment of the normal strain, \(s_j\) is the change in spacing, and \(n_j\) is the unit vector normal to the \(j\)th set of fractures.

Using the proposed methodology, the hydraulic properties of a single fracture subjected to normal and shear stresses could be characterized with a closed-form solution, the validation of which could be verified by comparing with the existing results of coupled shear-flow tests [Esaki et al., 1999]. Zhou et al. [2008] developed a new analytical model to calculate the coupled flow-stress permeability tensor for fractured rock masses, written as:

\[
[K] = [\tilde{K}] + \frac{8\pi}{12\mu V_p} \sum_j \sum_{i=1}^{n_i} W_{ij} f^{*} (\beta^*)_{ij} b_{ij0}^3 [M]_{ij}
\]  \hspace{1cm} (2-32)

where \([K]\) is the equivalent permeability tensor, \(r\) and \(b_0\) represent the scale of fracture, \([M]_{ij}\) is a parameter to reflect the effect of the orientation of fractures on the fluid flow for the \(j\)th fracture of the \(i\)th set, \(W\) is a parameter to show the impact of the connectivity of the network on the fluid flow, and \(f(\beta^*)\) is a function demonstrating the coupling effect between fluid flow and deformation.

This model considered the influences of pre-peak shear dilation and shear contraction on the hydraulic properties of fractured rock masses, and could be utilized to determine whether the continuum approach is applicable in the hydromechanical analysis. By neglecting the permeability of the rock matrix and the connectivity of the
fractures, their model (Eq. (2-32)) is totally equivalent to Snow’s model written as [Snow, 1969]:

$$[k] = \frac{g}{12\mu} \sum_{k=1}^{n} \frac{b'(k)}{B(k)} \left[ \delta_{ij} - n_i \right] n_j (k)$$

(2-33)

where $B(k)$ is the average spacing of the $k$th set of fractures, $b'$ is the half aperture, and $\delta_{ij}$ is the Kroneker delta, vanishing when $i \neq j$, and unity when $i = j$.

Following this line, Oda proposed another classic formulation as [Oda, 1985]:

$$k_{ij} = \lambda''(P_{kk} \delta_{ij} - P_{ij})$$

(2-34)

where $P_{ij}$ is the fracture geometry tensor, with $P_{kk} = P_{11} + P_{22} + P_{33}$, and $\lambda''$ is the dimensionless scalar to penalize the permeability of real fractures with roughness and asperities.

Based on the normal loading experiments on fractures [Iwai, 1976; Bandis et al., 1983], Baghbanan and Jing [2008] assumed that the ratio of the maximum fracture closure to the initial aperture is 0.9, and derived a function to link the normal stiffness of each fracture to its normal stress as

$$k_n = \frac{(10\sigma_n + \sigma_{nc})}{9\sigma_n e_i}$$

(2-35)

where $k_n$ is the normal stiffness, $\sigma_n$ is the normal stress, $\sigma_{nc}$ is the critical normal stress, and $e_i$ is the aperture of the $i$th fracture.

The stress ratio (horizontal/vertical stress) varied from 1 to 5, and the aperture was assumed to follow a lognormal distribution. Their results showed that the stresses applied on the boundaries would reduce the equivalent permeability of fracture networks within 2 orders of magnitude. With the increasing stress ratio, it becomes more difficult to establish the REV of a fracture network, comparing to an unstressed model, because the contribution of smaller fractures becomes more significant during shear.

Open questions for Section 2.2.7:

- In laboratory experiment, investigating the effect of stress on the permeability of a fractured rock mass with multiple fractures still faces several challenging issues, such as how to apply confining stress and hydraulic pressure simultaneously on a boundary, how to seal the impermeable boundaries, and how to observe the real-time channelization of fluid flow inside of the model.
- The cracking process in rocks will give rise to the connectivity/permeability of a
fracture network subjected to boundary stresses. How to precisely quantify the competitive contributions of fracture closure and fracture propagation to the equivalent permeability of stressed rock masses?

In 2-D DFNs, the out-of-plane stress is commonly neglected. How much difference on permeability will be produced by neglecting this stress?

2.2.8 Permeability and anisotropy

Primarily due to the existence of fractures, fractured rock masses commonly exhibit anisotropic properties [Nur, 1971; Leary and Henyey, 1985; Schoenberg, 1995; Koyama et al., 2006; Barton and Quadros, 2014]. Anisotropic shear behavior of jointed rock masses has been investigated in both laboratory experiments [Hayashi and Fujiwara, 1965; Kawamoto, 1970; Nagayama and Katahira, 1989; Nagayama et al., 1994; Li et al., 2014] and numerical simulations [Jing and Stephansson, 2007]. The permeability of a fracture network also shows anisotropic properties [Snow, 1969; Long et al., 1982; Vu et al., 2013]. Oda et al. [1987] established a permeability tensor in terms of two tensors and a nondimensional scalar depending on the geometric aspects (i.e., spacing, orientation, and aperture) of fractures. The prediction of the proposed formulation (Eq. (2-34)) on permeability tensor agrees well with the results of a large-scale hydraulic conductivity test. Shapiro et al. [1999] proposed an approach for estimating the permeability tensor utilizing seismic emission. The results showed that the rock volume is characterized by a significant permeability anisotropy depending on the oriented crack systems. Chen et al. [1999] presented an analytical model to predict the permeability tensor of natural fractures nested in parallel-plate type configurations. For a fracture network containing multiple sets of fractures, the permeability tensor \( (k_{ij}) \) can be obtained as follows:

\[
 k_{ij} = \frac{1}{12} \sum_{m=1}^{M'} \phi_i^{(m)} e^{(m)} \Omega_{ij}^{(m)}
\]

where \( \phi_i \) is a set of equivalent porosity, \( e^{(m)} \) is the \( m \)th fracture aperture, \( \Omega_{ij}^{(m)} \) are the \( m \)th fracture coordinate transformation coefficients, and \( M' \) is the total number of fracture sets.

Zhang et al. [1996] established an approach to evaluate the 2-D permeability tensor of a natural fractured rock mass using modified distinct element method (DEM). They found that the permeability tensor only depends on the statistical features of fracture pattern geometry for the DFNs with fixed hydraulic aperture, while the permeability tensor strongly depends on both the geometry of fracture distributions and the applied
stress with variable hydraulic apertures. Min and Jing [2003] provided a methodology to characterize the equivalent elastic properties of fractured rock masses. Verification of the methodology led to good agreements with closed-form solutions for a regularly fractured rock mass. They successfully incorporated the complex fracture geometry and various constitutive relations of fractures and rock matrix, and their interactions into the calculation. Chen et al. [2008] established fracture networks utilizing the Monte Carlo method, and estimated the permeability tensor of fractured rock masses, using the composite element method (CEM). Their results showed that the proposed algorithm is theoretically valid for both 2-D and 3-D fracture networks. Baghbanan and Jing [2008] investigated the directivity of the equivalent permeability, considering the effects of fracture aperture distributions. It was found that the pattern of fracture aperture distributions can robustly affect the directivity of the equivalent permeability. However, there are a large number of factors (i.e., fracture density, length, aperture, and orientation) that could affect the anisotropic properties of permeability of fracture networks, and to date, no generic mathematical expressions that synthetically take into account these factors for characterizing anisotropic hydraulic behaviors have been established.

*Open questions for Section 2.2.8:*

- The distributions of fracture orientation and the magnitude of fracture aperture are two most important factors that govern the anisotropy of permeability. Does the aperture distribution also have anisotropic properties?
- Can the anisotropy of permeability be represented by the fractal dimensions of different fracture sets? For example, each fracture set has a fractal dimension, and the directivity of the equivalent permeability is a function of these directional fractal dimensions.
- How does the scale effect influence the anisotropy of permeability?
- The anisotropy of permeability of a 2-D DFN may be totally different from that of a 3-D DFN [Lang et al., 2014]. How to establish mathematical expressions for anisotropic permeability of 3-D DFNs based on geometric parameters discussed above, e.g., length, aperture, and roughness?
2.2.9 Permeability and scale

The measurement scale of permeability was distinguished in three levels: the laboratory scale (1 ~ 10 cm), the borehole or in-situ scale (1 m ~ 1 km), and the regional scale (1 ~ 100 km) [Clauser, 1992; Lei et al., 2015a]. The influence of model scale on the permeability based on in-situ measurements have been extensively studied during the past several decades [Brace, 1980, 1984; Neuman, 1994; Renshaw, 1998; Wang et al., 2002]. The permeability increases with the increment of model scale from the borehole to the regional scale, according to the works by Brace [1980, 1984] and Neuman [1994], whereas Clauser [1992] and Renshaw [1998] reported that permeability holds constant values or decreases with the increasing scale. Davy et al. [2006] concluded that for a fixed fracture density, with the increment in fracture network scale, the connectivity would be enhanced, leading to the increased permeability. In numerical simulations, fracture networks with randomly distributed fractures are typically established using the Monte Carlo methods, and consequently, random numbers used for network construction would affect the geometry of networks and subsequently influence the mechanical and hydraulic properties of established numerical models [Rouleau and Gale, 1987; Xu et al., 2013; Cadini et al., 2013; Liu et al., 2015d]. To diminish the effect of random numbers, a large number of DFN realizations are required to determine the REV [Bear, 1972; Oda, 1988; Khaleel, 1989; Kulatilake and Panda 2000; Brown et al., 2000; Esmajeli et al., 2010]. When the REV does exist, it can be used to define the minimum volume of a sampling domain of a fracture network, beyond which the permeability of the sampling domain remains essentially constant [Long et al., 1982; Zhou and Yu, 1999; Chalhoub and Pouya, 2008; Maghous et al., 2008].

Wang et al. [2002] estimated the REV and the hydraulic conductivity tensor of a fractured rock mass by performing a single well packer test and numerical simulations. A block size of 15 m was obtained as the REV to represent the hydraulic behavior of the rock mass. Min and Jing [2003] calculated the normalized elastic modulus and Poisson’s ratio with varying side lengths of square DFNs. The REV was determined by analyzing the variations of “coefficient of variation”, which was defined as the ratio of standard deviation over the mean value of a certain property (i.e., elastic modulus or Poisson’s ratio) at a given scale. Baghbanan and Jing [2007] studied the effects of fracture aperture distributions on the size of REV, based on numerical simulations of 640 DFN models. When the aperture of each fracture is correlated with its length as Eq. (2-19), it is more difficult to reach a REV, comparing with the fractures having a
constant aperture. Wu and Kulatilake [2012] proposed a procedure to investigate the size effect and the REV of a fractured rock mass, considering the influences of number of fracture sets, intensity, distribution of orientation, and size of fracture sets. Based on the datasets of Yujian River Reservoir in China, a reasonable REV size of around 25 m was identified.

The determination of REV depends on both of the geometric and geological (e.g., stress history and shear dilation) properties of fractures, therefore the value of REV may vary remarkably location by location. Establishment of models that systematically take into account all of those influential factors is expected in future studies.

Open questions for Section 2.2.9:
✧ How to establish mathematical functions that relate the REV to the fracture geometries (i.e., trace length, aperture, and density)?
✧ The REV of 2-D DFN will be different from that of 3-D DFN of a rock mass. Could we establish mathematical functions to link the REV of 2-D models to that of 3-D models?
✧ How much does the scale effect of single fractures (i.e., the ratio of fracture aperture to length) affect the permeability of a DFN?

2.3 3-D fracture networks

Although many technical issues still exist for accurately measuring and representing the geometry of natural 3-D fracture networks, some efforts have already been devoted to understanding their mechanical and hydraulic properties, and their correlation with the geometric distribution of fractures [Clifton and Abou-Sayed, 1979; Advani et al., 1990; Erhl et al., 2009; Meheust et al., 2011]. The 3-D fracture networks have the outstanding advances of describing the orientation, the connectivity, and the permeability tensor of the natural rock masses [Gonzalez-Garcia et al., 2000; Bogdanov et al., 2003; Yao et al., 2013; Violela et al., 2014; Hyman et al., 2015]. Understanding the hydraulic properties of 3-D DFNs would help to gain an insight into the real flow mechanism of fluid, because the 2-D DFN, which is a cut plane of the 3-D models, trends to underestimate the equivalent permeability [Min et al., 2004; Lang et al., 2014]. The permeability of 3-D fracture networks with complex topology is commonly determined using a boundary element method (BEM) [Andersson and Dverstorp, 1987],
and a finite volume method (FVM) [Koudina, 1998; Khamforoush et al., 2008; Mourzenko et al., 2011].

Hsieh [1985] proposed a method to calculate the 3-D hydraulic conductivity tensor of an anisotropic medium, and analyzed the effect of planar no-flow and constant-head boundaries on the response. Andersson and Dverstorp [1987] developed a 3-D flow model using BEM to predict the hydraulic properties of fractured rock masses, by treating fractures as circular discs with arbitrary sizes, orientations, transmissivity, and locations. Based on a series of hypothetical examples, they found that large fractures and high fracture density lead to good connectivity in the networks. Cacas et al. [1990] conducted a large-scale investigation of fracture flow in a granite uranium mine at Fanay-Augeres, France, and developed a 3-D DFN model assuming negligible matrix permeability, which was calibrated based on the in-situ measurements. However, the proposed model is restricted to idealized channels within the fracture planes. Durlofsky [1991] presented a pressure gradient and flux averaging technique for modeling steady state fluid flow in 3-D fracture networks, with no a priori assumptions regarding the orientation of the eigenvectors. Wen et al. [2003] subsequently applied this technique to structured grids. Koudina et al. [1998] systematically estimated the permeability of 3-D DFNs by triangulating the network and solving the 2-D Darcy equation in each fracture, in which fracture shape was assumed to be polygonal and the heterogeneity characteristics of aperture distribution were negligible. Bour and Davy [1998] studied the connectivity of 3-D fault networks, and found that faults larger than a critical length scale may form a well-connected fracture network, while smaller faults may be not connected on average. Dershowitz and Fidelibus [1999] established DFN models using the pipe network elements, and developed a boundary element procedure for derivation of pipe properties. Their results agreed well with those of polygonal-element models. Guvanasen and Chan [2000] presented a 3-D finite-element solution to the thermohydromechanical deformation with hysteresis in a fractured rock mass. The results indicated that non-isothermal deformation plays an important role in fluid flow and solute transport in fractured rock masses. Vesselinov et al. [2001a; 2001b] described a 3-D numerical inverse model for the interpretation of cross-hole tests in unsaturated fractured tuffs. They used this model to describe the effect of boreholes on pressure propagation through the rock. Wang et al. [2002] calculated the hydraulic conductivity tensor of a 3-D fracture network model, and the results are congruent with the existing fracture systems. Ivars [2006] performed numerical simulations to address the hydro-mechanical properties of a 3-D fractured rock mass. The results showed that the stress - permeability coupling may be one of the reasons for the usually less than
expected inflow into larger diameter holes. Mustapha and Mustapha [2007] simulated fluid flow through 3-D DFNs comprised of thousands of fractures, by applying a local transformation to simplify the highly complex geometry of fractures. Zhou et al. [2008] established a 3-D fracture network model based on the field mapping data of the Laxiwa Hydraulic Power Station, and then calculated the equivalent permeability in different flow directions. Erhel et al. [2009] presented an original conforming mesh generation method of 3-D DFN networks and solved the flow by using a mixed hybrid finite element (MHFE) method. The mesh generation method could address the penalizing configurations stemming from close fractures and acute angles between fracture intersections. The interface conditions for all intersections were considered to make up a consistent system of equations. However, the matrix permeability was not incorporated in their proposed model. Ishibashi et al. [2012] developed GeoFlow, a DFN simulator in which fractures have a heterogeneous aperture distribution, to illustrate the formation of 3-D preferential flow paths (channeling flow). This model explained the general trend of the uneven outflows in the experiments conducted on a granite sample containing two intersecting fractures. The fracture permeability in a matrix element was calculated by counting the number of fracture elements that were intersected with matrix elements. This simplification might reduce the precision of hydraulic property predictions, especially for the model comprising a large number of fractures. De Dreuzy et al. [2012] studied the influences of fracture scale heterogeneity on the properties of 3-D fracture networks, based on numerical simulations of $2 \times 10^6$ DFN models. They concluded that at the fracture scale, considering aperture heterogeneities would reduce the equivalent fracture permeability of up to a factor of six, comparing with the parallel plate model with an identical mean aperture. De Dreuzy et al. [2013] later proposed a numerical benchmark for modeling single phase flow in stochastic DFNs. Lang et al. [2014] developed a method to compute the equivalent permeability tensor of a 3-D fracture and matrix model. Their technique made no assumptions regarding the orientation of principal permeability, allowing for accurate quantification of block permeability with arbitrary anisotropy. This new method was applied to compare with the measured effective permeability of volumetric domains and their seemingly equivalent 2-D counterparts. The results showed that the equivalent permeability calculated from the 2-D cut plane is underestimated by up to 3 orders of magnitude when the fracture distributions are near the percolation threshold, and gradually converges toward a factor of three with increasing the fracture density. Hyman et al. [2015] developed DFNWORKS, a parallelized computational suite that can simulate single/multiphase flow and transport in 3-D DFN networks. Lei et al. [2015b] presented a stress-induced
variable aperture model using the finite-discrete element method (FEMDEM) to characterize the effect of polyaxial stress conditions on the fluid flow in 3-D fracture networks. They found that the components of the permeability tensor could vary more than 3 orders of magnitude with respect to the change of stress ratio from one to four. These works highlighted the importance of establishing 3-D models to accurately estimate the permeability of rock masses because the 2-D models which are merely cut planes of the real models could underestimate the permeability for several orders.

When solving fluid flow in fracture networks, a notable difference between 2-D and 3-D models is the geometric meshing [Vohralik et al., 2007; Benedetto et al., 2016a]. In a 2-D model, the fractures are commonly treated as line segments in which the meshing is considerably simple; however, in a 3-D model, especially when the local variation of aperture in fractures is considered for solving the local cubic law, the fractures are represented by planes that have zero to several intersections with others, which complicates the structure of meshing [Benedetto et al., 2016b]. The accuracy of solving fluid flow in 3-D models significantly depends on the quantity of meshing, especially the meshing of the intersection between two fractures [Kalbacher et al., 2007; Li et al., 2014]. This raises a problem that when a large number of fractures are involved in a 3-D model, the number of meshes may be too enormous to compute. Therefore, some simplifications were applied to the 3-D modelling of rock masses, for instance, in the works of Cacas et al. [1990] and Dershowitz and Fidelibus [1999], mono-dimensional pipes, instead of traces, were utilized to represent fracture faces. Higher levels of meshing will be required if the nonlinear flow behavior is investigated for 3-D models. In the future works, more efficient meshing techniques and computational algorithms for solving linear and nonlinear flow in 3-D models will be required to meet the increasing demand on solving the problems associated with fluid flow in fractured rock masses with greater scales and complexities. Meanwhile, mathematical expressions for calculating permeability of 3-D DFNs involving important geometric parameters need to be established.

Open questions for Section 2.3:

- Directly calculating the equivalent permeability of 3-D DFNs is a challenging and time-consuming work. Is it possible to simplify the 3-D DFNs to 2-D models to facilitate the computation, while maintaining the accuracy of prediction of permeability?
- How to precisely establish 3-D fractured rock mass models and conduct flow tests
in laboratory? How to measure and assess fluid flow distributions inside of these models?

- The influence of dead-end on permeability of 2-D DFNs is typically negligible. Does this apply to the fluid flow property of 3-D DFNs?

2.4 Concluding remarks and future works

We reviewed the mathematical expressions of equivalent permeability of 2-D fracture networks reported in the past a few decades, and presented recent developments of 3-D fracture networks to highlight the direction of future research. The impacts of nine important parameters on the equivalent permeability of DFNs were summarized, including (i) fracture length distribution, (ii) aperture distribution, (iii) fracture surface roughness, (iv) fracture dead-end, (v) number of intersections, (vi) hydraulic gradient, (vii) boundary stress, (viii) anisotropy, and (ix) scale. Among these parameters, the fracture length distribution has received extensive research because it is the most important single parameter that governs the permeability of fracture networks. The fracture length distribution mainly follows power law, exponential, and lognormal functions, and different forms of mathematical expressions have been proposed based on these functions. We noticed substantial developments in the application of fractal theories to characterize the length distribution of fractures, which may also be useful for quantifying other geometric aspects of fracture networks. The aperture of fractures is found to be correlated to their length; therefore, mathematical expressions for aperture could be established based on those for the length. Notable is that the aperture of a fracture is sensitive to its geological history and the stresses it is subjected to, and when wetted, it also evolves with dissolution of propping asperities and precipitation/dissolution on void surfaces. Consequently, coupled thermal-hydraulic-mechanical-chemical (THMC) processes need to be considered when establishing mathematical models for the aperture. These processes become increasingly complex as the introduction of the surface roughness of fractures into a model, which is one of the most intensively studied parameters but many aspects of its characters and effects still remain poorly understood. Representation of fractures by parallel-plates is commonly adopted in 2-D DFNs, with or without the hydraulic aperture being modified based on its correlation with the mechanical aperture. The influence of surface roughness on the magnitude of permeability is much less than that of the length and aperture of fractures, which justifies the usage of parallel-plates in most models. The fracture dead-ends have negligible influence on the magnitude of permeability, however,
it can increase the travel time of particles through a network. When reaching an intersection, the fluid will be redistributed into the branches that connect to this intersection, with the volume of fluid in each branch linearly correlated with the cubic of its aperture when the flow is in linear regime. The fluid flow transits from linear to nonlinear regime with the increase of the hydraulic gradient imposed on a model. Such transition will be accelerated by the geometric variations as surface roughness and intersections that trend to render the streamlines nonlinear. In the nonlinear regime, the ratio of the volume of fluid between branches connected to an intersection will change with the varying hydraulic gradient due to the inertial effects, which however was seldom considered in DFNs. Fracture networks typically consist of several sets of fractures that have different lengths, apertures, spacings, and particularly orientations, which are the sources of their anisotropic hydraulic behavior. The permeability tensor is extensively utilized to quantify the anisotropic hydraulic properties of DFNs, which trends to achieve constant values and directions when the scale of a studied model is larger than a sufficiently large volume, i.e., the REV. The REV size is different for different fractured rock masses according to their alternative geometric and geological properties of fractures. In a 2-D model, fluid flow is solved based on that the fractures are treated as line segments that could be easily meshed; however, the fractures in a 3-D model that contains a series of planes connected with each other have to be complicatedly meshed, in which the quantity of the meshing dominates the accuracy of flow rate calculation. The current works commonly make a number of simplifications to facilitate the meshing; therefore, it is still a challenging task to predict precise flow rate through 3-D DFNs where some new algorithms for quick and easy meshing are needed.

In the following, we list a few potential future works that are important but have not been thoroughly understood.

- **Hydraulic fracturing has been widely implemented for the exploitation of oil, gas, and geothermal resources** [Hubbert and Willis, 1957; Warpinski et al., 2009; Beckwith, 2010]. The current numerical simulations are only focused on the propagation of single fractures [Paris and Erdogan, 1963; Hoek and Bieniawski, 1965; Germanovich et al., 1994; Olson, 2004; Yang et al., 2013] and several fractures [Bobet and Einstein, 1998; Sagong and Bobet, 2002; Yang et al., 2012; Li et al., 2013], yet investigating the hydraulic fracturing behavior of complex fracture networks is still a great challenge because of the complex rock-fluid mechanisms involved and the time-consuming computation [Zhao et al., 2014b]. At high hydraulic gradient, the competitive roles of inertial effects that trend to decrease the permeability and hydraulic fracturing that trends to increase the permeability
need to be quantitatively estimated via laboratory experiments and/or numerical simulations.

- Although the coupled effects of shear stress and dilatancy on hydraulic properties of single fractures have been extensively investigated [Esaki et al., 1999; Jiang et al., 2004; Auradou et al., 2005; Indraratna et al., 2015], the effect of shear-induced dilation of fractures on the equivalent permeability of fracture networks is still unclear [Sullivan, 2007; Farahmand et al., 2015]. Future works need to focus on the influence of potential shearing plane, particularly those of faults, on the variation of permeability of DFNs subjected to different stresses.

- The previous works have shown that the fracture surface roughness can significantly affect the hydraulic properties of single fractures [Brown 1987; Zimmerman and Bodvarsson, 1996; Li et al., 2008; Xiong et al., 2011; Zou et al., 2015], however, limited works have taken into account the surface roughness in 2-D and 3-D DFNs. More realistic representation of the geometric properties of fractures in both macro (fracture distribution) and micro (void geometries of pores and single fractures) scales in 3-D DFNs is required in future explorations.

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3 Governing equations of fluid flow in rock fractures

3.1 Continuity equation of fluid flow

Fluid is a continuous media that is composed by infinite particles that fill all the void space. Flow field is defined by these fluids. Euler method is utilized to describe fluid flow and is focused on the whole state of flow field, rather than the motion of the individual particle. For example, velocity field, pressure field and density field can be presented by a series of functions of coordinate \((x, y, z)\) and time \((t)\).

\[
\begin{align*}
    u &= u(x, y, z, t) \\
    v &= v(x, y, z, t) \\
    w &= w(x, y, z, t) \\
    p &= p(x, y, z, t) \\
    \rho &= \rho(x, y, z, t)
\end{align*}
\] (3-1)

where, \(u, v, w\) are the velocities in \(x-, y-\) and \(z-\)directions, respectively. \(p\) and \(\rho\) is the pressure and density of point \((x, y, z)\) at time \(t\), respectively.

Differentiating Eq. (3-1) with respect to time \(t\) results in the expression of acceleration. Notable is that the three velocity components \((u, v, w)\) are three functions of coordinate and time, and these motion particles are correlated with the change of time, too. Therefore, the acceleration should be derived by derivation rules of compound function.

The \(x\)-directional component of acceleration can be expressed as:

\[
a_x = \frac{Du}{Dt} = u \frac{\partial u}{\partial t} + u \frac{\partial}{\partial x} \frac{\partial u}{\partial x} + u \frac{\partial}{\partial y} \frac{\partial u}{\partial y} + u \frac{\partial}{\partial z} \frac{\partial u}{\partial z}
\] (3-4)

where, \(\frac{Du}{Dt}\) is a total derivative of flow velocity \(u\) respect to time \(t\).

Due to the definition of particle’s velocity, the three velocity components can be described as:

\[
\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{dz}{dt} = w
\] (3-5)

As a result,

\[
a_x = \frac{du}{dt} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}
\] (3-6)

Similarly,

\[
a_y = \frac{dv}{dt} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}
\] (3-7)
\[ a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \]  \hspace{1cm} (3-8)

An arbitrary point \( C \) in the void space is selected with the components of velocities \((u, v, w)\) and fluid density \((\rho)\). For simplification, a hexahedral infinitesimal element is chosen as the control volume, as shown in Fig. 2.1. Point \( C \) is the center of the control volume and \( dx, dy, dz \) stand for the three lengths in \( x, y \)- and \( z \)-directions, respectively.

\[ \frac{2}{\rho} \int dydzdx \left[ \rho u \frac{\partial (\rho u)}{\partial x} \right] dydz \]

Because the control volume is very small and the flow elements in space are continuously differentiable functions, the distributions of flow elements on all surfaces can be approximately regarded as uniform distributions. Additionally, the component of momentum \((\rho u)\) is also a continuously differentiable function in pace, and can be depicted by similar expressions. The mass flow-rate of the left surface is:

\[ Q_{\text{left}} = \left[ \rho u - \frac{\partial (\rho u)}{\partial x} \right] dydz \]  \hspace{1cm} (3-11)

The mass flow-rate of the right surface is:
\[ Q_{mb} = \left[ \rho u + \frac{\partial (\rho u)}{\partial x} \frac{dx}{2} \right] dydz \]  

(3-12)

Thus, the inflow of the hexahedral infinitesimal element in the \( x \)-direction is:

\[ Q_{mx} = Q_{mu} - Q_{mb} = -\frac{\partial (\rho u)}{\partial x} dxdydz \]  

(3-13)

Similarly, the inflows of the hexahedral infinitesimal element in the \( y \)- and \( z \)-direction are:

\[ Q_{my} = -\frac{\partial (\rho v)}{\partial x} dxdydz \]  

(3-14)

\[ Q_{mz} = -\frac{\partial (\rho w)}{\partial x} dxdydz \]  

(3-15)

On the base of mass conservation, the variation of total mass flow-rate \( (\rho dxdydz) \) in unit time equals to that flows into the control volume in unit time. That is:

\[ \frac{\partial (\rho dxdydz)}{\partial t} = Q_{mx} + Q_{my} + Q_{mz} \]  

(3-16)

Substituting Eqs. (3-13), (3-14) and (3-15) into Eq. (3-16) and canceling \( dxdydz \) yield

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \]  

(3-17)

Eq. (3-17) is the three-dimensional flow continuity equation.

For steady-state flow, \( \frac{\partial \rho}{\partial t} = 0 \) and Eq. (3-17) can be expressed as

\[ \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \]  

(3-18)

For incompressible flow, Eq. (3-17) can be simplified to

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  

(3-19)

Fluid particle follows Newton’s second law of motion.
3.2 Derivation of Navier-Stokes equation

Fig. 2.2 Schematic view of a differential hexahedron

Fig. 2.2 depicts a differential hexahedron as representative elementary volume to analyze fluid flow in x-, y-, and z- coordinate. Each side is parallel to the coordinate axis with side lengths of $dx$, $dy$ and $dz$. $A(x,y,z)$ is a vertex of the hexahedron and the fluid pressure of point $A$ is $p$. Assuming that all the surfaces connected to point $A$ have the same fluid pressure $p$, and the fluid pressures of other surfaces are

$$p + \frac{\partial p}{\partial x} dx, p + \frac{\partial p}{\partial y} dy, p + \frac{\partial p}{\partial z} dz$$

(3-20)

Subsequently, the forces applied on the surfaces of hexahedron can be expressed as

(a) Fluid pressure on the surfaces

In $Ox$ direction: $pdxdydz - \left(p + \frac{\partial p}{\partial x} dx\right)dydz$

In $Oy$ direction: $pdxdzdy - \left(p + \frac{\partial p}{\partial y} dy\right)dxdz$

In $Oz$ direction: $pdx dydz - \left(p + \frac{\partial p}{\partial z} dz\right)dxdy$

(b) Mass force

The unit mass forces imposed on fluid are presented by $X$, $Y$ and $Z$, respectively. Thus, the total mass forces of the hexahedron are

In $Ox$ direction: $Xpdxdydz$

In $Oy$ direction: $Ypdxdydz$

In $Oz$ direction: $Zpdxdydz$
where, \( X, Y, Z \) have positive values when their directions walk along the coordinate axis.

Beside the above external forces, the differential hexahedron also has accelerations. The accelerations in different directions can be expressed by \( \frac{Du}{Dt}, \frac{Dv}{Dt}, \frac{Dw}{Dt} \), respectively.

According to Newton’s second law of motion, the correlation of the force and the acceleration of particles can be obtained. For example, in the \( x \)-direction

\[
 p dx dy dz \left( \frac{\partial p}{\partial x} + \frac{\partial \rho}{\partial u} \right) dy dz + \rho dx dy dz \frac{\partial u}{\partial t} = \rho dx dy dz \frac{\partial u}{\partial t} 
\]

Eq. (3-21) can be further abbreviated to

\[
 X - \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{du}{dt} 
\]

Similarly, an equation set can be written as

\[
 \begin{aligned}
 \frac{Du}{Dt} &= X - \frac{1}{\rho} \frac{\partial p}{\partial x} \\
 \frac{Dv}{Dt} &= Y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\
 \frac{Dw}{Dt} &= Z - \frac{1}{\rho} \frac{\partial p}{\partial z} 
\end{aligned}
\]

Substituting Eqs. (3-6), (3-7) and (3-8) into Eq. (3-23) gives

\[
 \begin{aligned}
 \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= X - \frac{1}{\rho} \frac{\partial p}{\partial x} \\
 \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= Y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\
 \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= Z - \frac{1}{\rho} \frac{\partial p}{\partial z} 
\end{aligned}
\]

The equation set Eq. (3-24) is the ideal differential equations of fluid motion, which was raised by Euler in 1755. It is also called the Euler equation.

Euler Eq. (3-24) and the three-dimensional flow continuity (see Eq. (3-17)) consist of a system of partial differential equations to describe ideal fluid motion. For incompressible fluid, the density \( \rho \) is a constant and there are four equations and four unknown quantities \( p, u, v, w \). Therefore, these equations can be solved. If fluid is compressible, the density \( \rho \) is an unknown quantity. There are four equations and five unknown quantities so that the equations can’t be solved theoretically. Simultaneous
equations of continuity equation, Euler equation, energy equation and fluid state equation should be utilized to obtain accurate solutions.

For viscous fluid, the shear stress should be considered. The viscous forces in x-, y- and z-directions can be written as $\mu \nabla^2 u$, $\mu \nabla^2 v$ and $\mu \nabla^2 w$, respectively. Thus, the differential equations of fluid flow change to

$$
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= X - \frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \nabla^2 u \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= Y - \frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \nabla^2 v \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= Z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \mu \nabla^2 w
\end{align*}
$$

The equation set Eq. (3-25) is the famous Navier-Stokes (NS) equation.

### 3.3 Governing equations for fluid flow in fractures

#### 3.3.1 Navier-Stokes equation

The Navier-Stokes equation can be expressed in a tensor form

$$
\rho \left( \frac{\partial u_i}{\partial t} + u_i u_{i,j} \right) = \rho f_i - P_j + \mu u_{i,j} \quad (3-26)
$$

where $u_i$ is the velocity of fluid in the $i$-direction with $i=x,y,z$ respectively, the body force is $f_i = g_i = [0,0,-g]$.

For steady-state flow, the parameters related to time $t$ can be ignored.

$$
\rho u_i u_{i,j} = \rho f_i - P_j + \mu u_{i,j} \quad (3-27)
$$

Hydraulic head is defined as the summation of height and hydraulic pressure head with

$$
h = z + \frac{P}{\rho g} \quad (3-28)
$$

Hydraulic gradient is defined by

$$
J_i = h_i = \frac{1}{\rho g} (-\rho g_i + P_j) \quad (3-29)
$$

Substitution of Eq. (3-29) into Eq. (3-27) leads to a more abbreviate form

$$
\rho u_i u_{i,j} = -\rho g h_i + \mu u_{i,j} \quad (3-30)
$$
3.3.2 Stokes equation

When fluid flows slowly, or Reynolds number \((Re)\) is very small, the left part of Eq. (3-30) can be assumed to be zero and hence deserve the Stokes equation as follows.

\[
h_{,ij} = \frac{\mu}{\rho g} u_{,ij}
\]  \hspace{1cm} (3-31)

Eq. (3-31) can be unfolded to the following forms:

\[
h_{,x} = \frac{\mu}{\rho g} \left( u_{,xx} + u_{,xy} + u_{,xz} \right)
\]  \hspace{1cm} (3-32-a)

\[
h_{,y} = \frac{\mu}{\rho g} \left( u_{,yx} + u_{,yy} + u_{,yz} \right)
\]  \hspace{1cm} (3-32-b)

\[
h_{,z} = \frac{\mu}{\rho g} \left( u_{,zx} + u_{,zy} + u_{,zz} \right)
\]  \hspace{1cm} (3-32-c)

3.3.3 Reynolds equation

Dimensional analysis and some definitions are made: \(A\) is the dimension of fracture in \(xy\)-plane, \(U\) is the dimension of flow velocity in both \(x\)- and \(y\)-directions and \(V\) is the dimension of flow velocity in \(z\)-direction.

\[
\text{mag} \left[ \frac{\partial^2 u_x}{\partial x^2} \right] \approx \text{mag} \left[ \frac{\partial^2 u_y}{\partial y^2} \right] \approx \frac{U}{A} \text{, mag} \left[ \frac{\partial^2 u_z}{\partial z^2} \right] \approx \frac{U}{a^2}
\]  \hspace{1cm} (3-33-a)

\[
\text{mag} \left[ \frac{\partial^2 u_y}{\partial x^2} \right] \approx \text{mag} \left[ \frac{\partial^2 u_z}{\partial y^2} \right] \approx \frac{U}{A} \text{, mag} \left[ \frac{\partial^2 u_x}{\partial z^2} \right] \approx \frac{U}{a^2}
\]  \hspace{1cm} (3-33-b)

\[
\text{mag} \left[ \frac{\partial^2 u_z}{\partial x^2} \right] \approx \text{mag} \left[ \frac{\partial^2 u_x}{\partial y^2} \right] \approx \frac{U}{A} \text{, mag} \left[ \frac{\partial^2 u_y}{\partial z^2} \right] \approx \frac{U}{a^2}
\]  \hspace{1cm} (3-33-c)

where \(a\) is the aperture of fracture.

When the dimension of fracture aperture is much less than the dimension of fracture in \(xy\)-plane, the first two terms of Eq. (3-32) on the right side is much less than the last term and can be neglected.

\[
h_{,x} = \frac{\mu}{\rho g} u_{,xx}
\]  \hspace{1cm} (3-34-a)

\[
h_{,y} = \frac{\mu}{\rho g} u_{,yy}
\]  \hspace{1cm} (3-34-b)
\[ h_z = \frac{\mu}{\rho g} u_{z,zz} \quad (3-34-c) \]

If aperture of fracture is very small and there is almost no change on hydraulic head in the z-direction, Eq. (3-34-c) can be simplified to

\[ h_z \approx 0 \quad (3-35) \]

Substitution of Eq. (3-35) into Eq. (3-34-c) gives

\[ u_x(x, y, z) = \frac{\rho g}{2\mu} \left( h_x \left( z + b_1 z + c_1 \right) \right) \quad (3-36-a) \]

\[ u_y(x, y, z) = \frac{\rho g}{2\mu} \left( h_y \left( z^2 + b_2 z + c_2 \right) \right) \quad (3-36-b) \]

\[ h = h(x, y) \quad (3-36-c) \]

Where, \( b_1, b_2, c_1 \), and \( c_2 \) are the integration constants.

Importing non-slip boundary conditions results in:

\[ u_x = 0, \quad \text{when } z = \pm \frac{E}{2} \quad (3-37) \]

Substitution of Eq. (3-37) into Eq. (3-36) leads to the values of integration constants \( b_1, b_2, c_1 \) and \( c_2 \), and then

\[ u_x(x, y, z) = \frac{\rho g}{2\mu} \left( h_x \left( z + \frac{E}{2} \right) \left( z - \frac{E}{2} \right) \right) \quad (3-38-a) \]

\[ u_y(x, y, z) = \frac{\rho g}{2\mu} \left( h_y \left( z + \frac{E}{2} \right) \left( z - \frac{E}{2} \right) \right) \quad (3-38-b) \]

\[ h = h(x, y) \quad (3-38-c) \]

Integrations of Eq. (3-38-a) and (3-38-b) lead to

\[ q_x = \int_{\frac{E}{2}}^{E} u_x dz = -\frac{\rho g E^3}{12\mu} \frac{\partial h}{\partial x} \quad (3-39-a) \]

\[ q_y = \int_{\frac{E}{2}}^{E} u_y dz = -\frac{\rho g E^3}{12\mu} \frac{\partial h}{\partial y} \quad (3-39-b) \]

Importing the law of conservation of mass of flow rate

\[ q_{x,x} + q_{y,y} = 0 \quad (3-40) \]

Then, the Reynolds equation can be given by
\[
\frac{\partial}{\partial x} \left( \frac{\rho g E^3}{12 \mu} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho g E^3}{12 \mu} \frac{\partial h}{\partial y} \right) = 0
\]  
(3-41)

3.4 Derivation of the Cubic law

3.4.1 Parallel plate model

Assumptions are made that fluid flows through two infinite smooth parallel plate, with a constant aperture \( E \), as shown in Fig. 2.3.

![Fig. 2.3 Flow velocity distribution in a parallel plate model](image)

3.4.2 Expression of the Cubic law

Because fluid flows along the \( x \)-coordinate, the flow velocities in the \( y \)- and \( z \)-directions should be equal to zero with

\[ u_y = y_z = 0 \]  
(3-42)

The continuity equation of fluid is

\[ \frac{\partial \rho}{\partial t} + (\rho u_i)_j = 0 \]  
(3-43)

For incompressible fluid, Eq. (3-43) can be simplified to

\[ u_{i,j} = 0 \]  
(3-44)
Substitution of Eq. (3-44) into Eq. (3-30) could reduce to the equation in the \( x \)-direction.

\[
\rho u_x u_{,x} = -\rho g h_{,x} + \mu u_{,xx} \tag{3-45}
\]

\[
0 = -\rho g h_{,x} + \mu u_{,xx} \tag{3-46}
\]

Importing the non-slip boundary conditions leads to

\[
q_x = \int_0^E \frac{E}{2} u_x \, dz = -\frac{\rho g E^3}{12 \mu} \frac{\partial h}{\partial x} \tag{3-47}
\]

This is the expression of the famous Cubic law.
4 Fluid flow properties at single intersections

4.1 Introduction

Significant efforts are being devoted to the understanding of fluid flow behaviors in fractured rock masses (i.e., granite and basalt), using the discrete fracture network (DFN) modelling techniques [Long et al. 1982; Jing 2003; Min and Jing 2003; Miao et al. 2015]. These DFNs usually contain hundreds/thousands of fracture segments and intersections, and assume that the flow rate is linearly related with the pressure drop, which follows the local cubic law (LCL) [Zimmerman and Bodvarsson 1996]. However, in the karst systems [Gale, 1984] or in the vicinity of wells during pump tests [Kohl et al. 1997] where the flow rate is high, it is found that the flow rate is nonlinearly related with the pressure drop, emphasizing that the LCL is not suitable. To clearly understand the nonlinearity of fluid flow in DFNs and modify the LCL to accurately estimate the permeability in such a condition, the hydraulic properties of fluid flow through single fracture intersections should be primarily characterized.

Previous studies have focused on flow behaviors of fluid through single fractures, suggesting that the nonlinear relationship of the flow rate and the pressure drop is affected by the joint surface roughness (JRC) [Barton 1973], the Reynolds number ($Re$) [Zimmerman et al. 2004; Xiong et al. 2011], the contact ratio and contact shapes [Zimmerman and Bodvarsson 1996; Li et al. 2008], the shear displacement [koyama et al. 2008a; javadi et al. 2014], etc. The nonlinear flow induces the energy loss that can reduce the permeability of a fracture, resulting in the hydraulic aperture ($e$) less than its mechanical aperture ($E$), where $e$ can be back calculated from the LCL and $E$ is the mean distance of the two walls of a fracture [Li and Jiang 2013]. Although the Navier-Stokes (NS) equations have been utilized to model fluid flow through single fractures or single fracture intersections, it is still a challenging and time-consuming work to apply it to the complex large scale fracture networks, because the NS equations have to solve a set of coupled nonlinear partial derivatives of varying orders [Brush and Thomson 2003; Javadi et al. 2010]. Instead, the simplified forms of the NS equations such as the LCL and the modified LCL are usually utilized to describe the linear and nonlinear relationships of the flow rate and the pressure drop. Previous works (see Table 4-1) have shown that the value of $e$ is smaller with larger JRC and $Re$, as well as smaller $L$, where $L$ is the straight length of a fracture. However, there are no identical mathematical or empirical expressions to describe the variations of $e$ for single fractures with different Reynolds numbers, JRCs and fracture lengths.
### Table 4-1 Empirical expressions of the hydraulic aperture in previous studies

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lomize</td>
<td>1951</td>
<td>( e^2 = \frac{E^2}{C} \cdot \left( 1 + m \left( \frac{y}{2E} \right)^{\frac{3}{2}} \right), \ m = 17 )</td>
<td>( y ) is the magnitude of the discontinuity surface roughness, ( m ) is a coefficient.</td>
</tr>
<tr>
<td>Louis</td>
<td>1969</td>
<td>( e^2 = \frac{E^2}{C} \cdot \left( 1 + m \left( \frac{y}{2E} \right)^{\frac{3}{2}} \right), \ m = 8.8 )</td>
<td></td>
</tr>
<tr>
<td>Quadros</td>
<td>1982</td>
<td>( e^2 = \frac{E^2}{C} \cdot \left( 1 + m \left( \frac{y}{2E} \right)^{\frac{3}{2}} \right), \ m = 20.5 )</td>
<td></td>
</tr>
<tr>
<td>Patir and Cheng</td>
<td>1978</td>
<td>( e = E \left[ 1 - 0.9 \exp \left( -0.56 \frac{E}{\sigma_E} \right) \right] )</td>
<td>( \sigma_E ) is the standard deviation of the varying aperture over mechanical aperture.</td>
</tr>
<tr>
<td>Witherspoon et al.</td>
<td>1980</td>
<td>( e = e_0 + f\Delta E )</td>
<td>( e_0 ) is the initial hydraulic aperture, ( f ) is an exponent function, ( \Delta E ) is the variation of mechanical aperture during shear.</td>
</tr>
<tr>
<td>Walsh</td>
<td>1981</td>
<td>( e = E \left[ 1 - \frac{1}{1 + C} \right] )</td>
<td>( C = 0.25 ) in the study of Zimmerman et al. [1992]</td>
</tr>
<tr>
<td>Barton et al.</td>
<td>1985</td>
<td>( e = \frac{E^2}{JRC^{2/3}} \left( e^2 = \frac{E^2}{C} \right) ) ( C = \frac{JRC}{e^2} )</td>
<td></td>
</tr>
<tr>
<td>Hakami</td>
<td>1995</td>
<td>( e^2 = \frac{E^2}{C}, \ C = 1.1 - 1.7 )</td>
<td>( E = 100 - 500 \ \mu m )</td>
</tr>
<tr>
<td>Renshaw</td>
<td>1995</td>
<td>( e = E \left[ 1 + \frac{\sigma_E}{E} \right]^{\frac{1}{2}} )</td>
<td></td>
</tr>
<tr>
<td>Zimmerman and Bodvarsson</td>
<td>1996</td>
<td>( e' = E \left[ 1 - 1.5 \frac{\sigma_{\text{appr}}}{E} + \ldots \right] \left( 1 - 2C \right) )</td>
<td>( C ) is the contact ratio</td>
</tr>
<tr>
<td>Waite et al.</td>
<td>1999</td>
<td>( e = E \langle E \rangle / \tau^{1/3} )</td>
<td>( \langle E \rangle ) is the harmonic mean of true aperture; ( \tau ) is tortuosity</td>
</tr>
<tr>
<td>Olsson and Barton</td>
<td>2001</td>
<td>( e = \frac{E^2}{JRC^{2/3}} \left( u_s \leq 0.75u_p \right) ) ( e = E^{1/2} JRC_{\text{mob}} \left( u_s \geq u_p \right) )</td>
<td>( JRC_{\text{mob}} ) is the mobilized value of ( JRC ); ( u_s ) is the shear displacement that does not exceed 75% of the peak shear displacement ( u_p ) (the shear displacement at the peak shear stress).</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Year</td>
<td>Equation/Formula</td>
<td></td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------</td>
<td>-----------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Yu and Cheng; Liu et al.</td>
<td>2002;</td>
<td>( e = \left( \frac{L_v}{L_s} \right)^{0.05} )</td>
<td></td>
</tr>
<tr>
<td>Olson; Klimczak et al.</td>
<td>2003;</td>
<td>( e = \frac{\pi}{4} E_{\max} = \frac{\pi}{4} L^0.5 )</td>
<td></td>
</tr>
<tr>
<td>Baghbanan and Jing</td>
<td>2008</td>
<td>( e(\mu \eta) = \frac{\sigma_{nc} (\text{MPa})}{2.51} )</td>
<td></td>
</tr>
<tr>
<td>Xiong et al.</td>
<td>2011</td>
<td>( e^3 = E \left( 1 - 1.0 \frac{\sigma_{apert}}{E} \right) \left( 1 - \frac{\sigma_{apert}}{E} \frac{1}{10} \sqrt{10 \sqrt{\text{Re}}} \right) )</td>
<td></td>
</tr>
<tr>
<td>Rasouli and Hosseinian</td>
<td>2011</td>
<td>( e = E \left( 1 - 0.03d_{mc}^{0.565} \right) \text{JRC} ) or ( e = E \left( 1 - 2.25 \frac{\sigma_{apert}}{E} \right) )</td>
<td></td>
</tr>
<tr>
<td>Li and Jiang</td>
<td>2013</td>
<td>( e = \frac{E}{1 + Z_2^{1.25} + \left( 0.00006 + 0.004Z_2^{2.25} \right) \text{Re} - 1} )</td>
<td></td>
</tr>
</tbody>
</table>

- \( L \) is the straight length of a fracture from the starting point to the ending point; \( L_t \) is the tortuous length along the rough fracture surface; \( D_T \) is the fractal dimension of the streamline of fluid flow through a fracture.
- \( E_{\max} \) is the maximum mechanical aperture of an opening-mode fracture.
- \( \sigma_{nc} \) is the critical normal stress.
- \( \sigma_{apert} \) is the standard deviation of mean mechanical aperture.
- \( \sigma_{slope} \) is the standard deviation of local slope of fracture surface.
- \( d_{mc} \) is minimum closure distance; \( \text{JRC} \) is the average of joint roughness coefficients for upper and lower rock fracture profiles.

**Z_2** is the root mean square of the first deviation of the profile.
Fig. 4-1 A DFN model containing lots of fracture intersections and segments. The schematic drawings of fluid flow through fracture intersections are also given for the cases of (a) two inlets and two outlets, (b) one inlet and two outlets, (c) one inlet and one outlet, and (d) two inlets and one outlet.

For fluid flow in rock fracture networks, the flow paths will be changed at fracture intersections according to the numbers of fracture segments connected to the intersections (see Fig. 4-1). Many researchers have studied the nonlinear flow regimes at fracture intersections. For example, Wilson and Witherspoon [1976] conducted a series of laboratory experiments to determine the magnitude of laminar flow interference effects at fracture intersections. Their results indicated that head loss at the intersection was equivalent to a length of about five pipe diameters with flow at a $Re$ of 100, and that the inertial effects at intersections were negligibly small when flow was in the laminar flow region. Kosakowski and Berkowitz [1999] numerically calculated flow patterns through intersecting parallel plates with more realistic intersection geometries, by solving NS equations, and concluded that even for $Re > 10$, the non-linear inertial effects became important. Johnson et al. [2006] characterized fluid flow and mixing in rough-walled fracture intersections, both experimentally and numerically. They found that flow channelization through rough-walled intersection fractures significantly enhanced physical mixing compared with the parallel plate model. However, there are few works focused on the fluid redistribution at fracture intersections and the calculation of the hydraulic aperture of each fracture segment. The hydraulic properties of fluid flow at fracture intersections have not been systematically reported with mathematical expressions.
To clearly understand the mechanisms of fluid redistribution and nonlinear flow characteristics, as well as the mathematical descriptions of the hydraulic aperture for each segment connected to the fracture intersections, in this study, fluid flow tests were carried out and corresponding numerical simulations by solving the NS equations were performed to exhibit the existence of nonlinear relationship of the flow rate and the pressure drop at fracture intersections. Then, large scale numerical models were established to extensively study the roles of hydraulic gradient, fracture surface roughness, intersecting angle, and scale effect. Finally, an empirical expression was proposed to calculate the hydraulic aperture mathematically.

4.2 Flow test

4.2.1 Flow test at intersections considering fracture surface roughness

Two square glasses were cracked to conduct flow tests through fracture intersections as shown in Fig. 4-2. One of them, the smooth fracture surface model, was designed with a mechanical cutting to form smooth fracture surfaces and the second one, the rough fracture surface model, was cut artificially to generate rough fracture surfaces. The two glasses both have two crossed fractures that intersect at a degree of 60° and have the same size with a side length of 50 cm and a thickness of 5 mm. Another two glasses and four inflow/outflow tanks were glued to the fractured glasses to seal the
model. The two walls of each fracture were manufactured with a distance of 1 mm by using regular rulers, which may produce considerable errors. Each inflow/outflow tank was controlled with a valve to guide fluid flowing into the model or out of the model. Basic assumptions are made that the glasses are impermeable and fluid only flows through the cracks between every two fractured glasses, and the temperature is 20°, while the density of the fluid is 998.2 kg/m³. The two walls of each fracture have well-mated surfaces, except for a few damaged parts induced by splitting in the rough model.

Fig. 4-3 shows the experimental system of fluid flow test. To avoid the influences of impurities in the water, which can deposit within the fractures and change the value of aperture, a filter was attached at the end of the inlet pipe. A high precision syringe pump with an accuracy of ±0.001 mL/min was utilized to conduct fluid into the model through inlet_1. The flow rate of inlet_1 was controlled in the range of 1 – 100 mL/min with the syringe pump, corresponding to 1.67×10⁻⁸ - 1.67×10⁻⁶ m³/s. The effluents at the outlets were collected by the electronic balances with an accuracy of ±0.01 g. The electronic balance was connected to a computer with a R232 data line, to record the
weight of the effluents in real time. A differential manometer with an accuracy of ±10 Pa was used to measure the pressure difference between the inlet and outlets. A high resolution CCD camera with an accuracy of 100 lp/mm was utilized to take the images of fracture geometry and the detailed descriptions will be illustrated in the following Section 4.3.

Before tests, the tested models were placed on a horizontal table to diminish the pressure difference between the inlet and outlets. The connecting tubes, the inflow tanks, the outflow tanks and all the fractures were fulfilled with distilled water, and the air bubbles within the water and fractures were also removed using a vacuum pump. Fluid was introduced into the model through one fixing inlet (inlet_1), and out of the model through variable outlets (outlet_2, outlet_3, outlet_4, outlet_2&3, outlet_2&4, and outlet_3&4, respectively).

4.2.2 Flow test at intersections considering intersecting angles

To experimentally visualize the fluid flow and solute transport processes within fracture intersections, each of the three large transparent glass plates with a same size of 0.5 m × 0.5 m × 0.005 m was cut into four parts, and these parts were then sandwiched and sealed between another two large plates. The aperture between each two parts was fixed as 0.01 mm±0.00002 mm. Two inflow tanks were installed on the two inlets to guide the fluid from a high-precise syringe pump to the model, and another two outflow tanks were installed to export the fluid from the model to other measuring equipment. Each tank was equipped with a valve that can be opened or closed independently. The models with intersecting angles of 60°, 90° and 120° were prepared, respectively. The schematic view of the models is shown in Fig. 4-4, and one of the specimens with an intersecting angle of 90° is shown in Fig. 4-5. Dye solute, which is made by mixing water and food red with a concentration of 5 g/L, was used in the tests to easily visualize the fluid flow through the intersections. The food red can dissolve sufficiently in water, therefore, its density is 1005 g/L and it does not change the viscosity of water (0.001 Pas). The extremely low permeability of glass guaranteed that fluid only flowed through the fractures. Distilled water was used as fluid with a density of 998.2 kg/m³ and a dynamic viscosity of 0.001 pa·sec at a room temperature of 20°.
The schematic view of the system for fluid flow test is the same as shown in Fig. 4-3. The cases of one inlet and two outlets intersected at 60°, 90° and 120° (see Fig. 4-4) were tested, respectively. The flow rate of the inlet_1, $Q_1$, had a constant value of 18 ml/min. As a reference, the corner formed by the segments 3 and 4 was set as a 0 point, and when the solute arrived at this point, the CCD camera started to capture the solute transport images within the intersection with a time interval of 0.05 s. By means of
these images, the distance parallel to the wall of each segment between the tip of the dye solution (the fastest velocity) and the 0 point was measured and calculated as demonstrated in Fig. 4-7(e). The pressure difference between each inlet and outlets was also measured with a differential manometer with an accuracy of ±0.01 kPa. The experimental model was sealed by the impervious strong glue, so when conducting fluid flow tests, the model was assumed to be undeformable and the aperture of the fracture was assumed to be constant values with no relationship with the associated confining pressure.

4.3 Numerical simulation

For the case of incompressible and steady Newtonian fluid, flow is usually governed by the following NS equations, which are derived from Newton’s second law, written as

\[
\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla P + \nabla \cdot \mathbf{T} + \mathbf{f}
\]

(4-1)

where \( \mathbf{u} \) is the flow velocity, \( \rho \) is the fluid density, \( P \) is the hydraulic pressure, \( \mathbf{T} \) is the shear stress tensor, \( t \) is the time, and \( \mathbf{f} \) is the body force. For laminar flow in natural single fractures, the convective acceleration terms, \( (\mathbf{u} \cdot \nabla) \mathbf{u} \), are the only source of nonlinearity that is strongly affected by the geometric variation of fractures [Xiong et al. 2011]. In a parallel-plate model at a low flow rate, the NS equations can be simplified to the LCL as follows, which is commonly adopted in the calculation of fluid flow in each fracture in DFNs [e.g., Long 1982; Min et al. 2004],

\[
Q = \frac{-g e^3 J}{12 \mu}
\]

(4-2)

where \( Q \) is the flow rate, \( g \) is the gravitational acceleration, \( J \) is the hydraulic gradient and \( \mu \) is the dynamic viscosity, \( e \) is the hydraulic aperture. Eq. (4-2) implies that \( Q \) is linearly related with \( J \) when the flow rate is sufficiently low. By increasing the flow rate, Eq. (4-2) becomes inapplicable, and a new mathematical expression of the nonlinear relationship of \( Q \) and \( J \) was proposed [Bear 1972; Moutsopoulos 2009; Qian et al. 2011; Cherubini et al. 2012; Adler et al. 2013], written as

\[
J = AQ + BQ^2
\]

(4-3)

where \( A \) and \( B \) are two coefficients that are related to fracture geometries, such as aperture and roughness. Eq. (4-3) is the famous Forchheimer’s law. Here, \( J \) is linearly proportional to the pressure gradient, \( \nabla P \), as follows
\[ J = \frac{h_2 - h_1}{\Delta L} = \frac{1}{\rho g} \frac{P_2 - P_1}{\Delta L} = \frac{1}{\rho g} \frac{\Delta P}{\Delta L} = \frac{1}{\rho g} \nabla P \]  

(4-4)

where \( h_1 \) and \( h_2 \) are the hydraulic heads of the two tips of a fracture. The numerical simulations were performed based on a FVM (finite volume method) code ANSYS FLUENT module [Fluent, 2006], and no-slip boundary conditions were assigned to the walls of the fractures.

The numerical models were established using a visualization technique based on the geometries of the experimental models. First, the experimental models were fulfilled with the dye solutions, and the CCD camera was utilized to take photographs. Second, these images were imported into AutoCAD and the dimension system of AutoCAD was configured to meet each image by referencing to the scale bar on the experimental figures. During image processing, the fracture geometries were re-depicted with a resolution of 0.01 mm. Finally, the inflow and outflow tanks were added to the models to form connected flow paths, which were the same as those of the experimental models. The detailed information of visualization techniques and the measurement of fracture surfaces can refer to Koyama et al. [2008b] and Liu et al. [2014]. Fig. 4-6 presents the two numerical models and enlarged views of intersections. The results show that for the smooth fracture surface model, the mechanical apertures of the four segments were \( E_1 = 1.37 \, \text{mm} \), \( E_2 = 1.40 \, \text{mm} \), \( E_3 = 1.20 \, \text{mm} \) and \( E_4 = 0.95 \, \text{mm} \), while for the rough
fracture surface model, \( E_{1} = 0.94 \, \text{mm} \), \( E_{2} = 1.33 \, \text{mm} \), \( E_{3} = 1.41 \, \text{mm} \) and \( E_{4} = 0.98 \, \text{mm} \). The fracture surface roughness was quantified by a dimensionless factor \( Z_{2} \), which was defined as the root mean square of the first deviation of the profile and was calculated using the following equation [Myers 1962; Tse and Cruden 1979]:

\[
Z_{2} = \left[ \frac{1}{M} \sum \left( \frac{z_{i} - z_{i-1}}{x_{i} - x_{i-1}} \right)^{2} \right]^{1/2}
\]

where \( x_{i} \) and \( z_{i} \) represent the coordinates of the fracture surface profile, and \( M \) is the number of sampling points along the length (i.e., \( x \) coordinate) of a fracture. Each fracture segment of the smooth fracture surface model has the same \( Z_{2} = 0 \), yet for the rough fracture surface model, the average values of \( Z_{2} \) were 0.18, 0.25, 0.42, and 0.15, corresponding to the four segments, respectively.

4.4 Results and analyses

4.4.1 Flow characteristics considering fracture surface roughness

The tested and simulated results were exhibited in Fig. 4-7 with one fixing inlet (inlet_1) and variable outlets (outlet_2, outlet_3, outlet_4, outlet_2&3, outlet_2&4, and outlet_3&4, respectively). The results show that \( J \) is quadratically related with \( Q \), following the form of Eq. (4-3) with the correlation coefficient \( R^{2} > 0.99 \). The simulation results by solving the NS equations agree well with the flow test results, indicating that the fracture geometries obtained by using the visualization techniques are reliable and the FVM code is capable of simulating nonlinear flow behaviors at fracture intersections. The nonlinearity of the smooth fracture surface model arises from fracture intersection, aperture variation, hydraulic gradient and inflow/outflow tanks, while the nonlinearity of the rough fracture surface model is also caused by fracture surface roughness, besides the above mentioned aspects. The larger \( Q \) would cause the stronger nonlinearity and more significant deviations with respect to the results calculated by solving the LCL. The conductivity of the smooth model is stronger than that of the rough model, because its average mechanical aperture (1.23 mm) is larger than that of the rough model (1.17 mm). The variations of \( J \) for the cases of one inlet and one outlet are more rapid than the ones for the cases of one inlet and two outlets, corresponding to the same variations of \( Q \).
Fig. 4-7 Comparisons of experimental and numerical results for the cases of one fixing inlet (inlet_1) and variable outlets (outlet_2, outlet_3, outlet_4, outlet_2&3, outlet_2&4, outlet_3&4, respectively). (S: smooth fracture surface model, R: rough fracture surface model, E: experiment result, N: numerical)

Attendances should be paid that in this study, $J$ is used to quantify the nonlinearity of fluid flow in crossed fractures, rather than $Re$, which is typically utilized in previous works [i.e., Zimmerman et al. 2004; Koyama et al. 2008; Javadi et al. 2014]. The reasons are that in a DFN model (see Fig. 4-1) or in a crossed fracture model (see Fig.
(4-6), there are more than one fracture segment. However, each fracture segment contributes to only one $Re$, and it is a challenging work to calculate the $Re$ of each segment. In the in-situ pumping tests or in the general numerical simulations of fluid flow in fractured rock masses, the dimensionless hydraulic gradient, $J$, usually has a known value, which is linearly proportional to the pressure gradient according to Eq. (4-4). For fluid flow in single fractures, the magnitude of $J$ is also proportional to $Re$ of single fractures, therefore, $J$ may be a more robust parameter to estimate the hydraulic properties of fluid flow at fracture intersections.

4.4.2 Flow characteristics considering intersecting angles

Fig. 4-7 Laboratory experiment of solute flow patterns through fracture intersections at different intersecting angles (60°, 90° and 120°) under different time intervals ($T = 0$ s, 2 s, 4 s and 6 s).
Fig. 4-7 shows the flow images within the intersections captured by a CCD camera and Fig. 4-8 shows the corresponding numerical simulation results of flow in single intersections, corresponding to the models shown in Fig. 4-4 with the intersecting angles of the two crossed fractures as 60°, 90° and 120°, respectively. In these models, \( Q_1 \) was assigned a constant value of 18 ml/min, corresponding to a Reynolds number of 40. The Reynolds number is defined as \( Re = \frac{\rho V D}{\mu} \), where \( V \) is the flow velocity, and \( D \) is the characteristic length (or hydraulic diameter) that can be calculated by the following equation for the rectangular duct [Aharwal et al., 2008]:

\[
D = \frac{4A}{\chi} = 2we/(w+e)
\]  

(4-6)

where \( A \) is the cross-sectional area of the fracture, \( \chi=2(w+e) \) is the length of the wetted periphery, and \( w \) is the width.

In Fig. 4-8, the blue color stands for the existing distilled water, while the red color stands for the 100% dye solution, and other colors correspond to the mixed solutions with concentrations ranging from 12.5 to 87.5%, as shown in the legend. The results show that the travel distance and distribution pattern of solution interfered by the intersection are almost identical between the captured images and the simulation results. After the solute reached the 0 point, it is redistributed into the two outlet segments, exhibiting a hyperbolic frontline in each outlet branch. The flow velocity at the tip of the hyperbolic frontline is significantly larger than the mean flow velocity. The concentrations of the dye solute at the tip and near the boundary of the hyperbolic frontline are lower than those inside the hyperbolic curves. This is because solute transport through a fracture intersection is bounded by the streamline routing and complete mixing. The experimental observations confirmed that the solute transport primarily followed the streamline model with negligible mixing [Park and Lee, 2001]. The streamline routing is capable of providing a good approximation to the solute transport. The influences of intersecting angles on the variation of flow patterns are negligible at the current \( Re \), and more remarkable changes are expected at a higher \( Re \). The numerical results in Fig. 4-8 show that the mixing of solute and water only happens at the small region of the interface of the two phases.
The travel distance of solution in segment 3, which is parallel to the segment 1, is larger than that in the segment 4 at the same elapsed time due to the inertial effects of flow. Direct comparisons of the travel distances in the two segments are shown in Fig. 4-9. The distance is linearly proportional to the time and the travel velocity (travel distance/time) increases proportionally with the increasing inflow rate. The intersecting angle has negligible influences on nonlinear flow in our experimental models, because its influence relies significantly on the relative scale of fracture length and aperture. When the length (≥ 25 cm) of the fracture segment is much greater than the aperture (1
cm) of fracture, the effect of intersecting angle vanishes. The good agreements between experimental and simulation results confirm the validity of the numerical models that solve the NS equations and the assumptions of no-slip boundary and viscous Newtonian fluid, which allows us to explore further conditions with different flow rates and intersection geometries. Fig. 4-10 shows the relationships of the inflow rate and the pressure drop obtained from experiment and numerical simulation gathering all of the flow rates ranging from 18 to 200 ml/min, based on the model with an intersecting angle of 90°. The discrepancy between experimental and numerical results is considerably small at a low flow rate, which gradually becomes large when the flow rate is larger than 120 ml/min. This may be attributed to the energy losses caused by the shape changes when the fluid flows in or out of the model through the tanks, which may become remarkable at large flow rate. The numerical models including the inflow and outflow tanks have considered the head loss between the tank and the intersection model. The fitting curves exhibit an initial linear regime and a subsequent nonlinear regime in which the cubic law is no longer applicable. The nonlinear regime is induced by the inertial effects of fluid flow influenced by the intersection. The impacts of the inertial effects on the pressure-flow rate relationship need to be extensively estimated to improve the accuracy of calculating fluid flow in DFN models.

Fig. 4-9 Comparisons of solute travel distances in the fractures along with the elapsed time. (E: Experimental result; N: Numerical simulation result)
Fig. 4-10 Comparison of flow rates with varying static pressure drops.

In order to quantitatively investigate the inertial effects of fluid at the intersection for different interesting angles, the flow rate of inlet_1, corresponding to the Re in the range of 0.1 to 200, was changed and the normalized flow rates at outlet_3 and outlet_4 were calculated, respectively.

\[ N_3 = \frac{Q_3}{(Q_3 + Q_4)} \]  \hspace{1cm} (4-7)  
\[ N_4 = \frac{Q_4}{(Q_3 + Q_4)} \]  \hspace{1cm} (4-8)

where \( N_3 \) is the normalized flow rate at outlet_3, \( N_4 \) is the normalized flow rate at outlet_4, \( Q_3 \) is the flow rate at outlet_3 and \( Q_4 \) is the flow rate at outlet_4.

Fig. 4-11 Normalized flow rates at outlet_3 and outlet_4 with varying Re in the range of 0.1 to 200.

Fig. 4-11 shows the distributions of the normalized flow rate with \( Re = 0.1 \sim 200 \). When the \( Re \) of inlet_1 is small (i.e., less than 1), the normalized flow rates at both outlet_3 and outlet_4 hold constant values, meaning that the inertial effects of fluid are negligible. When \( 1 < Re < 10 \), the inertial effects are relatively weak and the normalized
flow rates of the two outlets vary slightly. For large Re (i.e., Re > 100), the inertial effects become remarkable and the normalized flow rates change significantly. When the Re is large (i.e., Re > 100), the influences of intersecting angle seems to be important, however, the lengths of segment_4 for models with different intersecting angles are different, emphasizing that they cannot be compared with each other. Based on the models in Fig. 4-4 and by fitting these calculated results, it is found that the normalized flow rate is a function of the Re of inlet_1 and the intersecting angle. The expressions can be written as:

\[ N_3 = 0.5 + (0.023\theta + 0.42) \times 10^{-4} Re - (0.024\theta + 6.42) \times 10^{-8} Re^2 \]  
\[ N_4 = 0.5 - (0.023\theta + 0.42) \times 10^{-4} Re + (0.024\theta + 6.42) \times 10^{-8} Re^2 \]

where \( \theta \) is the intersecting angle of the two crossed fractures.

The above two equations imply that for Re < 1, the local cubic law (see Eq. (4-2)) applies, while for Re > 1 (or Re > 100), the Forchheimer equation (see Eq. (4-3)) applies.

4.5 Quantification of hydraulic properties in crossed fractures

Although the flow tests and corresponding numerical simulations of fluid flow at fracture intersections have shown that the hydraulic properties of crossed fracture models are robustly influenced by hydraulic gradient, fracture surface roughness, intersection, and aperture variation, their individual influences have not received quantitative assessments and no identical mathematical expressions have been established. So, more detailed parametric and quantitative studies are expected.

4.5.1 Generation and extraction of crossed fracture models

Two original large crossed fracture models were established with a constant \( E = 1 \) mm for each segment and an intersecting angle of 60°. As shown in Fig. 4-12, the smooth surface fracture model was consist of smooth fractures with \( Z_2 = 0 \) and the rough fracture surface model was consist of rough fractures with \( Z_2 = 0.42 \). \( Z_2 = 0 \) corresponds to JRC = 0, and \( Z_2 = 0.42 \) corresponds to JRC = 20, which can be calculated as follows [Tse and Cruden 1979]:

\[ JRC = 32.2 + 32.47 \log Z_2 \]  

Here, JRC = 20 is typically regarded as the roughest fracture surface in engineering practices according to the work of Barton et al. [1973]. Scale effects were also
considered by extracting the numerical models with a series of circles truncating the original large models with different radius, $R_r$, which ranged from 500 mm to 5 mm.

Fig. 4-12 Numerical models with varying scales and joint roughness coefficients.

Fig. 4-12 shows six models of them to show the procedures of extractions. For simplification, only the cases of one inlet (inlet_1) and two outlets (outlet_2&3, outlet_2&4, and outlet_3&4, respectively) were investigated. Thus, the values of $J$ for the three cases (outlet_2&3, outlet_2&4, and outlet_3&4) were identical if the models were truncated with the same circles and the same pressure drop was applied between the inlet and outlets. Previous works assumed constant values of $J$ to estimate the hydraulic properties of DFNs, for example, $J = 0.001$ [Cvetkovic et al. 2004; Zhao et al. 2013], $J = 0.1$ [Zhang and Sanderson 1996], and $J = 1$ [Long 1982; Klimczak et al. 2010; Zhao et al. 2011; Liu et al. 2015]. This work assigned a wider range of values as $J = 10^{-5}$-$10^{0}$, to sufficiently cover the cases encountered in natural ground water systems and in engineering practices. The value of $R_r$ ranges three orders of magnitude from 500 mm to 5 mm. If $R_r > 1000$ mm, it is out of the calculating power of a common PC, because the larger value of $R_r$ will generate more meshes with a fixing mesh spacing of 0.1 mm. When the flow rate at the inlet was equal to the flow rates at all outlets, the
flow was considered to have reached a steady state, and the corresponding \( Q \) and \( J \) were recorded for further analysis.

4.5.2 Effects of \( J \) and \( R_r \) on \( Q/J \) and \( \delta \)

Fig. 4-13 shows the relationships of \( Q/J \sim J \), \( Q/J \sim R_r \), and \( \delta \sim J \), respectively, with \( J \) ranging from \( 10^{-5} \) to \( 10^0 \) and \( R_r \) ranging from 5 mm to 500 mm. Here, only the case of inlet_1 and outlet_2&3 for fluid flow through the smooth fracture surface model was considered. The relative deviation, \( \delta \), is defined as follows:

\[
\delta = \left( \frac{Q/J}_{\text{LCL}} - \frac{Q/J}_{\text{NS}} \right) \times 100\% \tag{4-12}
\]

where, \( (Q/J)_{\text{LCL}} \) is the value of \( Q/J \) calculated by solving the LCL, and \( (Q/J)_{\text{NS}} \) is the value of \( Q/J \) calculated by solving the NS equations.

Fig. 4-13(a) presents that with the increment of \( J \), \( Q \) increases approximately linearly, resulting in constant values of \( Q/J \), when \( J \) is less than \( 10^{-3} \). For \( J > 10^{-2} \), the inertial forces of fluid play a more significant role, compared with those when \( J < 10^{-3} \), emphasizing that the nonlinear terms in Eq. (4-3) cannot be neglected. The larger value of \( J \) would cause the more energy loss of fluid flow and the smaller value of \( Q/J \). Therefore, as \( J \) increases, \( Q/J \) varies from constant values to variables, indicating that fluid flow changes from linear flow regimes to nonlinear flow regimes, and the nonlinearity is stronger with a larger \( J \). In the linear flow regimes, i.e., \( J < 10^{-3} \), the value of \( Q/J \) decreases with the increment of \( R_r \), yet in the nonlinear flow regimes, i.e., \( J > 10^{-2} \), the value of \( Q/J \) increases with the increment of \( R_r \), corresponding to the same \( J \). Fig. 4-13(b) shows the variations of \( Q/J \) with varying values of \( R_r \) from 5 mm to 500 mm. When the model scale is small, i.e., \( R_r < 50 \) mm, \( Q/J \) varies significantly, while with increasing \( R_r \), especially when \( R_r > 200 \) mm, \( Q/J \) gradually becomes stable and holds constant values. Fig. 4-13(c) depicts the relationship of \( \delta \) and \( J \), in which \( \delta \) is approximately zero when \( J < 10^{-3} \) and increases with the increase of \( J \). The larger value of \( \delta \) means the stronger nonlinearity of fluid flow and overestimates the value of \( Q/J \) more significantly when solving the LCL, compared with that by solving the NS equations. When \( R_r \) changes from 5 mm to 500 mm, which experiences a variation of 3 orders of magnitude for \( J = 10^0 \), \( \delta \) reduces from 74.17\% to 31.89\%.
4.5.3 Effects of intersecting angle and fracture surface roughness

In the numerical simulations, the inlet (inlet_1) was fixed and the combinations of the two outlets (outlet_2&3, outlet_2&4, and outlet_3&4) were changed to consider the effects of intersecting angle. Table 4-2 lists the variations of $Q/J$ and $\delta$ for the different cases with $J$ varying from $10^{-5}$ to $10^{0}$ and $R_r = 500$ mm. The results show that both $Q/J$ and $\delta$ have almost the same values for different combinations of inlet and outlets,
indicating that the influences of intersecting angle on both $Q/J$ and $\delta$ can be negligible. This is why only the case of inlet_1 and outlet_2&3 was considered, rather than all the cases, in the above Section 4.5.2. Table 4-3 shows the corresponding results for the rough fracture surface model. Considering fracture surface roughness with JRC = 20 would reduce the value of $Q/J$ by 26.55% and increase the value of $\delta$ by 4.24% in spite of the value of $J$, compared with the smooth surface fracture model with JRC = 0. Therefore, it just needs to simply multiply by two constant coefficients, i.e., $\alpha$ and $\beta$, to the results of $Q/J$ and $\delta$, respectively, when taking account of fracture surface roughness. Here, for fluid flow through crossed fractures with JRC = 0 ~ 20, $\alpha$ = 100% ~ 73.45% and $\beta$ = 100% ~ 104.24%.

Table 4-2 Simulation results of $Q/J$ and $\delta$ for the smooth fracture surface models

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<td>9.29E-04</td>
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Table 4-3 Simulation results of $Q/J$ and $\delta$ for the rough fracture surface models

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4.5.4 Effects of $J$ on $R_a$

Fig. 4-14 $R_a$ distributions with varying $J$ and different combinations of outlets.

Since there are two outlets for each case as shown in Fig. 4-12, the ratio of flow rate at one outlet to the flow rates at all outlets may change depending on the magnitude
of $J$. Here, for example, for the case of inlet_1 and outlet_2&3, the normalized flow rates at outlet_2 and outlet_3 are defined, respectively, as follows:

$$R_a\text{(at outlet}_2\text{)} = \frac{Q_2}{Q_2 + Q_3}$$

(4-13)

$$R_a\text{(at outlet}_3\text{)} = \frac{Q_3}{Q_2 + Q_3}$$

(4-14)

where, $R_a$ is the normalized flow rate, $Q_2$ is the flow rate at outlet_2, and $Q_3$ is the flow rate at outlet_3.

Fig. 4-14 shows the distributions of $R_a$ with increasing $J$ from $10^{-5}$ to $10^0$ for different combinations of outlets and model scales, based on the smooth fracture surface model. The results show that when $J < 10^{-3}$, $R_a$ holds an approximately constant value of 0.5, indicating that the inertial effects can be negligible. For $10^{-3} < J < 10^{-2}$, the inertial effects are relatively weak and $R_a$ varies slightly, while when $J > 10^{-2}$, the inertial effects become remarkable and $R_a$ changes significantly, especially for $R_f = 5 \sim 50$ mm. Therefore, the scale effect can be neglected when $J < 10^{-3}$, however, for $J > 10^{-2}$, it plays an important role in calculating $R_a$ and cannot be negligible. Notable is that for different combinations of outlets with $J > 10^{-2}$, the values of $R_a$ are changed, too, exhibiting that the intersecting angle has non-negligible influences on the distributions of $R_a$. When $J < 10^{-3}$, the influences of intersecting angle are very weak and can be neglected, too.
4.5.5 Effects of $J$ on $e/E$

The $e$ of each fracture segment was back calculated using Eq. (4-2), in which the $Q$ of each segment was determined by solving the NS equations (see Eq. (4-1)) for any $J$. The ratio of hydraulic aperture to mechanical aperture, $e/E$, was adopted to depict the evolutions of hydraulic aperture for different $J$. The effects of $J$ on $e/E$ are shown in Fig.
When $J < 10^{-3}$, the values of $e/E$ hold a constant value of approximating 1, indicating that $e = E$. With increasing $J$, $e/E$ gradually deviates the value of 1. Due to the existence of inertial forces at fracture intersections, for a high $J, R_a$ of one segment will increase and $R_a$ of the other one will decrease, resulting in that the value of $e/E$ of one segment is larger than 1 and the value of $e/E$ of the other segment is less than 1. The larger model scale is, the less significant variations of $e/E$ exhibit. The results also show that $e/E \geq 1$ for the segment with a smaller deviation angle along the flow direction of inlet, and $e/E \leq 1$ for the other segment with a larger deviation angle. For example, for the case of inlet_1 and outlet_2&3, $e/E \geq 1$ for the segment of outlet_3 (deviation angle = 0°), while $e/E \leq 1$ for the segment of outlet_2 (deviation angle = 120°). Therefore, when $J < 10^{-3}$, the effects of both $J$ and $R_r$ are very small and can be neglected, yet for $J > 10^2$, their influences have to be considered.

### 4.5.6 An empirical expression of $e/E$

The 30 cases with different values of $J$ and $E/R_r$ are shown in Table 4-4, and the values of $e/E$ of single segments are also listed for different combinations of inlet and outlets. Here, the dimensionless parameter, $E/R_r$, was utilized to study the effects of model scale, rather than the above mentioned $R_r$. The reasons are that in this study $E$ is a constant parameter and its influences on the hydraulic properties of fluid flow at fracture intersections are not considered, however, when $E$ changes, it would alter the flow behaviors. By fitting these parameters to numerically calculated results of $e/E$, a multi-variable regression algorithm was utilized to establish a mathematical expression. Here, $J$ and $E/R_r$ are two dimensionless and independent variables and $e/E$ is the dimensionless and dependent variable. The best-fitted expression is as follows:

$$e/E = a + bJ + c(E/R_r) + dJ^2 + f(E/R_r)^2 + gJ(E/R_r) + hJ^2(E/R_r)^2 + iJ^3 + j(E/R_r)^3$$

(4-14)

where, $a, b, c, d, f, g, h, i, j, k$ are coefficients that are related to the deviation angle of each segment and different combinations of inlet and outlets. Table 4-5 shows the basic parameters of Eq. (4-14). Substituting Eq. (4-14) into Eq. (4-2) would result in a modified LCL, in which $e$ is a function of $J, E$, and $R_r$, and $Q$ is nonlinearly related with $J$, when calculating the fluid flow of each segment in crossed fracture models or in DFN models. Comparisons of the predictions by using Eq. (4-14) and simulation results by solving the NS equations are demonstrated in Fig. 4-16. The results show that Eq. (4-14) gives good predictions to the numerical simulation results with $J = 10^{-5} ~ 10^0$ and $E/R_r = 0.002 ~ 0.2$ for different single segments of different combinations of inlet and outlets. The correlation coefficients for all the cases are $R^2 > 0.99$, indicating that Eq. (4-14) is sufficiently reliable to characterize nonlinear flow regimes at fracture intersections.
Fig. 4-16 Comparisons of the predictions by using Eq. (4-14) and simulation results by solving the NS equations.
<table>
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<th>S_outlet_3&amp;4</th>
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R² = 0.9960  R² = 0.9866  R² = 0.9934  R² = 0.9976  R² = 0.9976  R² = 0.9991
Table 4-5 Parameters of Eq. (4-14) for the smooth fracture surface models

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<td>3.79</td>
<td>-3.28</td>
</tr>
<tr>
<td>$k$</td>
<td>-38.77</td>
<td>49.00</td>
<td>-25.64</td>
</tr>
</tbody>
</table>

4.5.7 Scale effect

Due to the limitations of computing power of a common PC, the maximum $R_e$ is limited to 500 mm for the crossed fractures as shown in Fig. 4-12. In Fig. 4-13(b), even when $R_e = 500$ mm, the values of $Q/J$ did not receive a sufficiently stable state. To clearly understand the influences of scale effect, Eq. (4-14) is used to extend the model scale with $R_e$ ranging from 10 mm to 10000 mm, and the results are shown in Fig. 4-17. With the increment of $E/R_e$, the value of $e/E$ decreases for the segment with a smaller deviation angle, and increases for the other segment with a larger deviation angle. When $E/R_e < 10^{-3}$, $e/E$ gradually convergences to a small range of $[0.98, 1.03]$ for all cases, indicating that the hydraulic aperture is approximately equal to the mechanical aperture and the scale effect plays a negligible role on the evolution of hydraulic aperture of each segment. However, for $E/R_e > 10^{-2}$, especially when $J$ is large (i.e., $J = 1$), $e/E$ varies significantly and the scale effect has to be considered. With the decrease of $J$, $e/E$ deviates less slightly than that with a larger $J$. 
Fig. 4-17 Scale effects on the variations of $e/E$ for different $J$.

4.6. Conclusions

In this study, flow tests and corresponding numerical simulations were performed on the crossed fracture models consist of smooth and rough fracture surfaces to give an
insight into the hydraulic properties of fluid flow at fracture intersections. Then, the roles of hydraulic gradient, fracture surface roughness, intersecting angle, and scale effect were independently investigated on large scale numerical models. Finally, an empirical expression of calculating the hydraulic aperture of each fracture segment connected to the intersections was proposed, which was a function of the hydraulic gradient, the radius of truncating circles, and the mechanical aperture and could be utilized to modify the local cubic law. The main conclusions can be obtained as follows:

(1) The mechanical apertures of experimental models can be measured using the visualization techniques with a CCD camera. The numerical simulations examined the accuracy of the measured mechanical apertures, and the flow tests verified the reliability of the FVM code by solving the Navier-Stokes equations to quantify the hydraulic properties of fluid flow at fracture intersections.

(2) The flow tests show that the nonlinearity of fluid flow through crossed fracture models is potentially caused by fracture intersection, aperture variation, hydraulic gradient, inflow/outflow tanks, and fracture surface roughness.

(3) The numerical simulations based on the large scale models show that with the increment of the hydraulic gradient, the ratio of the flow rate to the hydraulic gradient, \( Q/J \), decreases and the relative deviation, \( \delta \), increases, due to the gradually increasing inertial effects. When taking account of the fracture surface roughness with JRC = 0 ~ 20, the values of \( Q/J \) and \( \delta \) would reduce by 0 ~ 26.55% and increase by 0 ~ 4.24%, respectively.

(4) The values of \( Q/J \) vary significantly when the radius of the truncating circles, \( R_r \), is less than 50 mm, and gradually become stable for \( R_r > 200 \) mm. In the linear flow regimes, i.e., \( J < 10^{-3} \), \( Q/J \) decreases with the increment of \( R_r \), yet in the nonlinear flow regimes, i.e., \( J > 10^{-2} \), \( Q/J \) increases with the increment of \( R_r \).

(5) The influences of intersecting angles of the two crossed fractures can be negligible on \( Q/J \) and \( \delta \) in spite of the value of \( J \). Their influences on the normalized flow rate, \( R_a \), and the ratio of the hydraulic aperture to the mechanical aperture, \( e/E \), can also be neglected when \( J < 10^{-3} \), however, the values of \( R_a \) and \( e/E \) are quite different for different intersecting angles when \( J > 10^{-2} \) and their influences have to be considered.

(6) By analyzing the relations of \( Q \sim J, R_a \sim J, \) and \( e/E \sim J \) based on the crossed fracture models, it can be found that they all have linear relationships where the inertial effects can be negligible, weak nonlinear relationships where the inertial effects are weak, and strong nonlinear relationships where the inertial effects are strong, corresponding to \( J < 10^{-3}, 10^{-3} < J < 10^{-2}, \) and \( J > 10^{-2} \), respectively.
(7) By fitting the results of the numerical simulations based on 30 cases, an empirical expression of calculating $e/E$ was proposed, which was a function of $J$ and $E/R_r$. The predictions of the proposed empirical expression agree well with the numerical simulation results with $J = 10^{-5} \sim 10^0$ and $E/R_r = 0.002 \sim 0.2$ for different combinations of inlet and outlets, indicating that this expression is sufficiently reliable to characterize hydraulic properties of fluid flow at fracture intersections. Using the proposed expression, $R_r$ was extended to 10000 mm to quantify the scale effects and the results depicted that for $E/R_r > 10^{-2}$, $e/E$ varies significantly and the scale effect has to be considered, while when $E/R_r < 10^{-3}$, the scale effect is less significant and can be neglected.

(8) It can be inferred that one of the conservative conditions to apply the local cubic law at fracture intersections is: $J < 10^{-3}$, and $E/R_r < 10^{-3}$, and JRC = 0. However, further works are needed to verify whether this criteria is also applicable for fluid flow in DFNs that contain hundreds/thousands of fracture intersections and segments.

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Nonlinear flow properties in two-dimensional rock fracture networks

5.1 Introduction

In recent years, discrete fracture network (DFN) modeling techniques have been extensively applied to investigations of hydro-mechanical and mass transport behaviors of fractured rock systems [Leung and Zimmerman 2012; Lang et al. 2014]. Many studies presumed that fluid flow in each single fracture in DFNs follows the cubic law, which suggests a linear relationship between the flow rate and the pressure drop in the calculations of fluid flow in fracture systems [Long et al. 1982; Jing 2003; Min et al. 2004]. However, accurate estimations by the cubic law that neglects the inertial effects can only be anticipated for sufficiently low Reynolds numbers (Re), and past studies have revealed that the flow rate could be nonlinearly related to the pressure drop when the applied pressure/flow rate becomes large [Cooke 1973; Holditch and Morse 1976; Gale 1984; Elworth and Doe 1986; Jung 1989; Kohl et al. 1997; Yeo and Ge 2001; Wen et al. 2006].

Previous studies have focused on the determination of the critical Re for the onset of nonlinear flow through single rock fractures, which suggested different ranges from 0.001 to 25 that vary with the surface roughness of fractures and applied normal and/or shear stresses [Oron and Berkowitz 1998; Zimmerman et al. 1996, 2004; Koyama et al. 2008; Xiong et al. 2011; Radilla et al. 2013; Javadi et al. 2014]. The surface roughness that contributes to complex void geometries and streamline structures can significantly reduce the critical Re for the flow regime transition [Parrish 1963; Zimmerman et al. 2004; Konzu and Kueper 2004; Ranjith and Darlington 2007; Javadi et al. 2010; Tzelepis et al. 2015]. Studies on fluid flow through crossed fractures revealed complex, nonlinear flow patterns when $1 < Re < 100$ and different mixing behaviors, bounded by complete mixing and streamline routing within fracture intersections [Stockman et al. 1997; Kosakowski and Berkowitz 1999; Mourzenko et al. 2002].

The Re was typically incorporated in some criteria for detecting the nonlinear flow through single fractures, which may not apply to DFNs, because flow in each single fracture can have a different Re, and the assessment of the values of localized Re in DFNs with large amounts of fractures would be a tough task. In contrast, the hydraulic gradient ($J$), defined as the ratio of hydraulic head to DFN side length, is typically a known parameter in many practices on fractured rock masses, such as hydraulic
pumping tests with prescribed hydraulic pressures. For a single fracture, the magnitude of \( Re \) is proportional to that of \( J \), and \( J \) is also a dimensionless parameter representing how fast a pressure drops over a given region. Therefore, \( J \) may be a more practical parameter for establishing a criterion for the onset of nonlinear flow in DFNs.

Although the mechanisms, such as the formation of vortices in the positions with dramatic geometric variations at a large \( Re \) that drive the nonlinear flow in rough-surfaced fractures and fracture intersections, have been extensively investigated [e.g., Kosakowski and Berkowitz 1999; Zimmerman et al. 2004; Koyama et al. 2008], the impacts of these micro-phenomena on macro hydraulic properties of DFNs have not been quantitatively estimated. Due to the enormous difficulty of establishing DFN models to consider the roughness of each single fracture and of solving the Navier-Stokes (NS) equations composed of a set of coupled nonlinear partial derivatives of varying orders [Zimmerman and Bodvarsson 1996; Brush and Thomson 2003; Javadi et al. 2010], most previous works presumed that the cubic law was always applicable, disregarding the magnitude of \( J \), such as \( J = 1 \) [Long 1982; Zhang et al. 1996; Zhang et al. 1999; Klimczak et al. 2010; Zhao et al. 2010; Zhao et al. 2011; Liu et al. 2015], \( J = 0.1 \) [Zhang and Sanderson 1996], \( J = 0.001 \) [Cvetkovic et al. 2004; Zhao et al. 2013], and \( J = \) unknown constants [Min et al. 2004; Baghbanan and Jing 2007; 2008; Parashar and Reeves 2012; Reeves et al. 2013; Latham et al. 2013]. It is therefore a crucial issue to determine the critical hydraulic gradient (\( J_c \)) for the onset of nonlinear flow in DFNs, below which the widely used cubic law is sufficiently applicable, and above which some nonlinear governing equations (e.g., Forchheimer’s law) need to be employed.

In this Chapter, a series of DFNs with different apertures, roughness, and numbers of intersections were established. Based on multi-variable regressions of their simulation results, mathematical expressions of \( J_c \) and the coefficients involved in Forchheimer’s law (\( A \) and \( B \)) were established. These expressions were applied to another series of DFNs with well-known geometric characteristics of fractures to verify their validity by comparing the predicted results with the fluid flow simulation results, and their nonlinear flow behaviors were analyzed and discussed.
5.2 Numerical setup

5.2.1 Governing equations

The numerical simulations were performed based on the FVM (finite volume method) code ANSYS FLUENT module [Fluent, 2006], by directly solving the NS equations, written as

\[ \rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \nabla \cdot \mathbf{T} + \rho \mathbf{f}, \]

where \( \mathbf{u} \) is the flow velocity, \( \rho \) is the fluid density, \( p \) is the hydraulic pressure, \( \mathbf{T} \) is the shear stress tensor, \( t \) is the time, and \( \mathbf{f} \) is the body force. For laminar flow in nature single fractures, the convective acceleration terms, \((\mathbf{u} \cdot \nabla) \mathbf{u}\), are the only source of nonlinearity that is strongly affected by the geometric variation of fractures [Xiong et al. 2011]. In a parallel-plate model at a low Re, the NS equations can be simplified to the cubic law as follows, which is commonly adopted in the calculation of fluid flow in single fractures in DFNs [e.g., Long et al. 1982; Min et al. 2004],

\[ Q = -\frac{\rho g h_H^3}{12 \mu} J, \]

where \( Q \) is the flow rate, \( g \) is the gravitational acceleration, and \( \mu \) is the dynamic viscosity, \( h_H \) is the hydraulic aperture. More details regarding modelling fluid flow through rough-walled fractures using ANSYS FLUENT module are referred to the work of Xiong et al. [2011].

5.2.2 Nonlinearity estimation

The most extensively adopted mathematical description of the nonlinear fluid flow in fractures is Forchheimer’s law, which in the simplest form is written as [Bear 1972; Ranjith and Darlington 2007; Ji et al. 2008; Moutsopoulos 2009; Nowamooz et al. 2009; Qian et al. 2011; Quinn et al. 2011; Ranjith and Viete 2011; Cherubini et al. 2012; Adler et al. 2013],

\[ J = AQ + BQ^2, \]

where \( A \) and \( B \) are two coefficients that are related to fracture geometries, such as aperture and roughness. In the linear regime, Eq. (5-3) simply reduces to \( J = AQ \), where \( A \) is linearly proportional to the reciprocal of the permeability of the fractures.
To quantitatively estimate $J_c$ for these models, a factor ($E$) was introduced to quantify the nonlinearity of fluids flowing through the crossed fractures and DFNs, which is written as [Zeng and Grigg 2006],

$$E = \frac{BQ^2}{AQ + BQ^2} \quad (5-4)$$

This factor represents the proportional contribution of the nonlinear term to the total pressure drop. Here, a critical value of $E = 0.01$ was defined to categorize the regime of fluid flow considering the magnitude of errors involved in the flow test and the numerical simulation of this study. At this value, the nonlinear term only contributes to 1% of the total pressure drop, which is reasonably low to eliminate the contribution of the nonlinear term. $E = 0.1$ was used in the literature [Zeng and Grigg 2006; Javadi et al. 2014] for similar tests or simulations. Selection of proper values for $E$ depends on the magnitude of estimated systematic errors of experiments, measurements and/or numerical simulations, and the required precision of the problem concerned.

5.3 Parametric study

5.3.1 Simulation cases

Fig. 5-1 Numerical models with varying numbers of intersections constituted by smooth fractures ((a) - (d) with JRC = 0) and rough-walled fractures ((e) - (h) with JRC = 20). The intersecting angle is 60° for all fractures. ($N_i$: the number of intersections).
Although it has been found that the nonlinearity of fluid flow is robustly influenced by surface roughness and aperture of single fractures and single intersections [Koyama et al. 2008; Xiong et al. 2011; Johnson and Brown 2001; Johnson et al. 2006], their independent influences on nonlinear fluid flow in DFN models with multiple fractures and intersections have not been quantitatively assessed. The effects of the mechanical aperture ($h_M$), the number of intersections ($N_i$), and the fracture surface roughness (JRC) on the nonlinearity of fluid flow were analyzed independently and quantitatively based on 40 geometrically simple DFN models, among which 8 examples are shown in Fig. 5-1. Each model consists of two sets of fractures that intersect at 60°. These numerical models have a dimension of 50 cm × 50 cm, and each smooth model (JRC=0) has corresponding rough models (JRC > 0). A code JRCGEN was developed by the authors to generate rough surfaces of fractures. For each fracture in a rough model, the original fracture walls (straight lines) were divided into 100 short segments with an identical length connected by nodes, and the position of each node was shifted away from the original straight lines. The magnitude of such shifts was assumed to follow the Gaussian distribution [Brown 1995]. JRC values of generated fracture surfaces were controlled to range from 0 to 20. In the present study, JRC was selected to represent the surface roughness, instead of other parameters, because it has been extensively accepted in the field of rock mechanics, which links the mechanical behavior to the hydraulic behavior of rock fractures [Barton 1985; Olsson and Barton 2001]. Wide ranges of values were assigned to these parameters in the numerical simulations, i.e., $h_M = 0.5 - 10$ mm, $N_i = 1 - 12$, JRC = 0 - 20, and $J = 10^8 - 10^9$, to facilitate the assessment of their mathematical relationships. JRC = 20 represents extremely rough rock fracture surfaces and was adopted here as an upper bound for roughness, while the value of the smooth model (JRC = 0) was used as a lower bound. In shallow rock masses, the mechanical aperture of rock joints and faults usually has a magnitude ranging 0.1 - 10 mm. We selected relatively large values within this range because (1) the number of meshes increases exponentially with the decreasing aperture at a fixed layer number, which would greatly increase the computational time; (2) fluid flow in fractures with small apertures typically have low $Re$ and flow rates, and thereby diminishing the significance of nonlinearity. If nonlinear flow did not appear in a model, then mathematical description of its nonlinear flow behavior could not be achieved. The range of $J$ covers most possible values encountered in natural ground water systems and engineering practices. A fixed layer number of 10 was used in the construction of meshes in all models because further increment of layer number did not improve the accuracy of calculation by more than 1% even for the models with the largest aperture. Hydraulic pressures
were imposed on the top and bottom boundaries, and the left and right boundaries were impermeable. When the flow rate at the bottom boundary became equal to the flow rate at the top boundary, the flow was considered to have reached a steady state and the flow rate was recorded for further analysis.

5.3.2 Simulation results

Simulation results of the relationships between $J$ and $Q$ were fitted to Eq. (5-3); then, $E$ and the corresponding $J_c$ were calculated using Eq. (5-4). For DFNs, $J$ was simply the ratio of hydraulic head difference between the inlet and outlet boundaries to the side length. Table 5-1 tabulates the values of $A$, $B$, and $J_c$ of all cases. Figs. 5-2 (a), (b), and (c) show the independent relationships of $J_c$ with $h_M$, $N_i$, and JRC, respectively. When one relationship is analyzed (e.g., $J_c$ vs. $h_M$ with varying $N_i$), the remaining parameter (e.g., JRC) was assigned a constant value. Fig. 5-2(a) shows that $J_c$ varies by approximately five orders of magnitude when $h_M$ changes between 0.5 mm and 10 mm in a smooth model ($JRC = 0$), suggesting that $h_M$ has a significant impact on $J_c$, which is three orders of magnitude larger than that of $N_i$. Fig. 5-2 (b) shows that $J_c$ decreases about five times faster with the increment of $N_i$, from 1 to 12 in a rough model ($JRC = 20$) than that of a smooth model ($JRC = 0$) when $h_M = 1.0$ mm. Curves of other JRC values (0 - 20) are bounded between these two curves. Fig. 5-2(c) shows that $J_c$ undergoes a more significant reduction when JRC increases from 0 to 5, comparing with the further increment of JRC from 5 to 20, when $h_M = 1.0$ mm. Among these three parameters ($h_M$, $N_i$, and JRC), $J_c$ is most sensitive to $h_M$, followed by $N_i$ and JRC. For each single fracture in a DFN, the flow rate changes proportionally with the cubic of the aperture; therefore, a small change of aperture would result in a great change in flow rate and $Re$, which would then largely impact on its nonlinear flow behavior. As demonstrated in the models with a single intersection, the fracture intersection and roughness enhance the inertial effects of fluid flow, thereby reducing the required $Re$ or $J_c$ for the onset of nonlinear flow. Their effects, however, are several orders of magnitude lower than that of $h_M$. This can also be confirmed in Table 5-1, where the values of $A$ and $B$ decrease by 3 - 4 orders of magnitude when $h_M$ increases by one order of magnitude from 0.5 mm to 5 mm. Similar changes of $N_i$ and JRC can only result in the changes of $A$ and $B$ less than one order of magnitude.
Table 5-1 Values of $h_M$, JRC, and $N_i$ used in the parametric study, and simulated and predicted results of $A$, $B$, and $J_c$. (S: simulation results)

<table>
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<tr>
<th>Cases</th>
<th>Mean $h_M$ (mm)</th>
<th>JRC</th>
<th>$N_i$</th>
<th>$A$  (S) (Eq. (7))</th>
<th>$B$  (S) (Eq. (8))</th>
<th>$J_c$ (S) (Eq. (9))</th>
<th>Cases</th>
<th>Mean $h_M$ (mm)</th>
<th>JRC</th>
<th>$N_i$</th>
<th>$A$  (S) (Eq. (7))</th>
<th>$B$  (S) (Eq. (8))</th>
<th>$J_c$ (S) (Eq. (9))</th>
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<td>4.76</td>
<td>4</td>
<td>5.86E+02</td>
<td>6.69E+02</td>
<td>1.33E+05</td>
<td>Case_38</td>
<td>10</td>
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<td>5.56E-01</td>
<td>7.35E-01</td>
<td>3.47E+02</td>
</tr>
<tr>
<td>Case_19</td>
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<td>4.76</td>
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<td>4.47E+02</td>
<td>5.54E+02</td>
<td>8.47E+04</td>
<td>Case_39</td>
<td>10</td>
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<td>6.09E-01</td>
<td>2.86E+02</td>
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<td>Case_20</td>
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<td>4.76</td>
<td>12</td>
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<td>5.42E+04</td>
<td>Case_40</td>
<td>10</td>
<td>0</td>
<td>12</td>
<td>3.27E-01</td>
<td>5.37E-01</td>
<td>2.08E+02</td>
</tr>
</tbody>
</table>

$R^2 = 0.96$  \quad  $R^2 = 0.94$  \quad  $R^2 = 0.99$
Fig. 5-2 Relationships between the critical hydraulic gradient $J_c$ with (a) mechanical aperture $h_M$, (b) number of intersection $N_i$, and (c) joint roughness coefficient JRC.

A multi-variable regression algorithm was adopted to establish the mathematical relationships between these parameters. $h_M$, $N_i$, and JRC are three independent variables, and $A$, $B$, and $J_c$ are three dependent variables. The best-fitted expressions are as follows:
\[ A = \left( \lambda h_M \right)^{-0.5} \cdot \exp\left\{ 310.5(\lambda h_M)^{-0.008} - 0.001JRC - 0.7N_i^{0.25} - 303 \right\}, \]  
(5-5)

\[ B = \left( \lambda h_M \right)^{-2} \cdot \exp\left\{ 719(\lambda h_M)^{-0.002} - 0.01JRC - 5N_i^{0.15} - 700.4 \right\}, \]  
(5-6)

\[ J_c = \left( \lambda h_M \right)^{-0.5} \cdot \exp\left\{ 800.8(\lambda h_M)^{-0.01} - 0.03JRC - 0.3N_i^{0.5} - 303.3 \right\}, \]  
(5-7)

where \( \lambda \) is a coefficient that quantifies the reduction of \( h_M \) due to the variation of fracture apertures in a DFN, and its unit is mm\(^{-1}\). If fractures in a DFN have an identical \( h_M \), then the value of \( \lambda \) becomes 1 mm\(^{-1}\). The unit of \( h_M \) is millimeters in these equations. Three predominant factors \((h_M, N_i, \text{and JRC})\) that significantly impact on the nonlinear flow behavior of DFNs are incorporated into these empirical equations, with an exponential form. Predicted values by these equations are tabulated in Table 5-1, which show high correlation coefficients with the simulation results. It is worth noting that the proposed expressions were obtained from DFN models of simple geometries, and their validity needs to be confirmed by applying them to DFN models with more realistic geometric characteristics.

5.4 Quantification of nonlinearity for DFNs

5.4.1 Generation of DFNs

To verify the validity of Eqs. (5-5) - (5-7) and to characterize the nonlinear flow in DFNs, another series of DFNs with more realistic geometric characteristics of fractures was established, and flow simulations by directly solving the NS equations were conducted. First, two relatively large, square DFN models with a side length of 5 m were generated using the Monte Carlo method, and then smaller DFNs with side lengths of 0.5 m, 1 m, 1.5 m, and 2 m were extracted from the two large models (see Fig. 5-3). Fractures in one of the large models have smooth surfaces (smooth model), and fractures in the other model (rough model) have rough surfaces. The values of \( Z_2 \) of single fractures in the rough model range from 0.30 to 0.44, corresponding to the JRC values of 15.22 to 20.62. The two models have an identical geometric distribution of fractures, except for the surface roughness. The values of \( N_i \) are 4, 42, 83, and 129 in the DFNs with side lengths of 0.5 m, 1.0 m, 1.5 m, and 2.0 m, respectively.
Fig. 5-3 Extraction of DFNs from larger models that have smooth surfaces ((a) - (c), JRC = 0) and rough surfaces ((d) - (f), mean JRC = 17.79), respectively. ($L_S$: DFN side length)

The well-known geometric distributions of fractures were applied to establish the DFN models. The straight length (from the starting point to the ending point) of each fracture followed a power law function, defined as [Davy 1993; Bour and Davy 1997; Odling 1997; De Dreuzy et al. 2002],

$$n(l) = \alpha d^{-\alpha}, l \in [l_{\text{min}}, l_{\text{max}}], \quad (5-8)$$

where $n(l)$ is the number of fractures with lengths in the range of $[l, l + dl]$. $\alpha$ is a normalization factor, $\alpha$ is the power law exponent, and $l_{\text{min}}$ and $l_{\text{max}}$ are the minimum and the maximum fracture lengths, respectively.

The orientation of fractures was presumed to follow the Fisher distribution [Min and Jing 2003; Min et al. 2004; Baghbanan and Jing 2007; 2008], in which the deviated angle $\theta$ from the mean orientation angle was calculated by

$$\theta = \cos^{-1}\left[K^{-1}\ln\left(e^K - UE^{K-K}\right)\right], \quad (5-9)$$
where $K$ is the Fisher constant and $U$ is a random number uniformly distributed in the range of $[0, 1]$.

The mechanical aperture of each fracture was presumed to follow the square root correlation with the fracture length $l$ as [Vermilye and Scholz 1995; Olson 2003; Klimczak et al. 2010],

$$h_M = \alpha' l^{0.5},$$ \hspace{1cm} (5-10)

where $\alpha'$ is a proportionality coefficient. For the two large models, $\alpha' = 0.0025$, $a = 1.7$, $l_{\text{min}} = 1.2$ m, $K = 8$, the number density of fractures is $3.6/m^2$, and the original orientations of the fractures are $0^\circ$, $60^\circ$, and $120^\circ$, which are typical values utilized in generation of DFNs.

Fluid flowed through the DFNs from the upper boundary towards the bottom boundary, and the left and right boundaries were impermeable. The applied $J$ varied in a wide range from $10^{-8}$ to $10^0$, which covered most possible ranges of $J_c$ for flow regime transitions in DFNs.

5.4.2 Results and analysis

Simulation results of the relationship between $J$ and $Q$ are shown in Fig. 5-4(a). $J$ exhibits a quadratic relationship with $Q$, which can be well-fitted by Forchheimer’s law. The fitted values of $A$, $B$, and $J_c$ are tabulated in Table 5-2. Their values diminish with increasing number of intersections, following a power law, where the value of $B$ decreases faster than the value of $A$, showing that the nonlinear term is more sensitive to $N_i$ than the linear term, which is more closely correlated with the permeability and $h_M$ of a model. With increasing model size, $Q$ becomes greater at a given $J$, due to the increased number of connected flow paths in the model. As mentioned before, rough surface geometries would reduce the permeability of fractures. Therefore, at a given $J$ and model size, the rough model has lower $Q$ than that of the smooth model. Fig. 5-4(b) shows the relationship of $Q/J$ and $J$ with $J = 10^{-8}$–$10^0$. When $J < 10^{-6}$, the value of $Q/J$ holds constants, indicating that fluid flow is in a linear regime and the cubic law is applicable. For $10^{-6} < J < 10^{-4}$, $Q/J$ changes slightly, revealing a weak inertia regime. While $J > 10^{-4}$, $Q/J$ varies significantly and $Q$ is nonlinearly related with $J$, in which a strong inertia regime exists and the cubic law is not applicable.
Fig. 5-4 Simulation results of the relationships of (a) $J$~$Q$ and (b) $Q/J$~$J$ of the DFNs with well-known geometric characteristics.
Table 5-2 Values of $h_M$, JRC, and $N_i$ of generated DFNs, and simulated and predicted results of $A$, $B$, and $J_c$, using $\lambda = 0.8$. (S: simulation results)

<table>
<thead>
<tr>
<th>Cases</th>
<th>Mean $h_M$ (mm)</th>
<th>Mean JRC</th>
<th>Mean $N_i$</th>
<th>$A$ (S) (Eq. (5-5))</th>
<th>$B$ (S) (Eq. (5-6))</th>
<th>$J_c$ (S) (Eq. (5-7))</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-0.5m</td>
<td>4.19</td>
<td>0</td>
<td>4</td>
<td>2.75E+01</td>
<td>1.85E+01</td>
<td>7.58E+03</td>
</tr>
<tr>
<td>R-0.5m</td>
<td>3.99</td>
<td>17.79</td>
<td>4</td>
<td>2.94E+01</td>
<td>2.09E+01</td>
<td>1.33E+04</td>
</tr>
<tr>
<td>S-1.0m</td>
<td>4.16</td>
<td>0</td>
<td>42</td>
<td>1.16E+01</td>
<td>8.54E+00</td>
<td>5.01E+02</td>
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<tr>
<td>R-1.0m</td>
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<td>17.79</td>
<td>42</td>
<td>1.09E+01</td>
<td>1.02E+01</td>
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<td>S-1.5m</td>
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<td>83</td>
<td>6.52E+00</td>
<td>5.35E+00</td>
<td>2.70E+02</td>
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<tr>
<td>R-1.5m</td>
<td>4.07</td>
<td>17.79</td>
<td>83</td>
<td>7.69E+00</td>
<td>6.43E+00</td>
<td>9.18E+02</td>
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<tr>
<td>S-2.0m</td>
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<td>2.97E+00</td>
<td>3.75E+00</td>
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<tr>
<td>R-2.0m</td>
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<td>17.79</td>
<td>129</td>
<td>4.53E+00</td>
<td>4.27E+00</td>
<td>1.37E+02</td>
</tr>
</tbody>
</table>

$R^2 = 0.99$ $R^2 = 0.69$ $R^2 = 0.95$

The relationships between $E$ and $J$ calculated by Eq. (5-4) are shown in Fig. 5-5. $E$ increases with the increase of $J$, approaching the threshold of 1 when $J = 1$, where the nonlinear term contributes to most pressure drops. For each pair of curves obtained from models of an identical $N_i$, $J_c$ for the rough model is always smaller than that for the smooth model, due to the pronounced inertial effects induced by surface roughness and intersections. The existence of vortices inside of the fractures and the intersections was observed in the numerical simulations, two examples of which are shown in Fig. 5-6, at $J = 10^{-2}$. Increments of $J$ would give rise to the volume of such vortices, narrowing down the effective flow paths, and consequently enhancing the nonlinearity of fluid flow. The difference between $J_c$ of each pair of curves decreases with increasing $N_i$, suggesting that the surface roughness becomes less important in the models with a large number of intersections. The meshed model of Fig. 5-3(d) contains 492892 nodes and 866120 zones, and it needs a simulation time of about 10 h to reach a steady state flow on a personal computer with 8 CPUs.
Fig. 5-5 Simulation results of the relationship between calculated $E$ and $J$ for the DFNs with well-known geometric characteristics.

Fig. 5-6 Numerically obtained meshes and streamlines of fluid flow through a rough-walled fracture (a–b) and an intersection (c–d), corresponding to points 1 and 2 in Fig. 5-3 (e) respectively, at $J = 10^{-2}$. 
Fig. 5-7 Comparisons of $A$, $B$, and $J_c$ between simulation results and predictions of Eqs. (7) - (9). The best fitted results were found when $\lambda = 0.8$ as tabulated in Table 5-2.

The predicted values of $A$, $B$, and $J_c$ by Eqs. (5-5), (5-6), and (5-7) are shown in Fig. 5-7 and are tabulated in Table 5-2. The mean values of $h_M$ and JRC of all fractures in each DFN were used in the calculations. The best-fitted values for $A$, $B$, and $J_c$ were found when $\lambda = 0.8$ mm$^{-1}$. Unlike the models shown in Fig. 5-1, $h_M$ in these DFNs (Fig. 5-3) is correlated with fracture length. Therefore, considerably smaller apertures could occur in a few shorter fractures connecting longer fractures with larger apertures, which may become “necks” for fluid flow, thereby reducing the permeability of a DFN [De Dreuzy et al. 2001a; 2001b]. It has been well known that $h_H$ (the hydraulic aperture) is
typically smaller than $h_M$ in rough-walled single fractures, due to the frictional loss induced by geometric variations of void structures, the effect of which on nonlinearity of flow is represented by JRC in Eqs. (5-5) - (5-7). For DFNs consisting of fractures with variable $h_M$, the representative $h_M$ (i.e., mean $h_M$) will be further reduced due to the macro-scale geometric variations among fractures with different apertures. As shown in Fig. 5-8, when $J$ is small (i.e., $J = 10^{-6}$), the flow patterns would hold the same, whereas, for larger $J$ (i.e., $J = 10^{-2}$), the variations of aperture would cause significant eddies, resulting in frictional losses and then reducing the hydraulic aperture. Such reduction is represented by $\lambda$ in Eqs. (5-5) - (5-7) with a best-fitted value of 0.8 for the DFNs shown in Fig. 5-3. Note that the value of $\lambda$ may be subjected to change when a different algorithm for generating aperture distributions in a DFN is employed.

Fig. 5-8 Flow velocity distributions through fractures with variable apertures.
Many previous studies used the cubic law as the governing equation to solve fluid flow in each single fracture in DFNs with hundreds or thousands of fractures and intersections but applied a larger $J$ than $J_c$ calculated by Eq. (5-7). To have confidence in using the cubic law in DFNs, the assigned $J$ needs to be first checked by comparing to the prediction of Eq. (5-7) before conducting further flow and mass transport simulations. If $J$ is larger than $J_c$, then the relationship between $Q$ and $J$ needs to be assessed by employing Forchheimer’s law, in which the two coefficients, $A$ and $B$, can be quantitatively estimated by Eqs. (5-5) - (5-6).

5.5 Effect of fracture dead-end on nonlinear flow and particle transport

5.5.1 DFN generation and basic assumptions

This Section is focused on the influences of fracture dead-end on fluid flow and particle transport in DFNs, and these DFN models were simply generated with randomly distributed fracture length, orientation, and aperture. The scale effects were not considered too. The size of each DFN was $1 \text{ m} \times 1 \text{ m}$, and the aperture was in the range of $2 - 10 \text{ mm}$. The boundary conditions were taken to be uniform pressures on the two opposing faces (left side and right side), with the other two faces (upper side and bottom side) impermeable, for both the models with the dead-ends and without the dead-ends as shown in Fig. 5-9. The hydraulic gradient, $J$, varied from $10^{-7}$ to $10^3$. 

![DFN model with dead-ends](image)

(a) DFN model with dead-ends

![DFN model without dead-ends](image)

(b) DFN model without dead-ends

Fig. 5-9 Two DFN models with dead-ends and without dead-ends.
experiencing five orders of magnitude. When the fluid flow was in a steady-state, the flow rate was calculated. The particles were injected at the left side boundary with a 1.0 mm interval, resulting in a total of 19 particles for each DFN model. When each particle arrived at the right side boundary, the corresponding travel time was recorded. Every fracture was assumed to be a smooth parallel plate model, and the fluid was assumed to flow through these fractures with rock matrix to be impermeable. The experimental model was sealed with the impervious strong glue, so when conducting fluid flow tests, the model was assumed to be fixed and the aperture of the fracture was assumed to be constant values with no relationship with the associated confining pressure.

5.5.2 Numerical simulation method

A finite volume method (FVM) code ANSYS FLUENT module that solves the NS equations [Fluent, 2006], was employed to simulate the fluid flow and solute transport in the experimental models considering water as a viscous incompressible Newtonian fluid. The results were compared with those solved by the cubic law. To differentiate the solute and distilled water in a flow, the volume of fluid (VOF) model was utilized to simulate the two different fluids in a system, by assuming that the solute diffusion was negligible. The trajectory of discrete phase particles was also simulated by integrating the force balance on the particles, which is written in a Lagrangian reference frame. The force balance equations in the local $x$-direction in Cartesian coordinates can be written as [Fluent, 2006]

$$\frac{du_p}{dt} = F_D(u - u_p) + \frac{gs}{\rho_p} \left( \rho_p - \rho \right) + F_x$$  \hspace{1cm} (5-11)

where $F_x$ is an additional acceleration term, $g_x$ is the gravitational acceleration in the local $x$-direction, $F_D(u - u_p)$ is the drag force per unit particle mass. The acceleration of the drag force, $F_D$, can be written by

$$F_D = \frac{18\mu}{\rho_p d_p^2} \frac{C_D Re}{24}$$  \hspace{1cm} (5-12)

where $u$ is the fluid phase velocity, $u_p$ the particle velocity, $\rho_p$ the density of the particle, and $d_p$ the particle diameter.

The integral time, $T'$, is used to describe the time spent along the particle path, $ds$, as
\[ T' = \int_0^\infty u_p(t') \frac{u(t')}{u_p} \, ds \]  \hspace{1cm} (5-13)

where \( t' \) is the reference time, \( s \) the time of particle transport, \( u_p' \) the variational particle velocity based on the mean particle velocity \( u_p \).

5.5.3 Fluid flow and particle transport in DFNs

Fig. 5-10 Variations of flow rate with varying hydraulic gradients.

Fig. 5-10 shows that \( Q \) and \( J \) have a nonlinear relationship for the two DFN models, especially when \( J \) is large (i.e., larger than \( 4.0 \times 10^{-4} \)), if the fluid flow is modelled by solving the NS equations. The quadratic function of \( Q \) and \( J \) of fluid flow in DFN models follows the same form of Eq. (5-3). However, when the fluid flow in each fracture is calculated by the cubic law, \( Q \) is linearly proportional to \( J \). The source of their differences only arises from fracture intersections, since the smooth parallel plate fracture model excludes the influences of fracture surface roughness. The relative flow rate deviation rate, \( \delta_1 \), was defined to analyze the influences of fracture intersection, as follows

\[ \delta_1 = \frac{Q_{\text{cubic}} - Q_{\text{NS}}}{Q_{\text{cubic}}} \times 100\% \]  \hspace{1cm} (5-14)

where \( Q_{\text{cubic}} \) is the flow rate calculated by solving the local cubic law in each fracture in a DFN model, \( Q_{\text{NS}} \) is the flow rate calculated by solving the NS equations.
The results are shown in Fig. 5-12, in which we can see that, $\delta_1$ is less than 2% when $J \leq 10^{-5}$, and less than 11% when $J \leq 10^{-4}$. Therefore, the critical condition of applying the local cubic law in each fracture in DFNs is: $J \leq 10^{-5}$. When $J$ is large (i.e., larger than $10^{-4}$), the influences of fracture intersection on fluid flow are more than 10%, indicating that the local cubic law is not applicable. Fig. 5-12 presents the results of $Q/J$ and the relative dead-end deviation rate, $\delta_2$, which is defined as

$$\delta_2 = \left( \frac{Q_{\text{no, dead-ends}} - Q_{\text{with, dead-ends}}}{Q_{\text{no, dead-ends}}} \right) \times 100\% \quad (5-15)$$

where $Q_{\text{no, dead-ends}}$ is the flow rate calculated for the DFN model without dead-ends, $Q_{\text{with, dead-ends}}$ is the flow rate calculated for the DFN model with dead-ends.

Even when $J$ is large (i.e., $J = 10^{-3}$), $\delta_2$ is less than 1.5%, indicating that the influences of the fracture dead-end on fluid flow can be negligible. Fig. 5-12 also shows that $J = 10^{-5}$ is the turning point to judge whether $Q/J$ is a constant or variable values against $J$. If the DFN model is considered as a macroscopic “single” fracture, a constant value of $Q/J$ means the local cubic law is applicable (see Eq. (5-2)), while a variable value of $Q/J$ means a nonlinear flow regime occurs in which the inertial effects caused by fracture intersections should not be neglected (see Eq. (5-3)).
Fig. 5-13 shows the results of breakthrough curves to estimate the influences of the fracture dead-end on the particle transport. The hydraulic gradient was controlled varying from $10^{-8}$ to $10^{-6}$. Most of the particles in the DFN model with dead-ends take a longer time to path through the model, compared with those without dead-ends. For further study on the influences of fracture dead-end, the relative time deviation rate, $\delta_3$, is defined as

\[
\delta_3 = \left( \frac{T_{\text{with dead-ends}} - T_{\text{no dead-ends}}}{T_{\text{with dead-ends}}} \right) \times 100\%
\]  

(5-16)

where $T_{\text{with dead-ends}}$ is the travel time of the particle through the DFN model with dead-ends, $T_{\text{no dead-ends}}$ is the travel time of the particle through the DFN model without dead-ends.
Fig. 5-13 Breakthrough curves of particle through the DFN models.

Fig. 5-14 presents the variations of $\delta_i$ against each particle injected at the left side boundary. The results show that most of the relative time deviation rates are in the range of $5 – 35\%$, exhibiting that the influences of fracture dead-end on particle transport cannot be negligible. However, for the last $3 – 5$ particles, the relative time deviation rates are very large, due to the stochastic properties of particle transport.
5.6 Conclusions

First, a parametric study was implemented in a series of DFNs with various combinations of mechanical aperture, surface roughness, and number of intersections to investigate the effects of these factors on the transition of flow regimes in DFNs. Three empirical expressions for predicting the coefficients $A$ and $B$ involved in Forchheimer’s law, and the critical hydraulic gradient $J_c$ were established based on multi-variable regressions of simulation results. Then, another series of DFNs were generated, based on well-known geometric distributions of fractures, and their nonlinear flow behavior was analyzed. Validity of the proposed equations was verified by comparing the predicted results with the simulation results of these DFNs.

Fluid flow in DFNs can be quantified by Forchheimer’s law, in which $AQ$ and $BQ^2$ represent the linear and nonlinear components, respectively. At sufficiently low hydraulic gradients or Reynolds numbers, the nonlinear term $BQ^2$ drops out and Forchheimer’s law reduces to the cubic law. Transition from the linear regime to the nonlinear regime would be found at a lower hydraulic gradient in a DFN with rougher fracture surfaces, greater mechanical apertures, and a larger number of intersections. The critical hydraulic gradient is most sensitive to the mechanical aperture, followed by the number of intersections and surface roughness, because the Reynolds number in each fracture in a DFN changes proportionally with the cubic change of the mechanical aperture, and the intersection and surface roughness that are the sources of the frictional loss and inertial efforts will only have significant influences at large Reynolds numbers.
Predicted values by the empirical equations fit well with simulation results of DFNs with correlated fracture length - aperture distributions, by introducing a parameter $\lambda$ to account for the reduction of aperture induced by aperture variations.

When the imposed hydraulic gradient on a DFN is below the predicted critical hydraulic gradient, application of the cubic law in conjunction with necessary modifications considering aperture reductions induced by surface roughness would give reasonable solutions. The use of Navier-Stokes equations is appreciated when the hydraulic gradient is larger than the critical value, which, however, is not always applicable due to the limitation of computational capacities when a large number of fractures are involved in a model. In such conditions, one may apply Forchheimer’s law to the studied models using predicted values of $A$ and $B$ based on the proposed equations, which will profoundly improve the reliability of obtained pressure - flow rate relationships, compared with those made by linear predictions.

The effects of fracture dead-end on fluid flow can also be negligible with a relative dead-end deviation rate less than 1.5% even when $J = 10^{-3}$. However, the effects of fracture dead-end on particle transport, corresponding to $J$ varying from $10^{-8}$ to $10^{-6}$, should be considered, because most of the relative time rates are located in the range of $5 – 35\%$.

References


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6 A fractal model for predicting EFNP (Part 1)

A fractal model for characterizing fluid flow in fractured rock masses based on randomly distributed rock fracture networks

6.1 Introduction

Permeability is a crucial hydro-mechanical property of rock masses and is important in many areas of geosciences and geoengineering, including dam foundations and petroleum reservoirs. The permeability of a rock mass is mainly governed by rock fractures that separate intact rock blocks with negligible matrix permeability (e.g., granite and basalt) [Barenblatt et al., 1960; Barenblatt and Zheltov, 1960]. A tremendous amount of effort has been exerted to understand the behavior of fluid flow in rock masses in recent decades [National Research Council, 1996; Baghbanan and Jing, 2007; 2008; Koyama et al., 2008; Zhao et al., 2011; Liu et al., 2014]. However, accurately estimating the permeability of rock masses is still challenging because of the complexities of fracture distributions at the macro-structural (i.e., geometry of the fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [Oliveira and Graca, 1987; Hudson and Harrison, 1997; Yu and Cheng, 2002; Zhang, 2005]. One key difficulty is that rock fractures typically have rough surfaces between which fluid flows non-uniformly. A particle will travel a longer distance along a tortuous path through a rough-walled fracture than through a parallel-walled fracture. Another difficulty is mathematically describing the geometric distributions of rock fractures in fracture networks, which usually contain several sets of fractures with different orientations, lengths and apertures. Fortunately, the distribution of fractures in fracture networks have been found to exhibit fractal characteristics [Barton and Larsen, 1985; Barton and Hsieh, 1989; La Pointe, 1988; Kulasilake et al., 1995; 1999], which provides a possible approach for describing the geometric characteristics of fracture networks while considering both the macro-scale and micro-scale properties of the fractures.

A few predictive fractal models have been developed to calculate the permeability of stochastic rock fracture networks. The purpose of their models and the outcome of their studies are summarized in Table 6-1.
Based on the fractal models proposed for the porous media [Yu and Cheng, 2002] and regular tree networks [Xu et al., 2006; 2008] to calculate the permeability of rock masses, the present study focused on extending this fractal model to the fractured media consisted of randomly generated stochastic discrete fracture networks (DFNs) using the Monte Carlo method. A probability density function was derived to depict the trace length $l$ of each rock fracture between a minimum and maximum trace length, and the apertures of the fractures were correlated with their trace lengths. Flow simulations of models with various fractal dimensions were conducted, and the relations between the fractal dimensions and the equivalent permeability were estimated.

Table 6-1 Review of fractal DFN models used to calculate the permeability of rock masses.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Purpose of the model</th>
<th>Outcome of the study</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>De Dreuz et al.</em></td>
<td>2001a; 2001b; 2002</td>
<td>Investigation of the hydraulic properties of 2-D fracture networks with random fracture geometries that follow a power law length distribution.</td>
<td>They analyzed the influence of the power law exponent $a$ in their models and found that if $a$ was greater than 3, the classical percolation model based on a population of small fractures was applicable, and the fluid flow appeared to be relatively homogeneous in the flow direction. In contrast, if $a$ was less than 2, the applicable model was made up of the largest fractures of the network, and the main flow paths were composed of a few large fractures. Between the two limits ($2 &lt; a &lt; 3$), relatively uniform fluid flow occurred in all of the fractures.</td>
</tr>
<tr>
<td><em>Yu et al.</em></td>
<td>2002</td>
<td>Development of a fractal model to calculate the equivalent permeability of bi-dispersed porous media.</td>
<td>They extensively evaluated the influences of the fractal dimension $D_f$ (which represents the fracture distribution) and $D_T$ (which represents tortuosity) on the equivalent permeability. They found that the equivalent permeability was a function of the tortuous fractal dimension, pore area fractal dimension, sizes of particles and clusters, micro-porosity inside clusters, and the effective porosity of a medium.</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Year</td>
<td>Description</td>
<td>Significance</td>
</tr>
<tr>
<td>--------------</td>
<td>------</td>
<td>------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Yu et al.</td>
<td>2005</td>
<td>Establishment of a 2-D fractal model to calculate the permeability of a porous media model generated by the Monte Carlo method.</td>
<td>Their model can predict the transport properties (i.e., permeability, thermal conductivity, dispersion coefficient and electrical conductivity) of saturated or unsaturated fractal porous media.</td>
</tr>
<tr>
<td>Xu et al.</td>
<td>2006</td>
<td>Development of a fractal model for fluid flow in porous media that are embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [2011] and Bejan and Zane [2013].</td>
<td>They found that the permeability of the model that incorporated the flow tortuosity was approximately 20% lower than the model that did not consider the tortuosity.</td>
</tr>
<tr>
<td>Zou et al.</td>
<td>2007</td>
<td>Establishment of a fractal model to analyze the 3-D surfaces of rock fractures.</td>
<td>Their results indicated that larger values of the fractal dimension $D$ of the profile of a rough surface or smaller values of the scaling constant $G$ signify a smoother surface topography.</td>
</tr>
<tr>
<td>Jafari and Babadagli</td>
<td>2012</td>
<td>Estimation of the equivalent permeability of fracture networks using numerical simulations</td>
<td>They derived a nonlinear multivariable regression to address the equivalent permeability by calculating three parameters: $X_1$ (the connectivity index), $X_2$ (the box-counting fractal dimension of the fracture lines) and $X_3$ (the hydraulic conductivity).</td>
</tr>
<tr>
<td>Zheng et al.</td>
<td>2012</td>
<td>Development of a fractal model for fluid flow in porous media that are embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [2011] and Bejan and Zane [2013].</td>
<td>They derived an analytical expression for the gas permeability in dual-porosity media based on the pore size of the matrix and the diameter of the mother channel of embedded fractal-like tree networks. They found that for a certain fracture network, the dimensionless permeability $K^*$, which is defined as the ratio of the porous matrix permeability $K_m$ to the fracture network permeability $K_f$, increases with increasing matrix porosity $e$.</td>
</tr>
</tbody>
</table>
6.2 Fractal characteristics of rock fractures

Mandelbrot [1982] verified that the cumulative size distribution of islands on the surface of the earth followed the power law

\[ N'(A' > a') \sim a'^{-(D/2)} \]  \hspace{1cm} (6-1)

where \( N' \) is the total number of islands with an area \( A' \) greater than a constant \( a' \) and \( D \) is the fractal dimension that represents the size distribution of the islands.

Based on this theory, Majumdar and Bhushan [1990] developed an equivalent equation to describe the distribution of islands by regarding \( a'_\text{max} \) as the largest island:

\[ N'(A' \geq a') = \left( \frac{a'_\text{max}}{a'} \right)^{D/2} \]  \hspace{1cm} (6-2)

Eq. (6-2) shows that there is only one largest island on the earth, which is true in the physical world. Xu et al. [2006; 2008] used Eq. (6-2) to describe the geometric distribution of pores in porous media that are embedded with randomly distributed 2-D fractal-like tree networks, where \( a'_\text{max} = g\lambda^2, \quad a' = g\lambda^2, \quad \lambda \) is the diameter of a pore, and \( g \) is a geometric factor. The distribution of fractures in 2-D rock masses is considered to be analogous to that of islands on the surface of the earth and that of pores in porous media, which yields

\[ N(L \geq l) = \left( \frac{l_{\text{max}}}{l} \right)^{D_{f}/2} \]  \hspace{1cm} (6-3)

where \( N \) is the total number of fractures with a length \( L \) greater than a constant fracture length \( l \); \( D_{f} \) is in the range of \([1, 2]\) for 2-D fracture networks; and \( l_{\text{max}} \) is the maximum trace length of fractures in a rock mass.

Differentiating Eq. (6-3) with respect to \( l \) leads to

\[ -dN = \frac{D_{f}}{2} l_{\text{max}}^{D_{f}/2} l^{-1(D_{f}/2+1)} dl \]  \hspace{1cm} (6-4)

The negative sign on the left side of Eq. (6-4) implies that the number of fractures decreases with increasing trace length. This equation gives the correlation of fracture number and trace length. The total number of fractures \( N_i \) can be calculated by setting \( l = l_{\text{min}} \), which yields
\[ N_t(L \geq l_{\text{min}}) = \left( \frac{l_{\text{max}}}{l_{\text{min}}} \right)^{D_f/2} \]  

(6-5)

where \( l_{\text{min}} \) is the minimum trace length of the fractures in a rock mass. Dividing Eq. (6-4) by Eq. (6-5) gives

\[ \frac{dN}{N_t} = \frac{D_f}{2} l_{\text{min}}^{D_f/2} l^{-\{D_f/2+1\}} dl = f(l) dl \]  

(6-6)

where \( f(l) = \frac{D_f}{2} l_{\text{min}}^{D_f/2} l^{-\{D_f/2+1\}} \) is the probability density function; according to probability theory, this function satisfies the following equation:

\[ \int_{l_{\text{min}}}^{l_{\text{max}}} f(l) dl = \int_{l_{\text{min}}}^{l_{\text{max}}} f(l) dl = 1 \cdot \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right)^{D_f/2} \equiv 1 \]  

(6-7)

By eliminating the constant value 1, Eq. (6-7) becomes

\[ \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right)^{D_f/2} \equiv 0 \]  

(6-8)

Eq. (6-8) implies that \( l_{\text{min}} \ll l_{\text{max}} \) should be satisfied for Eq. (6-7) to hold. Eq. (6-8) is therefore a necessary condition for a fracture distribution to exhibit fractal characteristics. In this study, \( l_{\text{min}}/l_{\text{max}} \leq 10^{-3} \) is used as a threshold to enable fluid flow in 2-D fracture networks to be effectively represented using a fractal model. For all of the fractures with trace lengths in the range of \([l_{\text{min}}, l_{\text{max}}]\), the cumulative probability \( R \) can be integrated as

\[ R(l) = \int_{l_{\text{min}}}^{l} f(l) dl = \int_{l_{\text{min}}}^{l} \frac{D_f}{2} l_{\text{min}}^{D_f/2} l^{-\{D_f/2+1\}} dl = 1 - \left( \frac{l_{\text{min}}}{l} \right)^{D_f/2} \]  

(6-9)

Eq. (6-9) implies that when \( l \to l_{\text{min}}, R = 0 \), and when \( l \to l_{\text{max}}, R = 1 \). As long as \( l \) is in the range of \([l_{\text{min}}, l_{\text{max}}]\), the value of \( R \) is in the range of \([0, 1]\). Therefore, by assigning random numbers between 0 and 1 to \( R \), the correlated trace length \( l \) can be back-calculated by

\[ l = \frac{l_{\text{min}}}{(1-R)^{2/D_f}} = \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right) \frac{l_{\text{max}}}{(1-R)^{2/D_f}} \]  

(6-10)
To facilitate the calculation, each fracture is labeled with an integer from 0 to \( N_t \). For the \( i \)th fracture, the trace length \( l_i \) can be calculated using a random number \( R_i \) as follows:

\[
l_i = \frac{l_{\text{min}}}{(1 - R_i)^{2/D_f}} = \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right) \frac{l_{\text{max}}}{(1 - R_i)^{2/D_f}}
\]

(6-11)

where \( i = 1, 2, 3, \ldots, N_t \) and \( N_t \) is the total number of fractures in the network.

Eq. (6-11) represents the fractal length distributions in 2-D rock fracture networks and is significantly correlated with the minimum trace length \( l_{\text{min}} \), the random number \( R \), the fractal dimension \( D_f \) and the total number of fractures \( N_t \). The validity of Eq. (6-11) will be verified in Sections 6.4.2 and 6.4.3 by comparing the correlation of the fracture number and fracture length with the results of other studies and by comparing the fractal dimension \( D_f \) of DFN models generated using Eq. (6-11) with their theoretical values.

Previous studies of fracture sizes from centimeters to meters at different sites have found that the aperture and the trace length of fractures have positive linear correlations [Stone, 1984; Vermilye and Scholz, 1995] or power law correlations [Hatton et al., 1994; Renshaw and Park, 1997]. Longer fractures usually have higher permeabilities and larger apertures than shorter fractures. In this study, assuming that the DFN models are composed of a large number of small fractures with small hydraulic apertures and a much smaller number of longer fractures with larger apertures, the trace lengths of the fractures are randomly generated according to Eq. (6-11) and are correlated with the fracture aperture as follows [Baghbanan and Jing, 2007; 2008]:

\[
\frac{l_i^{D_f} - l_{\text{min}}^{D_f}}{l_{\text{max}}^{D_f} - l_{\text{min}}^{D_f}} = \frac{g(e_i) - g(e_{\text{min}})}{g(e_{\text{max}}) - g(e_{\text{min}})}
\]

(6-12)

\[
g(e_i) = \text{erf} \left( \frac{\ln e_i - \bar{e}_{\log}}{\sqrt{2}b} \right)
\]

(6-13)

where \( \text{erf} \) is an error function; \( \bar{e}_{\log} \) and \( b \) are the first and second moments of the log-normal distribution of the apertures, respectively; \( e_{\text{min}} \) and \( e_{\text{max}} \) indicate the minimum and maximum apertures, respectively; \( e_i \) is the hydraulic aperture of the \( i \)th fracture; and \( l_i \) can be calculated from Eq. (6-11).

Finally, the aperture of the \( i \)th fracture can be obtained by forcing \( \bar{e}_{\log} = 0 \) and \( b = 1 \) as follows [Baghbanan and Jing, 2007; 2008]:
where $erfinv$ is the inverse error function.

Rough surfaces of fractures render the streamlines of fluid flow nonlinear, which can increase the end-to-end distances required for fluid flow through fractures and therefore reduce their equivalent permeability. Here, as shown in Fig. 6-1, $l$ is the straight length of the fracture pathways in the flow direction, $l_t$ is the tortuous length along the fracture profile, and $e$ is the hydraulic aperture of a single fracture [Olsson and Barton, 2001]. Generally, $l_t > l$, except for fractures with flat surfaces (parallel plate model), where $l_t = l$. The tortuosity is defined as a parameter to depict the ratio of $l_t$ to $l$, in order to describe their difference induced by fracture surface roughness in 2-D rock fractures [Cai et al., 2010]. The correlation of the tortuous length $l_t$ and the straight length $l$ of fractures is also considered to be analogous to that of pores in porous media [Yu and Cheng, 2002; Yu et al., 2002], as expressed by

$$l_t = e^{1-D_T} l^{D_T}$$

(6-15)

where $D_T$ is the fractal dimension of the non-linear streamline of fluid flow, with a value in the range of [1, 2] for a single fracture in a 2-D rock fracture network. When $D_T = 1$, the streamline is linear, resulting in $l_t = l$, corresponding to a 2-D fracture. As a consequence, the value of $D_T$ can depict the non-linearity of the streamline, as well as the effect of tortuosity of fluid flow.

Fig. 6-1 Schematic view of fluid flow through a rough fracture. Tortuosity induced by fracture surface roughness makes the actual traveling length $l_t$ of fluid flow larger than the straight length $l$. 

$$e_t = \exp \left\{ \sqrt{2erfinv} \left[ \frac{l_t^{D_T} - l_{min}^{D_T}}{l_{max}^{D_T} - l_{min}^{D_T}} \right] \left[ \text{erf} \left( \frac{\ln e_{max}}{\sqrt{2}} \right) - \text{erf} \left( \frac{\ln e_{min}}{\sqrt{2}} \right) \right] + \text{erf} \left( \frac{\ln e_{min}}{\sqrt{2}} \right) \right\} \right\} \tag{6-14}$$
6.3 A fractal model for permeability estimation

6.3.1 Fluid flow in a single fracture

Although real rock fractures have rough walls and variable apertures, fluid flow through rock fractures is usually described by the cubic law [Koyama et al., 2008], which assumes that fractures consist of two smooth parallel walls. Under these conditions and substituting Eq. (6-11) and Eq. (6-15) into the cubic law, the flow rate of the $i$th fracture can be written as

$$q(i) = \frac{e_i^3}{12\mu} \frac{\Delta P_i}{l_i(i)} = e_i^2 \frac{\Delta P_i}{l_i^{D_f}} = e_i^2 \frac{\Delta P_i}{l_{min}^{D_f}}(1 - R_i)^{D_f/D_f}$$  \hspace{1cm} (6-16)

where $q(i)$ is the fluid volume through the $i$th fracture, $\Delta P_i$ is the local hydraulic pressure difference applied between the tips of the fracture, and $\mu$ is the viscosity of the fluid.

If a model consists of a single fracture that satisfies the parallel plate model, then $D_T = 1.0$ and $D_f = 2.0$. Eq. (6-16) reduces to

$$q_{pp}(i) = \frac{e_i^3}{12\mu} \frac{\Delta P_i}{l_{min}}(1 - R_i) = e_i^2 \frac{\Delta P_i}{l_i}$$  \hspace{1cm} (6-17)

where $q_{pp}(i)$ is the fluid volume through the $i$th fracture based on the parallel plate model. Eq. (6-17) is the standard form of the cubic law, which is the most simplified case of Eq. (6-16). The fractal length distribution presented in Eq. (6-11) is similar to the power law length distribution in 2-D random fracture networks [De Dreuzy et al., 2001a; 2001b].

6.3.2 Fluid flow in a 2-D fracture network

Algorithms for DFN generation and fluid flow through fracture networks were extensively described in the manual of UDEC [Itasca Consulting Group Inc., 2004], and only a few principal features are presented here. Three aspects should be addressed to generate a DFN. The first involves the regularization of DFN models. Because fluid flows within connected fracture networks, the fracture elements outside the model, the isolated fracture elements, and the “dead-end” fracture elements do not contribute to the fluid flow and should therefore be deleted. Second, parameters (i.e., trace length,
hydraulic aperture, and orientation) should be assigned to each fracture, and the mass continuity equations at each fracture intersection should be established. The third aspect addresses the iteration scheme of these equations for given boundary conditions. Steady-state fluid flow was adopted in this study for calculating the equivalent permeability of DFNs, and a generic hydraulic boundary condition with a constant hydraulic gradient in the $x$-direction was assumed, indicating that the directivity of the equivalent permeability is horizontal (see Fig. 6-2).

Fig. 6-2 (a) Hydraulic boundary conditions applied to a fracture network. The quadrilaterals represent hydraulic pressures, and the hydraulic gradient was fixed at 1 kPa/m to generate horizontal fluid flow. (b) Fluid flow equilibrium of a node in a fracture network.
In a 2-D fracture network, fractures are line segments, and fracture intersection points are denoted as nodes. The summation of the flow rate of each fracture connected to a node is zero or equals the added source term. Similarly, the summation of the total flow rate of the entire fracture network equals zero or the summation of the source terms added at each node. Fig. 6-2(b) shows a region composed of a node \( m \) and several fractures from the DFN model shown in Fig. 6-2(a). Taking into account the balance of fluid flow, the equilibrium equation of node \( m \) can be written as

\[
\left( \sum_{n=1}^{M} q_n \right) + Q_m = 0, \quad (m = 1,2, \ldots, N_{\text{node}})
\]  

(6-18)

where \( q_n \) is the fluid volume through the fracture element \( n \), \( M \) is the total number of fracture elements connected to node \( m \), \( Q_m \) is a source term at node \( m \), and \( N_{\text{node}} \) is the total number of nodes in the entire fracture network. Generally, under steady-state flow without any source terms, \( Q_m = 0 \) and \( M \leq 4 \) (two fractures intersect at one node) given that the probability that more than two fractures intersect at the same node is nearly zero. Steady-state flow was assumed; therefore, the fluid volume that flows into a fracture network is equal to that flowing out of the network. The fluid rate at the outlet boundary was then utilized to estimate the equivalent permeability of the DFN model.

Using Darcy’s law as shown in Eq. (6-19), the equivalent permeability \( K \) is obtained in Eq. (6-20):

\[
Q = A \frac{K}{\mu} L' \Delta P' 
\]

(6-19)

\[
K = \frac{\mu L'}{A \Delta P'} Q(e, D_f, l_{\min}, D_f, R, \Delta P')
\]

(6-20)

where \( A \) is the cross-sectional area perpendicular to the fluid flow direction, \( L' \) is the length of the fracture network in the fluid flow direction, \( Q \) is the total fluid volume through the fracture network per second, \( \Delta P' \) is the hydraulic pressure difference of the DFN model between the inlet boundary and the outlet boundary, and \( K \) is the equivalent permeability of the fracture network.

Eqs. (6-16) and (6-20) are the fundamental equations of the proposed fractal model for calculating the equivalent permeability of fracture networks and are a function of \( e, D_f, l_{\min}, D_f, R \) and \( \Delta P' \). In this model, \( D_f \) can be calculated by applying the box-counting method to fracture networks [Jiang et al., 2006] and \( D_f \) is a given input
parameter based on in situ geological survey data. No empirical constants are involved in this fractal model, and all of the parameters have clear physical meanings.

6.3.3 Basic parameters

Baghbanan and Jing [2007] studied the hydraulic properties of DFNs based on field mapping results of a real rock mass. For simplification, the values of most of the parameters from their study were used. The maximum trace length $l_{\text{max}}$ and minimum trace length $l_{\text{min}}$ were 500 m and 0.5 m, respectively, with a ratio $l_{\text{min}}/l_{\text{max}}$ of 0.001, which satisfied the condition of Eq. (6-8). The range of $D_f$ was 1.3 – 1.8, below which the connectivity of the fracture networks becomes very poor and above which the rock masses are so fragmented that they rarely exist in the nature. Wakabayashi et al. [1995], based on the 10 classical single fractures suggested by Barton [1977], studied the relation of $D_T$ and JRC (joint roughness coefficient) and derived a regression equation, where the JRC is a commonly used parameter for describing the surface roughness of rock fractures [Jang et al., 2006]. Their results show that the range of $D_T$ was 1.000 – 1.018, which roughly corresponds to JRC = 0 – 20. The maximum and minimum apertures were 100 μm and 1 μm, respectively. These parameters were utilized to generate the fracture networks with the Monte Carlo method using a fractal probability density function for the trace length distribution according to Eq. (6-11). The fracture orientation and fracture center point distribution were assumed to be uniformly distributed because this study is only focused on the validity of the proposed fractal model and its influence on the equivalent permeability of 2-D fracture networks.

6.4 Results and analyses

6.4.1 Determination of the model size

The size of a DFN model should be determined prior to performing the fluid flow simulation. In Eqs. (6-3), (6-16) and (6-20), the model size was unknown. It is well known that $D_f$ is not correlated with the model size, which is a fundamental property of fractal models. For fracture networks, the fractal dimension is positively correlated with the mass density (total length per square meter) of the fractures. To obtain the mathematical relation between $D_f$ and the mass density $d_m$, a series of DFN models with different values of $d_m$ were generated, and their fractal dimensions $D_f$ were calculated.
using the box-counting method. The results are shown in Fig. 6-3, in which a regression function could be obtained as follows:

\[ D_f = a'' + b'' \ln(d_m + c'') \]  
(6-21)

where \( a'' \), \( b'' \), and \( c'' \) are three regression parameters equal to 0.4594, 0.2797, and 8.4968, respectively.

![Fig. 6-3 The relation between the fractal dimension \( D_f \) and the mass density of fractures.](image)

According to the definition of the mass density of fractures, \( d_m \) can be expressed as follows:

\[ d_m = \frac{\sum_{i=1}^{N_i} l_i}{\frac{1}{L_t}} = \frac{\sum_{i=1}^{N_i} l_i \cdot (L_i)^2}{N} \]  
(6-22)

where \( L_t \) is the theoretical model length.

Substituting Eq. (6-22) into Eq. (6-21), \( L_t \) can be calculated as follows:

\[ L_t = \left( \frac{\sum_{i=1}^{N_i} l_i \left( \frac{D_f - a''}{b''} - c'' \right) ^{0.5}}{N} \right) \]  
(6-23)

Using the calculated value of \( L_t \) in Eq. (6-23), the number density of fractures \( d_n \), which defines the total number of fractures in a given area of the fracture network, is obtained as follows:

\[ d_n = \frac{N_i}{L_t^2} \]  
(6-24)
Consequently, for a constant fractal dimension $D_f$, the mass density $d_m$ and the number density $d_n$ are also constants. For an arbitrary model length $L_n > L_t$, the new fracture number $N_i'$ can be calculated based on Eqs. (6-23) and (6-24) as follows:

$$N_i' = \frac{N_i L_n^2}{L_t^2} = N_i L_n^2 \left[ \exp \left( \frac{D_f - d'}{b'} \right) - c'' \right] / \sum_{i=1}^{N_t} l_i$$

(6-25)

Considering previous studies on the representative elementary volume (REV) of fracture networks [Baghbanan and Jing, 2007], a model size $L_n$ of 5.0 m was chosen for the calculations. The mass density $d_m$, the theoretical fracture number $N_t$, the theoretical model size $L_t$, and the fracture number $N_i'$ of any model can be calculated with Eqs. (6-22), (6-5), (6-23) and (6-25), respectively, as shown in Table 6-2.

<table>
<thead>
<tr>
<th>$D_f$</th>
<th>$d_m$ (m/m$^2$)</th>
<th>$L_n$ (m)</th>
<th>$N_t$</th>
<th>$L_t$ (m)</th>
<th>$N_i'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>11.6905</td>
<td>2.8276</td>
<td>89</td>
<td>5.0</td>
<td>278</td>
</tr>
<tr>
<td>1.4</td>
<td>20.3656</td>
<td>2.6331</td>
<td>125</td>
<td>5.0</td>
<td>450</td>
</tr>
<tr>
<td>1.5</td>
<td>32.7688</td>
<td>2.7023</td>
<td>177</td>
<td>5.0</td>
<td>606</td>
</tr>
<tr>
<td>1.6</td>
<td>50.5020</td>
<td>2.6907</td>
<td>251</td>
<td>5.0</td>
<td>866</td>
</tr>
<tr>
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<td>75.8559</td>
<td>2.7149</td>
<td>354</td>
<td>5.0</td>
<td>1200</td>
</tr>
<tr>
<td>1.8</td>
<td>112.1052</td>
<td>2.8814</td>
<td>501</td>
<td>5.0</td>
<td>1508</td>
</tr>
</tbody>
</table>

6.4.2 Validity of the fractal length distribution

The length of each fracture was generated using Eq. (6-11) until the index $i$ reached $N_i'$ for different fractal dimensions $D_f$ from 1.3 to 1.8 (Table 6-2). Higher values of the fractal dimension $D_f$ lead to higher probabilities of generating longer fractures. The fracture number $n(l, 0.5)$ represents the number of fractures with lengths in the range of $(l - 0.5, l + 0.5)$. Fig. 6-4 and Table 6-3 show the statistical results of the relation between fracture length and fracture number using the parameters shown in Table 6-2. For each $D_f$, three sets of random number seeds were utilized to generate different fracture length distributions using Eq. (6-11). Each fracture length-fracture number curve was fitted using a power law function in the form of
\[ n(l, 0.5) = \alpha l^a \]  

(6-26)

where \( n(l, 0.5) \) is the number of fractures with lengths in the range of \((l - 0.5, l + 0.5)\), \( \alpha \) is the coefficient of proportionality, and \( a \) is the power law exponent.

Fig. 6-4 Correlation of fracture number and fracture length with fractal dimensions \( D_f \) ranging from 1.3 to 1.8.
Table 6-3 Calculation results based on the proposed fracture length distribution (Eq. (6-11)).

<table>
<thead>
<tr>
<th>$D_f$</th>
<th>Case</th>
<th>Value $\alpha$</th>
<th>Average $\alpha$</th>
<th>Value $a$</th>
<th>Average $a$</th>
<th>Value $R^2$</th>
<th>Average $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>Case_1</td>
<td>155.41</td>
<td>-2.3231</td>
<td>0.8652</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case_2</td>
<td>157.12</td>
<td>145.25</td>
<td>-2.3820</td>
<td>-2.2888</td>
<td>0.9176</td>
<td>0.8851</td>
</tr>
<tr>
<td></td>
<td>Case_3</td>
<td>123.22</td>
<td>-2.1613</td>
<td>0.8414</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>Case_1</td>
<td>129.65</td>
<td>-1.8506</td>
<td>0.7713</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case_2</td>
<td>127.05</td>
<td>138.49</td>
<td>-1.9661</td>
<td>-1.9681</td>
<td>0.7851</td>
<td>0.8012</td>
</tr>
<tr>
<td></td>
<td>Case_3</td>
<td>158.78</td>
<td>-2.0876</td>
<td>0.8471</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>Case_1</td>
<td>207.02</td>
<td>-1.8478</td>
<td>0.8135</td>
<td></td>
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Fig. 6-5 Linear relation between the power law exponent $a$ and the fractal dimension $D_f$. 

$y = -x + 3.49$
$R^2 = 0.8378$
With increasing $D_f$, more fractures with relatively long lengths appear, and the length of the longest fracture increases, as shown in Fig. 6-4. Table 6-3 and Fig. 6-5 show that the average power law exponent $a$ ranges from 2.29 to 1.70, corresponding to fractal dimensions $D_f$ of 1.3 to 1.8. These parameters have a linear relationship that can be expressed by

$$a = -D_f + 3.49 \quad (6-27)$$

For 2-D fracture networks, $D_f$ ranges from 1 to 2, which results in values of the power law exponent $a$ from 2.49 to 1.49 according to Eq. (6-27). These results reveal that the fracture length also follows a power law length distribution with the power law exponent $a$ linearly correlated with the fractal dimension $D_f$. The theoretical value of the power law exponent $a$ ranges from 1.49 to 2.49, which agrees well with the values reported in the literature from in situ measurements and theoretical analyses (Table 6-4) [De Dreuzy et al., 2001a; 2001b; Dverstorp and Andersson, 1989; Tsang et al., 1996; Bour and Davy, 1997]. Therefore, the proposed fractal length distribution approach is reliable and is capable of describing the characteristics of fracture distributions using a fractal dimension method.

<table>
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<th>Authors</th>
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<td>This study</td>
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### 6.4.3 Fractal evaluation methods

The stochastic DFN modeling technique was utilized to generate and characterize fracture networks. Measuring length density is difficult because each model contains a large number of fractures. The geometric distribution of rock fractures in a fracture network has fractal characteristics [Jafari and Babadagli, 2012]. Therefore, the fractal dimension may serve as an effective approach for assessing the geometric characteristics of rock fractures.
Because it is a simple and precise approach, the box-counting method was used and improved in this study to calculate the fractal dimensions of the DFN models. First, the original image was converted to monochrome (Fig. 6-6(a)), and a binary image (Fig. 6-6(b)) was then obtained based on the adaptive threshold of the grayscale. The example shown in Fig. 6-6(b) is 280 pixels in length and 280 pixels in width, and each pixel has a value of either 1 or 0 (null). Second, the image was covered by square boxes with different dimensions from 280x280 pixels (1 box) to 1x1 pixel (280x280 boxes). If the pixels in a box were not null, the box was given a value of 1; otherwise, it was given a value of 0. Finally, a log-log plot of the box count ($N_c$) vs. the number ($N_b$) of total boxes (Fig. 6-6(c)) was drawn, and the slope represents the fractal dimension. To verify the validity of this program, a series of well-known fractal graphs were generated, and the results were compared with the theoretical values. The deviations between the calculated and theoretical values are less than 2%; therefore, the program is considered to reliably estimate the fractal dimensions $D_f$.

Fig. 6-6 Image processing and calculation of the fractal dimension.
For each fractal dimension $D_f$, 10 sets of DFN models (numbered from 1 to 10) were developed using the fractal length distribution of Eq. (6-11). The fractal dimensions ($D_f$) of these DFN models were then calculated using the box-counting method and compared with their theoretical values (Fig. 6-7). When $D_f$ was small (e.g., 1.3 and 1.4), the calculated fractal dimensions $D_f$ were underestimated compared to their theoretical values because many isolated fractures that did not connect with other fractures were deleted from the models. At higher values of $D_f$ (1.5 and 1.6), the calculated and theoretical values agreed with each other, and the discrepancies decreased because the fractures became denser and the connectivity of the models improved. These results are robust evidence of the validity of applying Eq. (6-3) from porous media to fractured rock masses and the validity of the fractal length distribution presented in Eq. (6-11).

![Fractal Dimensions Comparison](image)

Fig. 6-7 Comparisons of fractal dimension $D_f$ between calculated values and theoretical values using 10 sets of DFN models for each $D_f$.

6.4.4 Characteristics of the flow patterns

Several examples of DFN models with side lengths of 5.0 m and $D_f$ values from 1.3 to 1.6 are shown in the left column of Fig. 6-8. These models were generated using the Monte Carlo method based on the parameters presented in Section 3. In these models, the upper and lower boundaries were assumed to be impermeable, and fluid flowed from left to right (see Fig. 6-2) with a hydraulic head of 5 m. When the inflow volume was equal to that of the outflow, the fluid flow was regarded as a steady-state...
flow. The flow rate distributions at these conditions are shown in the central column of Fig. 6-8. The right column of Fig. 6-8 shows corresponding remarks. In Figs. 7-8(b) and 7-8(e), preferential flow paths exist along the relatively long fractures that are subparallel to the flow direction (horizontal) and particularly in those that intersect the inlet and outlet boundaries. With an increase of $D_f$ (see Figs. 7-8(h) and 7-8(k)), more short fractures achieve large flow rates and the flow rate distributions within the networks become more homogeneous. These observations were also reported by De Dreuzy et al. [De Dreuzy et al., 2001a; 2001b; 2002], who found that the connectivity and permeability of 2-D fracture networks are controlled by the largest fracture in the system when only a few fractures exist and controlled by the fractures that are smaller than average when a large number of fractures exist.
Fig. 6-8 Geometric distributions of DFN models (a, d, g, j; left column), correlated flow rate distributions and flow paths (b, e, h, k; central column) and remarks (c, f, i, l; right column) with varying fractal dimensions, $D_f$, from 1.3 to 1.6.

6.4.5 Influences of $D_f$ and $D_T$ on the equivalent permeability

As shown in Eq. (6-16), the proposed model includes a random number $R_i$ that introduces randomness to the fracture networks. Ten random numbers corresponding to each fractal dimension $D_f$ were generated and applied to the DFN modeling. The relation between the equivalent permeability $K$ and the value of $D_f$ calculated utilizing these random numbers is shown in Fig. 6-9. Due to the randomness of the trace length and the location and orientation of rock fractures induced by the random numbers, DFN models with the same $D_f$ value can have large differences in their geometric distributions, which results in variations in the calculated equivalent permeabilities (see Fig. 6-9) [De Dreuzy et al., 2002]. When $D_f$ is small (e.g., 1.3), the equivalent permeability varies by approximately 5 orders of magnitude. The range of variation decreases to less than 2 orders of magnitude when $D_f$ becomes large (e.g., 1.7–1.8). With increasing $D_f$, the influence of the random number decreases, which is reasonable because the permeability of a model with only a few fractures can be affected more by the randomness of the trace length, location and orientation of the fractures than models that contain many fractures (see Figs. 6-8(a) and 6-8(j)). By taking the mean value of the results for each $D_f$, an approximate curve that indicates that the changes in equivalent permeability with $D_f$ follows an exponential law was obtained.
Fig. 6-9 Equivalent permeabilities of models with $D_f$ values from 1.3 to 1.8 generated using 10 sets of random numbers.

Fig. 6-10 shows the relation between the equivalent permeability and $D_T$ with trace length ratios $l_{\text{min}}/l_{\text{max}}$ of $2.0 \times 10^{-4}$, $1.0 \times 10^{-4}$ and $0.5 \times 10^{-4}$, which were obtained using a constant $l_{\text{min}} = 0.05$ m and $l_{\text{max}}$ values of 250 m, 500 m and 1000 m, respectively. With an increase of $D_T$, the equivalent permeability decreases because higher values of $D_T$ represent greater tortuosity and thus greater resistance to fluid flow in the fractures, which results in a lower equivalent permeability. Fig. 6-10 also shows that as the trace length ratio $l_{\text{min}}/l_{\text{max}}$ increases, the equivalent permeability decreases. The increase of $l_{\text{min}}/l_{\text{max}}$ will decrease the total number of fractures $N_t$ for a given $D_f$ of a DFN model. The smaller total number of fractures $N_t$ results in worse connectivity of the network and decreases the equivalent permeability in turn. The three regression functions in Fig. 6-10 show that the changes in equivalent permeability with $D_T$ follow exponential laws, which are affected by the trace length ratio.
Fig. 6-10 Relations between equivalent permeability with the fractal dimensions $D_T$ and the trace length ratios $l_{min}/l_{max}$.

The cubic law in the form of Eq. (6-17) is usually utilized for fluid flow in fractures by assuming that each fracture consists of two parallel walls. According to Eq. (6-16), the modified cubic law that considers the effect of $D_T$ can be rewritten as

$$q(i) = \frac{e^{\gamma + D_T}}{12 \mu} \frac{AP}{l_i^{D_T}}$$ (6-28)

$D_T$ is the source of the difference between Eq. (6-17) and Eq. (6-28), and its effects on DFN models can be estimated by the equivalent relative error $\overline{\delta}$:

$$\overline{\delta} = \frac{Q_{cubic}}{Q_{tortuosity}}$$ (6-29)

where $Q_{cubic}$ is the fluid volume through the DFN model based on the parallel plate model (see Eq. (6-17)) and $Q_{tortuosity}$ is the fluid volume considering the tortuosity of the flow (see Eq. (6-28)).

Fig. 6-11 shows the relation between $\overline{\delta}$ and $D_T$ at different values of $D_f$. The equivalent relative error $\overline{\delta}$ increases with an increase in $D_T$ and follows an exponential law. The values of $\overline{\delta}$ vary little with different random numbers at a given $D_T$, which indicates that the random number mainly represents the macro-scale properties of the fracture network. The four figures in Fig. 6-11 show that the values of $\overline{\delta}$ vary little among cases with different values of $D_f$, which shows that $D_f$ is another macro-scale parameter that, along with the random number, determines the geometric characteristics.
of the fracture networks. In the cases with $D_T = 1.018$, which corresponds to a JRC value of 20, the maximum values of $\bar{\delta}$ range from 17.64% to 19.51%, which indicates that the effect of tortuosity is not negligible and should be included in fractal models to accurately estimate the hydraulic behavior of fracture networks.

![Graphs showing relative errors for different fractal dimensions](image)

**Fig. 6-11** Relative errors of DFN models with $D_T$ values from 1.000 to 1.018 and $D_f$ values from 1.3 to 1.6.

### 6.5 Conclusions

In this study, a fractal model was established to assess the equivalent permeability of 2-D rock fracture networks. The fractal dimension $D_T$ and the fractal dimension $D_f$ were used in the model to represent the effects of the tortuosity of fluid flow in the fractures (micro-scale) and the geometric characteristics of the fracture distributions (macro-scale), respectively. Fluid flow was simulated in the generated fracture network...
models, and the relation between the fractal dimension and the equivalent permeability was analyzed.

The results showed that the correlation of fracture number and fracture length based on the proposed fractal length distribution in this study agrees well with reported values from the literature, which confirmed the reliability of the proposed length distribution approach. Comparisons of the fractal dimension $D_f$ between the values calculated using the box-counting method and the theoretical values agree with each other, which verified the validity of the fractal DFN models that were developed using the proposed fractal length distribution. When $D_f$ is small (e.g., less than 1.5), fluid flow mainly occurs in a few long fractures that are subparallel to the flow direction and particularly in the fractures that intersect the inlet and outlet boundaries of the models. When $D_f$ exceeds a certain value (e.g., 1.5), the flow rate distribution becomes more homogeneous, and shorter non-persistent fractures dominate the preferential flow paths. The equivalent permeabilities of models generated using different random numbers vary significantly with changes of $D_f$ when $D_f$ is small (e.g., less than 1.5), and they become more stable when $D_f$ is relatively large (e.g., greater than 1.6). This behavior is consistent with the observations of the flow paths, which show that the models become more homogeneous at larger values of $D_f$. Therefore, a mathematical expression between the equivalent permeability $K$ and the fractal dimension $D_f$ (e.g., the exponential relationship presented in this study) can be expected to be applicable for models with large values of $D_f$. For models with small values of $D_f$, other parameters, such as connectivity, should be taken into account to improve the accuracy of the predictions. Compared with the parallel plate model, the maximum deviation of the calculated flow volume that considers the effect of tortuosity ($D_T$) can be as high as 19.51% when $D_T = 1.018$, which corresponds to a JRC value of 20. These results show that both the geometric characteristics of the fracture distributions and the geometric characteristics (surface roughness) of single rock fractures (the source of tortuosity) have significant influence on the hydraulic behavior of fracture networks. Further development of the proposed model is required to estimate its scaling effects, which might have important impacts on the equivalent permeability and were not considered in this study.
References:


Itasca Consulting Group Inc. UDEC User’s guide, ver 4.0, Minneapolis, Minnesota, 2004.


7 A fractal model for predicting EFNP (Part 2)

A fractal model based on a new governing equation of fluid flow in fractures for characterizing hydraulic properties of rock fracture networks

7.1 Introduction

Fluid flow in fractured rock masses is governed by connected conductive fractures, due to their significantly higher permeability comparing with the rock matrix. In the past few decades, the discrete fracture network (DFN) modelling techniques have been developed to establish discontinuous models of fractured rock masses based on the assumption that fluid only flows in the connected conductive fractures with negligible rock matrix permeability [Louis 1969; Long et al. 1982; Zhang et al. 1996; Min and Jing 2003; Min et al. 2004; Baghbanan and Jing 2007; Leung and Zimmerman 2012; Liu et al. 2014]. Field mapping provides information of geometric characteristics of fractures for establishing DFN models, which could then be used for assessing the fluid flow characteristics numerically. A number of parameters are involved in the description of fracture geometries in rock masses (e.g., length, aperture, and dip angle). It is still a challenging and time consuming task to accurately obtain the geometric information of each single fracture in a field at both the macro-structural (i.e., geometry of the fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [Oliveira and Graca 1987; Hudson and Harrison 1997; Yu and Cheng 2002; Liu et al. 2015]. As an alternative, a number of mathematical expressions have been proposed to represent the characteristics of fracture distributions and to describe fluid flow behavior in single fractures with complex void geometries. Jafari and Babadagli [2008; 2009] have showed that the density and length of fractures are the two most critical parameters for calculating the equivalent fracture network permeability (EFNP). De dreuzy et al. [2001a; 2001b; 2002] have semi-empirically found that the fracture lengths follow a power law distribution, yet it has to heavily account for the length of each fracture to determine the power law exponent. Other studies have shown that the geometric distributions of fractures in fracture networks, especially the length of fractures, exhibit fractal characteristics, which presented a promising approach to quantitatively estimate the characteristics of fracture distributions. [e.g., Vermilye 1995; Hatton 1994; Renshaw 1997; Jiang et al. 2006].
Barton and Larsen [1985], La Pointe [1988], and Barton and Hsieh [1989] found that natural fracture patterns exhibit fractal characteristics based on statistical analysis of natural cracks and fractures. Babadagli et al. [2001] mapped natural fracture patterns of 2-D fracture networks of geothermal reservoirs at different scales. They observed that the fracture networks exhibit scale-invariant properties, however, fractal dimensions might significantly differ when the mass dimensions were measured by different methods. Bagde et al. [2002] calculated the fractal dimensions of blasted fragments and in situ rock blocks using size distribution curves. They concluded that the change of fractal dimension is nominal beyond a uniaxial compressive strength (UCS) value of 20 MPa, while there is a sharp increase in fractal dimension for rock mass rating (RMR) greater than 40. Zhao et al. [2009] found that the random distribution of fractures in a geologic mass agrees well with the fractal law. Their observations and statistics based on the data of three sites demonstrated that fracture distribution of each group, classified by the strike of the strata, still follows the fractal law, although the fractal dimension varies with different strikes to some extent. Zheng and Yu [2012] established a fractal permeability model for gas flow through dual-porosity media by embedding fractal-like tree networks. Their calculation results showed that the porous matrix can be seen as a gas storage medium with negligible contributions to gas flow and the permeability of a dual-porosity medium may primarily be controlled by the fractures. Jafari and Babadagli [2012] analyzed the influences of fracture network characteristics (density, length, orientation, connectivity, and aperture) on permeability using different calculation methods of the fractal dimension. A nonlinear multivariable regression was derived to estimate the equivalent fracture permeability based on five independent variables. Jafari and Babadagli [2013] later presented the relationship between percolation-fractal properties and permeability of 2-D fracture networks. They found that the fractal dimension of fracture lines obtained using the box counting method yields a more accurate estimation, comparing with the fractal dimensions of intersection point, connectivity index, and scanning lines in X- and Y- directions, for EFNP. Kruhl [2013] reviewed the applications of fractal-geometry techniques in the quantification of complex rock structures considering the scale effect, inhomogeneity, and anisotropy of rock masses. Miao and Yu [2015] derived an analytical expression for permeability of fractured rocks involving fractal dimensions for representing the fracture area, area porosity, fracture density, the maximum fracture length, aperture, fracture azimuth, and fracture dip angle. In most previous 2-D DFN models, the fractures were typically treated as straight lines, without considering their geometric properties in the
out-of-plane orientations. The hydro-mechanical properties of single rough rock fractures have been extensively studied experimentally and numerically [e.g., Barton and Choubey 1977; Zimmerman et al. 2004; Xiong et al. 2011], however, fractures in DFN models are still treated typically as parallel-plate models to allow the application of the cubic law, in which the flow rate is proportional to the cube of aperture by assuming a unit value of fracture width that represents the out-of-plane geometry of fractures. Klimczak et al. [2010] has addressed the influences of fracture width and derived a modified cubic law showing that flow rate is proportional to the quantic of the aperture, yet they did not consider the effects of fracture surface roughness. Most of the previous studies [i.e., Barton and Larsen, 1985; Zhao et al., 2009; Jafari and Babadagli, 2012] found that fracture length distribution shows fractal properties by measuring field mapping data, rather than theoretically deriving some expressions for fracture length distribution. Miao and Yu [2015] has stressed the influences of tortuosity on the total equivalent permeability of a fracture network, yet their model assumed that each fracture cuts through the model without considering the orientations and intersections of the fractures, which, to some extent, deviates from the engineering practices. To solve these problems, in a previous study, a fractal model in which the length distribution of fractures follows fractal characters was theoretically developed to estimate the equivalent permeability of rock fracture networks [Liu et al. 2015], and have discussed the relationship between equivalent permeability with fractal dimensions $D_T$ and $D_f$, which represent the tortuosity of fluid flow in single fractures (micro-scale) and the geometric characteristics of fracture distributions (macro-scale), respectively. However, the side length of DNFs in the previous study was fixed to a constant value of 5 m, and the validity of the proposed fractal model has not been verified at different scales.

As a continuum work, the objectives of the present study are: (1) to derive a new governing equation for fluid flow in single fractures taking into account the out-of-plane geometry of fractures, (2) to extensively verify the validity of the proposed fractal model at different scales, and (3) to estimate the effects of fractal dimension, model size, and random number on the equivalent permeability of DFNs.

7.2 Fractal characteristics of rock fractures in DFNs

Based on the fractal theory established by Mandelbrot [1982], Majumdar and Bhushan [1990] developed an expression to describe the island distributions on the
surface of the earth. Yu et al. [2002] subsequently used the fractal theory to describe the geometric distributions of pores in porous media. Liu et al. [2015] later introduced this theory into the mathematical calculation of fracture length distributions in fractured media with a constant DFN side length of 5 m. For the $i$th fracture in a DFN,

$$l_i = \left( \frac{l_{\text{max}}}{(1 - R_i)^{D_f}} \right)^{D_f} \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right)$$ (7-1)

where $l_i$ is the trace length of the $i$th fracture and $i = 1, 2, 3, \ldots, N$, $N$ is the total number of fractures in the network, $l_{\text{max}}$ and $l_{\text{min}}$ are the maximum and minimum trace lengths of fractures, respectively, and $R_i$ is the $i$th random number in the range of (0, 1). The process of deriving Eq. (7-1) is given in Liu et al. [2015] in detail.

Eq. (7-1) gives the expression of the fractal length distribution of fractures and its validity is verified in the Section 7.5 with varying DFN sizes. Eq. (7-1) implies that the fracture length, $l$, is only related with $l_{\text{min}}$, $R$, and $D_f$. The total number $N_t$ is correlated with $l_{\text{max}}$, which can be specified by the size of a DFN. The determination procedure of $N_t$ and its updated value $N'_t$ for any side length $L_n$ of DFNs is described in the Section 7.4.1.

7.3 Fluid flow characteristics of fractures in DFNs

7.3.1 Cubic law

The Navier-Stokes equations expressing momentum and mass conservation over fracture void spaces can be utilized for accurately calculating fluid flow in single fractures [Javadi et al. 2010; Xiong et al. 2011]. However, modeling by solving Navier-Stokes equations is usually time-consuming that requires high-performance computers and sophisticated computing techniques. As an alternative, fluid flow in single fractures in DFNs is mostly characterized by the “cubic law” for calculating laminar flow between parallel plates [Snow 1965; Louis 1969; Tsang and Witherspoon 1981; Klimczak 2010;].

$$Q = \frac{\rho e^3 g e^3}{12\mu} \frac{\Delta h}{L} W$$ (7-2)

where, $Q$ is the flow rate, $e$ is the hydraulic aperture of fracture, $\mu$ is the dynamic viscosity, $g$ is the gravitational acceleration, $\Delta h$ is the hydraulic head difference, $L$ is the tortuous length of a fracture, and $W$ is the width of a fracture.
7.3.2 Correlation of fracture aperture and trace length

In nature, a fracture is generated by a force to enable a crack to propagate in three modes: opening mode, sliding mode, and tearing mode, according to the stress environment acting on the crack [Irwin 1957; Sih 1962; Paris and Sih 1965; Pollard and Segall 1987]. Previous studies on the correlation between aperture and length of fractures have revealed that for sliding-mode or tearing-mode fractures, the maximum shearing displacement \(e_{\text{max}}\) is linearly proportional to the length as [Scholz 2002; Schultz et al. 2008a]:

\[ e_{\text{max}} = \gamma L \]  \hspace{1cm} (7-3)

where, \(L\) is the straight length and \(\gamma\) is a proportionality coefficient related to the mechanical properties of surrounding rocks.

For opening-mode fractures, previous studies revealed a linear correlation between the maximum displacement and the length [Vermilye and Scholz 1995]. Based on in-situ measurements containing 2 sets of dikes and 1 set of veins, Olson [2003] and Klimczak et al. [2010] found a sub-linear, square root power-law distribution as:

\[ e_{\text{max}} = \alpha' L^{0.5} \]  \hspace{1cm} (7-4)

where \(\alpha'\) is a proportionality coefficient with a form of

\[ \alpha' = \frac{K_{\text{ic}} (1-\nu)^2 \sqrt{8}}{E \sqrt{\pi}} \]  \hspace{1cm} (7-5)

where \(K_{\text{ic}}\) is the intrinsic fracture toughness of the material, \(\nu\) is the Poisson’s ratio, and \(E\) is the Young’s modulus.

Here, \(e_{\text{max}}\) of the opening-mode fractures is related to the average displacement \(e_{\text{avg}}\) as [Olson 2003]:

\[ e_{\text{avg}} = \frac{\pi}{4} e_{\text{max}} \]  \hspace{1cm} (7-6)

In this case, \(e_{\text{avg}}\) is identical to the aperture \(e\) in the cubic law (Eq. (7-2)). The present study only focuses on the opening-mode fractures which may exist in nature during hydraulic fracturing [Chen et al., 2015]. The opening-mode fractures are also called the “penny-shaped” fracture [Lawn 1993], and for some rocks (i.e., igneous rocks), the fracture length \(L\) typically equals to the fracture depth \(W\) as shown in Fig. 7-1 [Jaeger 1969; Timoshenko and Goodier 1970; Jaeger and Cook 1979].

\[ W = L \]  \hspace{1cm} (7-7)
7.3.3 Tortuosity of fluid flow

Natural fractures have rough surfaces that render non-linear streamlines of fluid flow and increase the end-to-end distances required for a fluid flowing through a fracture [Kulatilake et al. 1995]. For a parallel-plate model without roughness, the tortuous trace length $L_t$ of a particle flowing through the model equals to the straight length $L$. In contrast, a natural fracture with rough surfaces would have a larger $L_t$ than $L$, and a rougher surface would result in a larger difference between $L_t$ and $L$. Here, a fractal dimension $D_T$ was introduced to describe the difference between $L_t$ and $L$ in single rock fractures [Cai et al. 2010]. $L_t$ can be obtained based on the fractal scaling law as [Yu and Cheng 2002; Yu and Lee 2002]:

$$L_t = e^{1-D_T} L^{D_T}$$ (7-8)

where $D_T$ is the fractal dimension that represents the nonlinearity of streamlines of fluid flow (tortuosity) induced by surface roughness for single fractures in DFNs with a range of [1, 2]. For $D_T = 1$, the streamline is linear, thus $L_t = L$. The value of $D_T$ can depict the nonlinearity of streamlines and the effect of tortuosity on the permeability of a fracture [Liu et al. 2015]. As shown in Fig. 7-2, $L_t$ increases with increasing $D_T$ for DFNs with different values of $L$ when $e = 1$ mm.
7.3.4 A new governing equation for fluid flow in single fractures

Substituting the above Eqs. (7-4) ~ (7-8) into the cubic law (Eq. (7-2)) yields a new governing equation for calculating fluid flow in single fractures as:

\[ Q = \frac{4^{3-2D_T} \rho g}{3\mu(\pi \alpha')^{4-2D_T}} e^{6-D_T} \nabla h \]  

(7-9)

where \( \nabla h = \Delta h / L \) is the hydraulic gradient. The detail derivation of Eq. (7-9) is shown in the appendix.

In this new equation, the flow rate \( Q \) is proportional to \( e^{6-D_T} \), which implies a more robust nonlinearity between the flow rate and the aperture, comparing to the classic cubic law (Eq. (7-2)) in which flow rate is proportional to \( e^3 \). This is because while fractures in ordinary 2-D DFNs are treated as straight lines, this new model takes into account the geometry of fractures in the out-of-plane orientations (Fig. 7-1), which could better represent the geometric nature of real rock fractures. Meanwhile, this new model still has a 2-D framework that could help improve the computational efficiency since complete 3-D modelling of DFNs requires large computational power and time.

For a single straight fracture, \( D_T = 1 \) and the flow rate is proportional to \( e^5 \), which is called the “quantic law” by Klimczak et al. [2010]. When calculating fluid flow in two-dimensional fractures, it is commonly assumed that \( W = 1 \). However, in this study, the geometry of fracture in the out-of-plane orientations was considered, where \( W = L = [4e/(\pi \alpha')]^2 \) (by substituting Eq. (7-4) and Eq. (7-6) into Eq. (7-7)). For ideal smooth
parallel plate model without considering the geometry of fracture in the out-of-plane orientations, $D_T = 1$ and $W = L = [4e/(\pi a')]^2 = 1$, thus, Eq. (7-9) turns into Eq. (7-2), where $Q$ is proportional to $e^3$.

7.4 Generation of DFNs and the boundary conditions

7.4.1 Determination of the model size

The model size of DFNs is another important factor that has significant impacts on their hydraulic properties, which is also the motivation of calculating the representative elementary volume (REV) for DFNs in many previous studies [e.g., Long et al. 1982; Zhou and Yu 1999; Koyama and Jing 2007]. In the former practices utilizing the fractal dimension, $D_f$ was only used to describe the fracture distribution/density, and no correlation involving the model size has been developed. Different model sizes lead to different numbers of fractures in the models with identical density or $D_f$, which need to be described quantitatively. By computing the fractal dimension and the fracture density of a great number of DFNs, $D_f$ was found to be correlated with the fracture density $d_m$ in a form of [Liu et al. 2015]:

$$D_f = a'' + b'' \ln(d_m + c'')$$

(7-10)

where, $a''$, $b''$, $c''$ are three regression parameters, equaling to 0.4594, 0.2797, and 8.4968, respectively, and $d_m$ is the mass density of fractures. Notable is that the values of $a''$, $b''$ and $c''$ are limited on the DFNs with fracture length distributions following Eq. (7-1) and fracture locations and orientations being uniformly and randomly distributed.

At a given fracture density, the number of fractures changes proportionally with changing model size represented by side length, which could be expressed by the following equation by means of Eq. (7-10) as:

$$N'_t = \frac{N_t L_t^2}{L'^2} = N_t L_n^2 \left[\exp\left(\frac{D_f - a''}{b''}\right) - c''\right] / \sum_{i=1}^{N_t} l_i$$

(7-11)

where $N'_t$ is the fracture number (amount) corresponding to a DFN side length of $L_m$, and $L_T$ is the DFN side length corresponding to the model having a total fracture number of $N_t$. Calculation results of these parameters at different model sizes ranging from 5 m to 43 m, and different $D_f$ ranging from 1.3 to 1.8 are tabulated in Table 7-1.
Table 7-1 Parameters associated with different model sizes

<table>
<thead>
<tr>
<th>$D_f$</th>
<th>$d_m$ (m/m$^2$)</th>
<th>$N_t$</th>
<th>$L_n$ (m)</th>
<th>$N_t'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>11.6905</td>
<td>89</td>
<td>43</td>
<td>2470</td>
</tr>
<tr>
<td>1.4</td>
<td>20.3656</td>
<td>125</td>
<td>28</td>
<td>2472</td>
</tr>
<tr>
<td>1.5</td>
<td>32.7688</td>
<td>177</td>
<td>19</td>
<td>2589</td>
</tr>
<tr>
<td>1.6</td>
<td>50.5020</td>
<td>251</td>
<td>11</td>
<td>1938</td>
</tr>
<tr>
<td>1.7</td>
<td>75.8559</td>
<td>354</td>
<td>7</td>
<td>1592</td>
</tr>
<tr>
<td>1.8</td>
<td>112.1052</td>
<td>501</td>
<td>5</td>
<td>1513</td>
</tr>
</tbody>
</table>

7.4.2 Generation of DFNs

In the present study, several parameters for generating the DFNs borrowed the values from the study on a real rock mass conducted by Baghbanan and Jing [2008]. $l_{min}$ and $l_{max}$ were 0.5 m and 500 m, with a ratio of 0.001 that satisfies the critical condition of the fractal length distribution [Liu et al. 2015]. The range of $D_f$ was [1.3, 1.8], below which rock fractures could not efficiently connect with each other and beyond which rock masses are so fragmented that they barely exist in the nature. The center point and the orientation of fractures were uniformly and randomly distributed within each model, utilizing another two sets of random numbers that are distinguished from the first set of random number used in Eq. (7-1). The orientation angles of the fractures range from 0° to 360°. The location of each fracture could be identified by the center point, orientation, and fracture length.

The procedure for generation of DFNs based on the proposed fractal method is described as follows. First, the values of the fundamental parameters (e.g., $D_f$, $l_{max}$, and $l_{min}$) that describe the fractal characteristics of rock fractures in rock masses need to be determined. These values can be obtained by applying fractal dimension calculation methods (e.g., box-counting method) to field mapping images [Jiang et al. 2006; Liu et al. 2013]. Second, the parameters related to a DFN located in a specific region (e.g., $N_t'$, $L_n$, and $l_i$) need to be determined. The number of fractures $N_t'$ with respect to any given $L_n$ can be calculated by Eq. (7-11). The length of each fracture with an ID $i$ from 1 to the updated $N'$ is calculated by Eq. (7-1). Third, a DFN can be generated after obtaining the values of essential parameters mentioned above based on the Monte Carlo method. The center points of fractures are uniformly and randomly distributed within a targeted region with a side length of $L_n$. Then the starting point and the ending point of each fracture are calculated by applying uniformly and randomly distributed fracture
orientations. For any given $L_0$, parts of a few fractures may be generated outside the model, although their center points are located within the model. Meanwhile, isolated fractures and fractures with dead-ends that do not contribute to fluid flow may also be generated. These segments of fractures are deleted from the model.

7.4.3 Extraction of DFNs and the boundary conditions

Three original large DFNs were firstly established, with side lengths of 28 m, 19 m, and 11 m, corresponding to $D_f$ of 1.4, 1.5, and 1.6, respectively. Then smaller DFNs were extracted from these large DFNs with varying sizes starting from $0.5 \times 0.5$ m to $25 \times 25$ m ($D_f = 1.4$), to $14 \times 14$ m ($D_f = 1.5$), and to $9 \times 9$ m ($D_f = 1.6$), respectively. Three different sets of random numbers were required to generate the center point, the orientation and the length of fractures (Eq. (7-1)) in a DFN. To investigate the influences of the random number on the equivalent permeability of DFNs, 10 groups of random numbers, each group consisting of three different sets were utilized for DFN generation, leading to a total of 1290 DFNs. The details for fluid flow simulation on the DFNs can be referred to Liu et al. [2015]. As an example, Fig. 7-3 exhibits one large DFN and two extracted smaller DFNs with different side lengths when $D_f = 1.5$.

For each DFN, the top and bottom boundaries were assumed to be impermeable and fluid flowed from left side to right side of the model under a constant hydraulic gradient imposed on the left and right boundaries as shown in Fig. 7-3. When the flow rate of the right boundary became equivalent to that of the left boundary, the fluid flow was considered to have reached a steady state, and then the flow rate was recorded for calculation of the equivalent permeability. This study only considered the cases with hydraulic gradient variations along the $x$-direction.

For the purpose of visual comparisons, Fig. 7-4 shows three DFNs with the same side length of 9 m and varying $D_f$ of 1.4, 1.5, and 1.6. Fluid flow in the model of $D_f = 1.4$ was dominated by relatively short and fragmented fractures that did not cut through the model, while that in the model of $D_f = 1.6$ was dominated by relatively long fractures that cut through the model together with some clustered smaller fractures. The model of $D_f = 1.5$ showed the mixed mode of both. The models with $D_f = 1.7$ and 1.8 had much denser fractures than the models shown in Fig. 7-4 and fluid flow simulations in these models were beyond the computational capability of ordinary personal computers. Therefore, although DFNs with $D_f$ ranging from 1.3 to 1.8 were generated, the fluid flow simulations were only conducted on the models with $D_f$ of 1.4, 1.5 and 1.6 in this study.
Fig. 7-3 Extraction of DFNs from an original large model and their flow rate distributions. The quadrilaterals represent hydraulic pressures and the hydraulic gradient was fixed at 10 kPa/m to generate horizontal fluid flow in the $x$-direction with unit line thickness $= 1.87\times10^{-8}$ m/s.
Fig. 7-4 Geometric distributions of fractures in DFN models (a, c, e), and flow rate distributions (b, d, f) with a constant side length of 9 m and varying fractal dimensions, $D_f$, from 1.4 to 1.6. The thickness of the line represents the magnitude of flow rates, and each line indicates a flow rate of $5.60 \times 10^{-9}$ m$^3$/s.
7.5 Results and analysis

The proposed model incorporated a new governing equation for fluid flow in single fractures and the fractal length distribution of fractures into DFNs. It needs to be verified against the results of other models reported in previous studies and/or in-situ measurements before it could be applied for estimating the hydraulic properties of fractured rock masses.

![Graphs showing the correlation between fracture number and fracture length with different values of Df ranging from 1.3 to 1.8 and Lm ranging from 43m to 5 m.](image)

Fig. 7-5 Correlation between the fracture number and fracture length with $D_f$ ranging from 1.3 to 1.8 and $L_m$ ranging from 43m to 5 m.
### 7.5.1 Verification of the fractal length distribution of fractures

Table 7-2 Calculation results of $\alpha$ and $a$ based on the proposed fractal length distribution of fractures (Eq. (7-1))

<table>
<thead>
<tr>
<th>$D_f$</th>
<th>$L_m$ (m)</th>
<th>Case</th>
<th>Value ($\alpha$)</th>
<th>Average ($\alpha$)</th>
<th>Value ($a$)</th>
<th>Average ($a$)</th>
<th>Value ($R^2$)</th>
<th>Average ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>43</td>
<td>Case_1</td>
<td>1739.56</td>
<td>-2.7075</td>
<td>-2.7075</td>
<td>0.9644</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>1645.83</td>
<td>1701.63</td>
<td>-2.7455</td>
<td>0.9497</td>
<td>0.9615</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_3</td>
<td>1719.50</td>
<td>-2.7996</td>
<td>0.9704</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>28</td>
<td>Case_1</td>
<td>1369.72</td>
<td>-2.4496</td>
<td>-2.4496</td>
<td>0.9588</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>1480.58</td>
<td>1375.30</td>
<td>-2.4665</td>
<td>0.9484</td>
<td>0.9381</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_3</td>
<td>1275.60</td>
<td>-2.4034</td>
<td>0.9072</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>19</td>
<td>Case_1</td>
<td>1504.44</td>
<td>-2.3130</td>
<td>0.9405</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>1351.33</td>
<td>1400.47</td>
<td>-2.3363</td>
<td>0.9385</td>
<td>0.9405</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_3</td>
<td>1345.63</td>
<td>-2.2789</td>
<td>0.9425</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td>11</td>
<td>Case_1</td>
<td>617.20</td>
<td>-1.9364</td>
<td>0.8753</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>746.55</td>
<td>740.21</td>
<td>-1.9921</td>
<td>0.9150</td>
<td>0.8984</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Case_3</td>
<td>856.87</td>
<td>-2.1629</td>
<td>0.9049</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1.7</td>
<td>7</td>
<td>Case_1</td>
<td>495.29</td>
<td>-1.8072</td>
<td>0.9229</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>335.98</td>
<td>458.11</td>
<td>-1.6652</td>
<td>0.8573</td>
<td>0.9078</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>Case_3</td>
<td>543.07</td>
<td>-1.9026</td>
<td>0.9432</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td>5</td>
<td>Case_1</td>
<td>304.85</td>
<td>-1.6348</td>
<td>0.8482</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_2</td>
<td>459.20</td>
<td>343.39</td>
<td>-1.7716</td>
<td>0.9040</td>
<td>0.8648</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case_3</td>
<td>266.13</td>
<td>-1.5093</td>
<td>0.8421</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DFNs with different sizes were considered and the side length of the models varied from 5 m to 43 m, while the length of fractures with a total number of $N'$ was generated by means of Eq. (7-1) for different models with $D_f$ varying from 1.3 to 1.8. The greater value of $D_f$ has higher probability to generate longer fractures. Fig. 7-5 and Table 7-2 show the statistical results of the relation between the length and the number of fractures using the parameters shown in Table 7-1. Here, for each $D_f$, different sets of random numbers were utilized to obtain different length distributions of fractures. Each set of length - number relation of fractures followed a power law function with a form of
\[ n(l, 0.5) = \alpha l^{-a} \quad (7-12) \]

where, \( \alpha \) is the coefficient of proportionality and \( a \) is the power law exponent. \( n(l, 0.5) \) stands for the number of the fractures with lengths in the range of \((l - 0.5, l + 0.5)\).

![Graph showing the linear relationship between the power law exponent \( a \) and the fractal dimension \( D_f \).](image)

**Fig. 7-6** Linear relationship between the power law exponent \( a \) and the fractal dimension \( D_f \).

**Table 7-3** Comparisons between the values of the power law exponent \( a \) in the present and other studies

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Value of ( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dverstop and Anderson</td>
<td>1989</td>
<td>1.7</td>
</tr>
<tr>
<td>Tsang et al.</td>
<td>1996</td>
<td>3.0</td>
</tr>
<tr>
<td>Bour and Davy</td>
<td>1997</td>
<td>1.0 ~ 3.0</td>
</tr>
<tr>
<td>De Dreuzy</td>
<td>2001</td>
<td>0.0 ~ 3.5</td>
</tr>
<tr>
<td>Present study</td>
<td>2015</td>
<td>1.17 ~ 3.39</td>
</tr>
</tbody>
</table>

With the increment of \( D_f \), the longest fracture in a model became increasingly longer and the length - number relation of fractures became more scattered as shown in Fig. 7-5. Table 7-2 and Fig. 7-6 show that \( a \) was in the range of \([1.64, 2.75]\), corresponding to the range of \( D_f \) from 1.3 to 1.8. They followed a linear function that can be expressed by

\[ a = -2.22D_f + 5.61 \quad (7-13) \]
For 2-D DFNs, the value of $D_f$ is in the range of [1, 2], therefore, the corresponding value of $a$ ranges from 1.17 to 3.39 according to Eq. (7-13). These results confirmed that the length of fractures in the proposed model followed a power law length distribution with a linear relation between $a$ and $D_f$. The power law exponent $a$ had a range of [1.17, 3.39], which fitted well with the values reported in previous studies including in-situ measurements as tabulated in Table 7-3 [Dverstop and Anderson 1989; Tsang et al. 1996; Bour and Davy 1997; De Dreuzy 2001a].

7.5.2 Verification of the proposed governing equation

![Graph showing comparisons of in situ measurement data with the prediction of cubic law.](image)

Fig. 7-7 Comparisons of in situ measurement data (after Klimczak et al. [2010]) with the prediction of cubic law. The flow rates were derived with the cubic law from field measurements of apertures and lengths of natural opening-mode fractures from Olson [2003] and Schultz et al. [2008a; b]. Best fitted slopes of the measurement data exhibited an exponent of ($6-D_f$) in the range of [4, 5], corresponding to a $D_f$ in the range of [1, 2].

Based on the datasets collected and measured from opening-mode fractures by Klimczak et al. [2010], the relations between the flow rate and aperture calculated by Eq. (7-9) and the cubic law, and those obtained from in-situ measurements were compared, as shown in Fig. 7-7. The cubic law suggests that the flow rate should be proportional to $e^3$, which only holds for a pure parallel-plate model. In the proposed equation, since the out-of-plane components of fracture geometry and the surface roughness were
introduced, the flow rate became proportional to $e^{-b_D}$. These datasets were measured on different kinds of fractures that essentially have different surface roughness, thereby leading to different $D_T$. By fitting these datasets individually, it was found that the flow rate was proportional to $e$ with the exponents ranging from 4.56 to 4.98, corresponding to the values of $D_T$ ranging from 1.02 to 1.44. By taking the mean value of these datasets, the best-fitted value of the exponent was found to be 4.96, corresponding to a mean $D_T$ of 1.04. Since $D_T$ has a theoretical value bounded between 1 and 2, these results indicated that $D_T$ can efficiently quantify the influence of surface roughness of fractures on their hydraulic permeability. The fractures with $D_T = 1.04$ are rougher than those with JRC = 20, because according to the work of Kulatilake et al. [1995], when JRC = 20, $D_T = 1.02$. The reason is that $D_T$ in this study considered the variations of fracture surface roughness in 3-D space (i.e., x-y and the out-of-plane geometry), revealing more tortuous streamlines of fluid flow than that without considering the out-of-plane geometry. Given the 3-D nature of fractures, application of the cubic law in DFNs may lead to remarkable discrepancies when evaluating their hydraulic permeability.

7.5.3 Effects of the random number

The random number is essential in the Monte Carlo method for generating DFNs with randomly distributed fractures. Different random numbers may result in DFNs with different equivalent permeability although other parameters are unchanged. Here, ten groups of random numbers, each one of which includes three sets of random numbers for generating the center point, the orientation and the length of fractures respectively, were utilized to generate ten large DFNs, and a number of smaller DFNs with different sizes were extracted from them. When analyzing the effect of each set of random number (e.g., the set used for generating the length of fractures), the other two sets of random numbers were fixed. The equivalent permeability can be determined by Darcy’s law in the form of [Zhang et al. 1996]

$$Q = K\nabla h$$

(7-14)

where $K$ is the equivalent permeability of rock fracture networks.
Fig. 7-8 Equivalent permeability of DFNs with $D_f = 1.4$ using different sets of random numbers for generations of (a) the center point, (b) the orientation and (c) the length of fractures, and (d) the corresponding $V_M$ values.
Fig. 7-9 Equivalent permeability of DFNs with $D_f = 1.5$ using different sets of random numbers for generations of (a) the center point, (b) the orientation and (c) the length of fractures, and (d) the corresponding $V_M$ values.
Fig. 7-10 Equivalent permeability of DFNs with $D_f = 1.6$ using different sets of random numbers for generations of (a) the center point, (b) the orientation and (c) the length of fractures, and (d) the corresponding $V_M$ values.
Figs. 7-8, 7-9 and 7-10 show the relations between the equivalent permeability and the side length of DFNs generated by different sets of random numbers ((a) the center point, (b) the orientation, and (c) the length of fractures) when \( D_f = 1.4, 1.5, 1.6 \), respectively. The maximum variance (\( V_M \)) was calculated for the plots at each side length (maximum value minus minimum value), which represents how much the equivalent permeability varies with the random number. When the side length is small (e.g., less than 5 m), the equivalent permeability fluctuates significantly, and \( V_M \) tends to be stable along with the increase of the side length, in which the equivalent permeability keeps almost constant values that no longer change with the further increase of the side length. The side lengths at which \( V_M \) keeps stable values are approximately 17 m for \( D_f = 1.4 \), 6 m for \( D_f = 1.5 \), and 3 m for \( D_f = 1.6 \), which can be considered as the REVs for these DFNs.

The results (Figs. 7-8(d), 7-9(d) and 7-10(d)) show that with the increase of \( D_f \), the \( V_M \) values of equivalent permeability increase. The \( V_M \) values of the set of random numbers for generating fracture lengths are larger than the other two sets, revealing that the equivalent permeability is more sensitive to the fracture length. For a model with a large \( D_f \) (e.g., 1.6), many long fractures that cut through the left to right boundaries exist (Fig. 7-4(e)), which together with the clustered shorter fractures form a more homogeneous distribution of fluid flow paths than the models with smaller \( D_f \). In such a condition, a stable equivalent permeability could be achieved at a small side length. In contrast, in a model with a smaller \( D_f \) (e.g., 1.4), the fluid flow paths are dominated by a few long fractures that require a larger model to achieve a steady state of equivalent permeability (Fig. 7-4(a)). This explains why a larger \( D_f \) leads to a smaller REV. Meanwhile, since the DFNs were generated based on a fractal length distribution model and the aperture was correlated with the fracture length, the equivalent permeability is more sensitive to the length distribution of fractures rather than distributions of the center point and orientation. The fracture length distribution is the first-order parameter that directly influences the geometry of fracture network structure, however, the fracture center point distribution and orientation distribution are the second-order parameters that indirectly affect the geometry of fracture network structure. The dependence comes from the random numbers used to generate fracture lengths or fracture locations.
Fig. 7-11 Relationship between the equivalent permeability with the fractal dimensions $D_T$ and $D_f$.

Fig. 7-11 shows the relationship between the equivalent permeability with $D_T$ for the DFN models with $D_f$ of 1.4, 1.5 and 1.6. The equivalent permeability decreases with increasing $D_T$, because higher values of $D_T$ represent greater tortuosity and resistance to fluid flow in the fractures. The change of $D_T$ from 1 to 2 could result in a change of the magnitude of permeability within 2 orders, revealing that both $D_f$ and $D_T$ have significant impacts on the permeability of DFNs. The changes of equivalent permeability with $D_T$ for the models with different $D_f$ follow exponential laws. The coefficients involved in the exponential equations that best fit the simulated results are primarily dependent on the value of $D_f$. Their mathematical relation needs to be established in future studies by applying fluid flow simulations on DFNs with various combinations of $D_f$, $D_T$ and model size.

7.6 Conclusions

In this study, an expression for generating fracture lengths was theoretically derived corresponding to different situations where the fracture density can be changed by inputting the required fractal dimension $D_f$. Based on the fractal length distribution model, a governing equation for fluid flow in fractures that considers the effects of tortuosity and takes into account the out-of-plane geometry of fractures was proposed.
Finally, the REV size of DFNs and the effect of random number on equivalent permeability were estimated.

The results showed that $D_f$ has a linear relation with the power law exponent $a$, revealing that the fractures in the proposed fractal model also follow power law distributions that are well-accepted distributions in previous studies. The value of $a$ ranges $[1.17, 3.39]$, which is consistent with the value of similar models reported in literature. The flow rate in the proposed governing equation for fluid flow in single fractures is proportional to $e^{6-D_f}$, where $D_f$ ranges $[1, 2]$. This model fits better with several datasets of in-situ measurements than the cubic law in which the flow rate is proportional to $e^3$. By taking into account the out-of-plane geometry of fractures, the proposed governing equation incorporated the 3-D geometry of opening-mode fractures into a 2-D framework to facilitate efficient solutions for the fluid flow in DFNs. The REV decreases with increasing $D_f$, because the flow paths become more homogeneous as increasing number of fractures in a DFN. The random number utilized to generate the fracture length has larger impacts on the calculated equivalent permeability than those for generating the orientation and center point of fractures.

In the proposed model, the geometric characteristics of fracture distribution and the tortuosity of fluid flow in single fractures induced by surface roughness are represented by two fractal dimensions, $D_f$ and $D_T$ respectively. $D_f$ of fracture networks could be obtained by applying fractal dimension estimation methods to field mapping data or artificially generated networks. $D_T$ of fractures can be obtained by translating other representative values (e.g., the joint roughness coefficient) into $D_T$ or by directly applying fractal dimension estimation methods to measured surface topographic data of fractures. This model therefore has high potential to serve as a platform for in-situ estimation of hydraulic properties of fractured rock masses. In the future, this model will be applied to a real site where field mapping data and hydraulic testing results are available to verify its applicability and to improve its performance by introducing other parameters (e.g., aperture distribution and number of intersections) if necessary.
Appendix:

The deviation process of Eq. (7-9) is as follows:

\[
Q = \frac{\rho g}{12\mu} e^3 \frac{Ah}{L_1} W = \frac{\rho g}{12\mu} e^3 \left( \frac{Ah}{L_1} \right) L = \frac{\rho g}{12\mu} e^3 \nabla h \frac{L^2}{L_1}
\]

\[
= \frac{\rho g}{12\mu} e^3 \nabla h \frac{L^2}{e^{1-D_r} L^{D_r}} = \frac{\rho g}{12\mu} e^{2+D_r} \nabla h L^{2-D_r}
\]

\[
= \frac{\rho g}{12\mu} e^{2+D_r} \nabla h \left( e_{max}^{4-2D_r} \right) = \frac{\rho g}{12\mu} e^{2+D_r} \nabla h \left( \frac{1}{\alpha'} \pi \right)^{4-2D_r}
\]

\[
= \frac{4^{4-2D_r} \rho g}{12\mu (\pi \alpha')^{4-2D_r}} e^{6-D_r} \nabla h = \frac{4^{3-2D_r} \rho g}{3\mu (\pi \alpha')^{4-2D_r}} e^{6-D_r} \nabla h
\]

References:


8 A fractal model for predicting EFNP (Part 3)

A multiple fractal model for permeability of dual-porosity media embedded with randomly distributed fractures

8.1 Introduction

Permeability of fractured porous media such as rock masses plays a significant role in many engineering practices, such as underground nuclear waste repositories, CO₂ sequestration, and risk assessment of water inrush in karst tunnels and coal mines [Long et al. 1982; Li et al. 2014; Lang et al. 2014; Liu et al. 2015a, 2016]. The permeability is primarily governed by both rock fractures and rock matrix that contains a bundle of capillaries [Van Golf-Racht 1982]. Porous materials embedded with fractures are called the dual-porosity media [Barrenblatt et al. 1960; Warren and Root 1963], which have attracted much attention during the past several decades [Streltsova 1976; Huyakorn et al. 1983; Zheng and Yu 2012]. The fractures and capillaries inside of rock masses are commonly invisible, therefore it is a tough task to accurately describe their geometric properties, such as location, length, tortuosity, aperture, and diameter [Parashar and Reeves, 2012]. Different approaches have been taken to characterize the geometric properties of rock masses, among which some have shown that both the porous and fractured media show fractal characters, and the fractal approach has achieved great success in the analysis of fluid flow and solute transport behaviors in rock masses [Yu and Cheng 2002; Cai et al. 2012; Miao et al. 2014; 2015a].

For porous media, the size distribution of pore/capillary exhibits fractal properties. Yu and Li [2001] proposed a unified model for describing the fractal characteristics of porous media. A criterion was proposed for determining whether a porous medium could be characterized by the fractal theory and technique. Yu and Cheng [2002] later developed a fractal permeability model for bi-dispersed porous media based on the fractal properties of pores. Their model did not contain any empirical constants, and could reveal more mechanisms of fluid flow and transport than previous models. Wu and Yu [2007] developed a fractal resistance model for fluid flow through porous media, as a function of the pore-throat ratio, porosity, pore/capillary size, fluid velocity, fluid property, and fractal dimension of porous media. Their model revealed the mechanisms of resistance of fluid flow in porous media. Yun et al. [2009] analyzed the plane-radial
and plane-parallel flows for non-compressible Newtonian fluid in porous media, and developed several mathematical expressions for calculating the porosity, flow rate, velocity, and permeability. They also derived the pressure distribution equations for plane-radial and plane-parallel flows, and estimated the pressure and velocity distributions. Cai et al. [2014] developed a generated spontaneous imbibition model, taking into account different sizes and shapes of pores, the tortuosity of imbibition streamlines, and the initial wetting-phase saturation. The proposed model could be used to characterize the spontaneous imbibition of many different porous media. Their results showed that assumption of cylindrical pore shape could produce large discrepancies, and more realistic geometry of pores is required for a better estimation on spontaneous imbibition. Tan et al. [2015] numerically simulated fluid flow in porous media based on their proposed multiple fractal model. The validity of the proposed simulation method was verified against experimental data.

For fractured media, the fracture distributions show fractal characters at both macro-structural (i.e., geometry of the fracture network) [Berkowitz and Hadad, 1997; Bagde et al., 2002; Babadagli, 2002; Kruhl, 2013] and micro-structural (i.e., void geometry of single fractures) [Odling, 1994; Askari and Ahmadi, 2007; Babadagli et al., 2015] levels. Watanabe and Takahashi [1995] used a “fractal geometry” to describe the geothermal water flow in 2-D and 3-D fracture network models. The results showed that the performance of geothermal energy extraction systems could be significantly affected by the fracture density of rock masses. Babadagli and Develi [2003] investigated the fractal dimension of rock fracture surfaces using an automated surface scanning device and a power spectral density (PSD) measurement. The results indicated that the profiles in the loading direction yield higher fractal dimensions, showing anisotropic fractal features. Jiang et al. [2006] developed a 3-D fractal estimation method of roughness characterization for fracture surfaces, which could accurately quantify the evolution of fracture surface roughness during shear and its influence on the hydro-mechanical behaviors of rock fractures. Jafari and Babadagli [2012] used different calculation methods of the fractal dimension to analyze the influence of fracture network characteristics on permeability, and derived a nonlinear multivariable regression to estimate the equivalent fracture permeability based on five independent variables. Miao et al. [2015a] developed a fractal geometry theory and technique to study the seepage characteristics and established analytical expressions for permeability of fracture networks. The proposed expressions systemically considered the influences of a series of fractal dimensions for fracture area, area porosity, fracture density, aperture, the
maximum fracture length, azimuth, and dip angle. Liu et al. [2015b] proposed a fractal model to link the fractal characteristics with the equivalent permeability of fracture networks. The results showed that with the increment of fractal dimension, the variation of permeability changes from approximately five orders of magnitude to less than two orders of magnitude. Considering the tortuosity of fluid flow would reduce the permeability by 17.64 ~ 19.51% when the fractal dimension of nonlinear streamline of fluid flow was 1.018, corresponding to JRC = 20 [Kulatilake et al. 1995], where JRC is the abbreviation of joint roughness coefficient [Barton 1973].

For dual-porosity media, the fluid flow behavior is strongly affected by the fractal properties of both fractures and porous matrix. Xu et al. [2008] proposed a dual-domain model and analyzed the radial flow in the heterogeneous porous media based on fractal and constructal tree networks. They derived analytical expressions for seepage velocity, pressure drop, and local/global permeability, and discussed the transport properties for optimal branching structures. A linear scaling law was found between the global permeability of networks with the fractal dimension of channel diameter. Wang et al. [2011] established a fractal model to estimate the starting pressure gradient for Bingham fluids in dual-porosity media. They concluded that the starting pressure gradient would decrease with the increasing matrix porosity, diameter ratio, permeability, and fractal dimension for mother diameters, and would increase with the increasing yield stress and length ratio. Zheng and Qian [2012] established a fractal permeability model for gas flow through matrix porous media embedded with randomly distributed fractal-like tree networks, and an analytical expression of permeability in dual-porosity media was derived. They found that the dimensionless permeability, defined as the ratio of porous matrix permeability to fracture network permeability, increases with the increasing matrix porosity. Gas permeability of matrix was found much lower than that of rock fractures. Miao et al. [2015b] presented an analytical expression for permeability of dual-porosity media embedded with randomly distributed fractures by assuming that the fracture length follows the fractal scaling law and the flow in each fracture obeys the cubic law. The results showed that the permeability of rock matrix is approximately four orders of magnitude lower than that of fractures, when porosity is larger than 0.1. The ratio of the maximum diameter of pores to the maximum aperture of fractures plays a robust role in the permeability of dual-porosity media.

Although the previous works have investigated the relationships between the permeability and the fractal properties of size distributions of diameter/tortuosity of porous media and of length/roughness of fractures, the fractal dimension for the size
distribution of fracture aperture has not been incorporated into the models. The fracture aperture has significant influences on the permeability of rock masses, since fluid flow in fractures is commonly assumed to obey the cubic law, in which, the permeability of a single fracture changes proportionally with the cubic change of aperture. In addition, most previous studies focused on the calculation of permeability of 2-D fracture networks, in which the influence of fracture width in the out-of-plane orientation is usually neglected by assigning a unit value. This simplification of 3-D physical model to 2-D is a source of biased estimation of permeability. To solve these two problems, in this Chapter, we originally derived a fractal scaling law for the size distribution of fracture aperture, and verified its validity by comparing with the in-situ measurement data. Using the proposed fractal scaling law for aperture distribution, a multiple fractal model for calculating the permeability of 3-D dual-porosity media was proposed by incorporating the effect of fracture width in the out-of-plane orientation. Analytical solutions of flow rate and permeability of dual-porosity media were derived, and the influences of structural parameters involved in the proposed model on the dimensionless permeability were estimated.

8.2 Fractal properties of fractures

Previous studies have reported that the fracture length follows power-law, exponential, and lognormal distributions, influenced by a number of issues such as stress history, linkage of faults, sampling bias, and size of the dataset [Watanabe and Takahashi 1995; Chelidze and Guguen 1990; Sahimi 1993; Andrade et al 2009; Torabi and Berg 2011; Kolyukhin and Torabi 2013; Bonnet et al. 2001; De Dreuzy et al. 2001a, 2001b, 2002]. When the fracture length follows the power-law distribution, the value of exponent is typically located in the range of [1.0, 3.5] [Dverstorp and Anderson 1989; Tsang et al. 1996; Bour and Davy 1997; Liu et al. 2015b]. Miao et al. [2015a] has successfully verified that the fracture length distribution exhibits fractal characteristics and the cumulative number of fractures can be expressed by the following scaling law (Eq. (8-1)), based on the assumption that fracture aperture is linearly correlated with fracture length (Eq. (8-2)) [Stone 1984; Vermilye and Scholz 1995; Schultz et al. 2008].

\[
N'(L' \geq l') = \left(\frac{l'_{\text{max}}}{l'}\right)^{D_f}
\]

(8-1)

\[
e = \beta l'
\]

(8-2)
where \( N' \) is the cumulative number of fractures with a length \( L' \) larger than a constant fracture length \( l' \), \( l'_\text{max} \) is the maximum fracture length, and \( D_f \) is the fractal dimension for the size distribution of fracture length, \( e \) is the fracture aperture, and \( \beta \) is a proportionality coefficient.

As mentioned above, given that the permeability of fractures changes proportionally with the cubic change of aperture, fracture aperture should not be absent in a fractal model for calculating the permeability of fractured rock masses. Therefore, here we attempt to verify that the fracture aperture distribution also follows the fractal scaling law.

Substituting Eq. (8-2) into Eq. (8-1) yields:

\[
N(E \geq e) = \left( \frac{e_{\text{max}}}{e} \right)^{D_e}
\]  

(8-3)

where \( N \) is the cumulative number of fractures with an aperture \( E \) larger than a constant aperture \( e \), \( e_{\text{max}} \) is the maximum fracture aperture, \( D_e \) is the fractal dimension for the size distribution of fracture aperture that equals to \( D_f \).

Differentiating Eq. (8-3) with respect to \( e \) gives:

\[
-dN = D_e e_{\text{max}}^{-D_e} e^{-(D_e+1)} de
\]  

(8-4)

The negative sign on the left side of Eq. (8-4) implies that the number of fractures decreases with the increment of fracture aperture. By setting \( e = e_{\text{min}} \), the total number of fractures, \( N_t \), can be calculated according to the following equation:

\[
N_t(E \geq e) = \left( \frac{e_{\text{max}}}{e_{\text{min}}} \right)^{D_e}
\]  

(8-5)

where \( e_{\text{min}} \) is the minimum fracture aperture. Dividing Eq. (8-4) by Eq. (8-5) yields:

\[
- \frac{dN}{N_e} = D_e e_{\text{min}}^{-D_e} e^{-(D_e+1)} de = f(e) de
\]  

(8-6)

where \( f(e) = D_e e_{\text{min}}^{-D_e} e^{-(D_e+1)} \) is the probability density function of aperture. According to the probability theory, this function has to satisfy the following equation:

\[
\int_{e_{\text{min}}}^{+\infty} f(e) de = \int_{e_{\text{min}}}^{e_{\text{max}}} f(e) de = 1 - \left( \frac{e_{\text{min}}}{e_{\text{max}}} \right)^{D_e} = 1
\]  

(8-7)

By eliminating the constant value 1, Eq. (8-7) becomes:
To hold Eq. (8-8), \( e_{\text{min}} \ll e_{\text{max}} \) should be satisfied, which is the necessary condition for the size distribution of fracture aperture to exhibit fractal properties. In this Chapter, \( e_{\text{min}} / e_{\text{max}} \leq 10^{-3} \) is adopted as a threshold to guarantee the fractal characters of aperture distribution. For the fractures with their apertures located in the range of \([e_{\text{min}}, e_{\text{max}}]\), the cumulative probability, \( R \), can be integrated as:

\[
R(e) = \int_{e_{\text{min}}}^{e} f(e)de = \int_{e_{\text{min}}}^{e} D_e e^{-D_e} e^{-4(D_e + 4)}de = 1 - \left(\frac{e_{\text{min}}}{e_{\text{max}}}\right)^{D_e} \tag{8-9}
\]

Eq. (8-9) implies that as long as \( e \) is in the range of \([e_{\text{min}}, e_{\text{max}}]\), \( R \) is in the range of [0, 1]. By assigning uniformly distributed random numbers between 0 and 1 to \( R \), fracture aperture can be back-calculated by:

\[
e = e_{\text{min}} \left(\frac{1-R}{1-R}\right)^{D_e} \cdot \left(\frac{e_{\text{max}}}{e_{\text{min}}}\right) \left(\frac{e_{\text{max}}}{e_{\text{max}}}ight)^{D_e} \tag{8-10}
\]

For the \( i \)th fracture, the corresponding aperture, \( e_i \), can be calculated using the random number, \( R_i \), as follows:

\[
e_i = \left(\frac{e_{\text{min}}}{e_{\text{max}}}\right) \left(\frac{e_{\text{max}}}{e_{\text{max}}}\right)^{D_e} \left(1-R_i\right)^{D_e} \tag{8-11}
\]

where \( i = 1, 2, 3, \ldots, N_i \), and \( N_i \) is the total number of fractures.

Eq. (8-11) is the expression for fractal aperture distribution in the fracture networks, which is a function of the minimum aperture \( e_{\text{min}} \), the random number \( R_i \), the fractal dimension for size distribution of aperture \( D_e \), and the total number of fractures \( N_i \). Verification of Eq. (8-11) will be presented in the following Section 8.5.1 by comparing the relationship between \( D_e \) and the exponent with those reported in previous works, where the exponent can be calculated from the fracture number – aperture curves.
In the present Chapter, we assumed that the porous matrix is represented by a bundle of capillaries. Fig. 8-1 shows the schematic view of a dual-porosity medium with rough fractures and rough capillaries. Assumptions were made that each fracture and each capillary cut through the model and the fractal dimension for the size distribution of aperture \( D_e \) is calculated from the right side face (y-z plane) of the model in the range of \([1, 2]\). The roughness of fracture surfaces was characterized using line segments appeared on the front face (x-z plane) of the model. The tortuous length, \( L_{tf} \), on the front face is equal to or larger than the straight length, \( L_{tf0} \), of a fracture, which follows the fractal scaling law [Liu et al. 2015b, 2015c]:

\[
L_{tf} = L_{tf0}^{D_f} \frac{1}{(\cos \theta)^{D_f}} e^{1-D_f} = \left( \frac{L_0}{\cos \theta} \right)^{D_f} e^{1-D_f}
\] (8-12)

where \( D_f \) is the fractal dimension of the nonlinear streamline of fluid flow on the front face (x-z plane) with a value in the range of \([1, 2]\), \( \theta \) is the dip angle of fracture plane with respect to the fluid flow direction, and \( L_0 \) is the length of the model. When the fracture surface is smooth (\( D_f = 1 \)) and the dip angle is 0 (\( \theta = 0 \)), the tortuous length...
equals to the straight length \((L_{tf} = L_0)\). Fluid flow in the fractures is assumed to obey the cubic law [Min et al. 2003, 2004; Baghbanan and Jing 2007, 2008; Li et al. 2008].

\[
q_f = \frac{W e^3 \Delta P}{12\mu L_{tf}} \quad (8-13)
\]

where \(q_f\) is the flow rate of each fracture, \(\mu\) is the dynamic viscosity, \(\Delta P\) is the pressure difference between the two tips of a fracture, and \(W\) is the width of a fracture that can be calculated by:

\[
W = \frac{W_0}{\cos \alpha} \quad (8-14)
\]

where \(W_0\) is the width of the model, \(\alpha\) is the angle between the orientation of outcrop of fracture on the right face (y-z plane) with the y-axis.

The total flow rate, \(Q_f\), of all fractures can be obtained by integrating Eq. (8-13) from the minimum aperture to the maximum aperture as follows:

\[
Q_f = \int_{e_{min}}^{e_{max}} q_f dN = \left(\cos \theta \right) \frac{D_f}{\cos \alpha} \frac{W_0}{12\mu} \frac{D_e}{D_{tf} - D_e + 2} \frac{\Delta P}{D_{max}^{e_{max}} e_{max}^{2+D_{f}}} \left[1 - \left(\frac{e_{min}}{e_{max}}\right)^{D_{f} - D_{e} + 2}\right] \quad (8-15)
\]

As mentioned above, \(1 \leq D_{tf} \leq 2, 1 \leq D_e \leq 2\), and \(e_{min}/e_{max} \leq 10^{-3}\); therefore, \(\left(\frac{e_{min}}{e_{max}}\right)^{D_{f} - D_{e} + 2} < 1\). Eq. (8-15) can then be simplified to:

\[
Q_f = \left(\cos \theta \right) \frac{D_f}{\cos \alpha} \frac{W_0}{12\mu} \frac{D_e}{D_{tf} - D_e + 2} \frac{\Delta P}{D_{max}^{e_{max}} e_{max}^{2+D_{e}}} \quad (8-16)
\]

Eq. (8-16) is a new governing equation of fluid flow in fractures, in which the total flow rate is mathematically correlated with the fractal dimensions \(D_{tf}\) and \(D_e\) for size distributions of tortuosity and aperture respectively, the orientations \(\alpha\) and \(\theta\) of the fractures, and the maximum aperture \(e_{max}\). At a constant hydraulic pressure, with the increment of \(D_e\), the fractures will be denser in a model and the total flow rate increases. The rougher the fracture surface represented by \(D_{tf}\), the smaller the flow rate. The total flow rate is linearly correlated with \(e_{max}^{2+D_{e}}\), which implies that the maximum aperture might be the most sensitive parameter in the calculation of flow rate. This is reasonable because fluid flow in natural rock masses is typically governed by a few major flow paths through fractures with large or maximum apertures.

For linear Darcy flow in fractured rock masses, the total flow rate is linearly proportional to the hydraulic pressure gradient as [Liu et al. 2014, 2015b]:

\[
Q_f = \left(\cos \theta \right) \frac{D_f}{\cos \alpha} \frac{W_0}{12\mu} \frac{D_e}{D_{tf} - D_e + 2} \frac{\Delta P}{D_{max}^{e_{max}} e_{max}^{2+D_{e}}} \quad (8-16)
\]
\[ Q_f = \frac{K_f A \Delta P}{\mu L_0} \quad (8-17) \]

where, \( K_f \) is the equivalent permeability of rock fracture networks, \( A \) is the cross-sectional area that equals to \( W_0 H_0 \) as shown in Fig. 8-1. By substituting Eq. (8-16) into Eq. (8-17), \( K_f \) can be back-calculated by

\[ K_f = \frac{\mu Q_f}{A} \frac{\Delta P}{L_0} = \left( \frac{\cos \theta}{\cos \alpha} \right)^{D_p} \frac{L_0^{D_f}}{12 H_0} \frac{D_e}{D_e - D_e + 2} \right) ^{2 + D_p} \quad (8-18) \]

Eq. (8-18) is the analytical expression for the equivalent permeability of rock fracture networks.

### 8.3 Fractal properties of porous matrix

Previous works have assumed that porous media (i.e., rock matrix) are comprised of a bundle of tortuous capillaries and the distribution of diameter of capillaries follows the fractal scaling law as [Yu and Li 2001; Yu 2008]:

\[ N(\Psi \geq \lambda) = \left( \frac{\lambda_{max}}{\lambda} \right)^{D_p} \quad (8-19) \]

where \( N \) is the cumulative number of capillary/pore with its diameter \( \Psi \) larger than a constant diameter \( \lambda \), \( \lambda_{max} \) is the maximum capillary/pore diameter, and \( D_p \) is the fractal dimension for the size distribution of capillary/pore diameter. Fluid flow through a single tortuous capillary is governed by the modified Hagen-Poiseuille equation [Denn 1980; Yu and Cheng 2002]:

\[ q_m = \frac{\pi}{128 \mu} \frac{\Delta P}{L_{tp}} \lambda^{4} \quad (8-20) \]

where \( q_m \) is the flow rate of a single capillary/pore, and \( L_{tp} \) is the tortuous length of a capillary/pore that can be calculated by [Yu and Li 2001; Yu 2008]:

\[ L_{tp} = L_0^{D_p} \frac{\lambda^{1 - D_p}}{\lambda_{min}} \quad (8-21) \]

where \( D_{tp} \) is the fractal dimension of the nonlinear streamline of fluid flow in the capillary/pore with a value in the range of [1, 2]. Substituting Eq. (8-21) into Eq. (8-20) and integrating Eq. (8-20) from the minimum capillary/pore diameter, \( \lambda_{min} \), to the
maximum capillary/pore diameter, \( \lambda_{\text{max}} \), the total flow rate, \( Q_m \), through the porous media can be written as:

\[
Q_m = \frac{\pi L_0^{1-D_p}}{128} \frac{\Delta P}{\mu} \frac{D_p}{L_0} \lambda_{\text{max}}^{3+D_p} \tag{8-22}
\]

According to the Darcy’s law, the equivalent permeability of the matrix, \( K_m \), can be back-calculated as:

\[
K_m = \frac{\mu Q_m}{A} = \frac{\pi L_0^{1-D_p}}{128 W_0 H_0} \frac{D_p}{L_0} \lambda_{\text{max}}^{3+D_p} \tag{8-23}
\]

Eq. (8-23) is the analytical expression for the permeability of porous matrix, which is a function of the model size \((W_0, L_0, \text{and } H_0)\) and the structural parameters \((D_y, D_p, \text{and } \lambda_{\text{max}})\) of the porous matrix.

### 8.4 A multiple fractal model

Since the fracture walls are permeable, the volume of fluid flowing into the fractures is assumed to be equal to that out of the fractures [Chen et al. 2007]. The total flow rate \((Q)\) of a dual-porosity medium is the summation of the flow rates in both fractures and porous matrix (see Eq. (8-24)). Analogously, the total equivalent permeability \((K)\) is the summation of the equivalent permeability of both fractures and porous matrix (see Eq. (8-25)).

\[
Q = Q_m + Q_f = \left(\frac{\cos \theta}{\cos \alpha}\right)^{D_y} \frac{W_0}{12 \mu} \frac{D_y}{D_y - D_e + 2} \frac{\Delta P}{L_0} \lambda_{\text{max}}^{3+D_p} + \frac{\pi}{128} \frac{L_0^{1-D_y}}{\mu} \frac{\Delta P}{L_0} \frac{D_e}{12 H_0} \frac{D_e}{D_y - D_e + 2} \lambda_{\text{max}}^{3+D_p} \tag{8-24}
\]

\[
K = K_m + K_f = \left(\frac{\cos \theta}{\cos \alpha}\right)^{D_y} \frac{L_0^{1-D_y}}{12 H_0} \frac{D_e}{D_y - D_e + 2} \lambda_{\text{max}}^{3+D_p} + \frac{3\pi}{128} \frac{L_0^{1-D_y}}{\mu} \frac{\Delta P}{L_0} \frac{D_e}{D_y - D_e + 2} \lambda_{\text{max}}^{3+D_p} \tag{8-25}
\]

Eqs. (8-24) ~ (8-25) are the analytical expressions for the total flow rate and the total equivalent permeability of dual-porosity media. To quantitatively investigate the individual contribution of fractures and porous matrix to the total equivalent permeability, a dimensionless permeability \((K^*)\) defined as the ratio of the equivalent permeability of porous matrix to that of fractures is utilized as:

\[
K^* = K_m / K_f = \frac{3\pi}{32 W_0} \left(\frac{\cos \theta}{\cos \alpha}\right)^{D_y} \frac{D_e}{3 + D_y - D_e} \frac{2 + D_y - D_e}{D_e} \lambda_{\text{max}}^{3+D_p} \tag{8-26}
\]
Eq. (8-26) is the analytical expression for the dimensionless permeability, which is a function of the multiple fractal dimensions of fractures and porous matrix.

8.5 Results and analysis

8.5.1 Validity of the proposed fractal aperture distribution

In order to obtain the relationship between fracture number and aperture in fracture networks, we primarily assumed that $e_{\text{min}} = 10 \mu m$ and $e_{\text{max}} = 10 \text{ mm}$, which are typical values observed in natural rocks and satisfy the fractal scaling law of aperture distribution as mentioned above (see Eq. (8-8)). By assigning different values from 1.1 to 1.9 to $D_e$, we calculated the total number of fractures according to Eq. (8-5). Then the fractal aperture distribution was generated using Eq. (8-11). The number of fractures with their apertures in the range of $[0, 2]$ mm were counted with an interval of 0.1 mm, leading to 20 times of counting for each $D_e$. The variation of number of fractures with the varying apertures is shown in Fig. 8-2. By fitting these data, it was found that the number of fractures decreases with the increment of aperture, following a power-law function as:

$$ n = a_1 e^{-b_1} \quad (8-27) $$

where $n$ is the number of fractures, $a_1$ is the proportionality coefficient, and $b_1$ is the exponent of the fracture number – aperture relationship.

![Fig. 8-2 Relationship between the number and the aperture of fractures obtained from the proposed fractal aperture distribution.](image)
The results showed that $a_1$ decreases and $b_1$ increases with the increasing $D_e$ as shown in Fig. 8-3. $a_1$ and $b_1$ follow a power-law distribution and a linear distribution, respectively, as shown in Eq. (8-28) and Eq. (8-29).

$$a_1 = 1.30 \times 10^6 D_e^{-1.528}, \quad R^2 = 0.9429$$  \hspace{1cm} (8-28)$$

$$b_1 = 1.86 D_e + 0.30, \quad R^2 = 0.9690$$  \hspace{1cm} (8-29)$$

where, $R^2$ is the correlation coefficient. Substituting Eqs. (8-28) ~ (8-29) into Eq. (8-27) yields:

$$n = 1.30 \times 10^6 D_e^{-1.528} e^{-(1.86 D_e + 0.30)}$$  \hspace{1cm} (8-30)$$

Eq. (8-30) is an empirical expression for the fracture number – aperture relationship, which has taken into account the fractal dimension, $D_e$, for the size distribution of aperture. For all cases in Figs. 8-2 and 8-3, the correlation coefficients, $R^2$, between the calculation results and the fitted curves are larger than 0.90, indicating that Eqs. (8-27) ~ (8-30) perform well in quantifying the fracture number – aperture relationship.
To verify the validity of the proposed fractal aperture distribution, we collected a series of datasets of in-situ measurements from the works of Li et al. [2010] and Xiong [2011], and compared the values of $a_1$ and $b_1$ estimated by this study to their studies. Li et al. [2010] conducted a field study on a compacted, cracked soil ground at a steady moisture condition to investigate the probability distribution of the geometric parameters of cracks. The obtained model from field measurement is shown in Fig. 8-4(a). Xiong [2011] generated 2-D fracture networks (see Fig. 8-4(b)) based on the measurement of fracture parameters on the northern slope of Three Gorges ship lock, Chongqing, China. Their results showed that the fracture aperture follows lognormal distributions. The mean aperture and the variance are 0.49 mm and 0.33 mm respectively for Li’s dataset, and are 0.27 mm and 0.13 mm for Xiong’s dataset. We counted the number of fractures with different values of aperture, and obtained the best-fitted fracture number – aperture curves as shown in Fig. 8-5. The values of $a_1$ and $b_1$ for Li’s dataset are 1.29E–07 and 2.55 respectively, while for Xiong’s dataset are 2.42E–09 and 3.42 respectively. Both datasets show that fracture aperture in the range of $[e_{min}, e_{max}]$ follows power-law distributions, which is congruent with those shown in Fig. 8-2. Here, aperture with the maximum number was selected as $e_{min}$ (= 0.2 mm) for each dataset, because the apertures smaller than this value do not follow the power-law distribution, that make negligible contribution to the total flow rate of fractures. Meanwhile, in Li’s dataset, the maximum aperture is 1.5 mm, which was selected as $e_{max}$ for both datasets to maintain the consistency. The fractal dimension, $D_f$, of the fracture networks of those two datasets was calculated utilizing the box-counting
method. Details regarding to the calculation process are described in Liu et al. [2015b]. After calculation, $D_f = 1.26$ was obtained for the fracture network of Li’s dataset with a side length of 500 mm, and $D_f = 1.56$ was obtained for the fracture network of Xiong’s dataset with a side length of 60 m. Comparison between Eq. (8-1) and Eq. (8-3) indicates that $D_e = D_f$; therefore, $D_e = 1.26$ and $D_e = 1.56$ were obtained for Li’s and Xiong’s datasets, respectively. These values were plotted on Fig. 8-3, and were compared with the fitting curves of Eqs. (8-28) and (8-29). The results showed that although the value of $a_1$ is slightly greater than that of the fitting curve, both of $a_1$ and $b_1$ exhibit good agreement with the predictions of Eqs. (8-28) and (8-29). These results have confirmed that the proposed fractal aperture distribution could properly represent the real distribution of fracture apertures in nature.

![Graphs showing the relationship between number of fractures and aperture for Li’s and Xiong’s models.](image)

(a) Li’s model: $a_1 = 2.55$

(b) Xiong’s model: $a_1 = 3.42$

Fig. 8-5 Relationship between the number and the aperture of fractures obtained from in-situ measurements.

### 8.5.2 Effect of structural parameters

The relationships between dimensionless permeability and structural parameters (i.e., $a$, $\theta$, $D_f$, $D_p$, $D_e$, $D_p$, $e_{\text{max}}$, $\lambda_{\text{max}}$) of the dual-porosity media were investigated and the results are presented in this Section. The values of fundamental parameters involved in the construction of dual-porosity models are tabulated in Table 8-1. When analyzing the sensitivity of one/two parameters, the other parameters were assigned their original values. The dimensionless permeability was calculated using Eq. (8-26) for each case.
Table 8-1 Parameters used for construction of the dual-porosity models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial value</th>
<th>Range</th>
<th>Parameter description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_0$</td>
<td>100 m</td>
<td>-</td>
<td>Length of the model</td>
</tr>
<tr>
<td>$H_0$</td>
<td>100 m</td>
<td>-</td>
<td>Height of the model</td>
</tr>
<tr>
<td>$W_0$</td>
<td>100 m</td>
<td>-</td>
<td>Width of the model</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0 °</td>
<td>0 ~ 80 °</td>
<td>Orientation of fracture width direction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0 °</td>
<td>0 ~ 80 °</td>
<td>Fracture plane dip with respect to the fluid flow direction</td>
</tr>
<tr>
<td>$D_e$</td>
<td>1.5</td>
<td>1.1 ~ 1.9</td>
<td>Fractal dimension for size distribution of fracture aperture</td>
</tr>
<tr>
<td>$D_p$</td>
<td>1.5</td>
<td>1.1 ~ 1.7</td>
<td>Fractal dimension for size distribution of pore/capillary diameter</td>
</tr>
<tr>
<td>$D_{tf}$</td>
<td>1.1</td>
<td>1.1 ~ 1.9</td>
<td>Fractal dimension for nonlinear streamline of fluid flow in a fracture</td>
</tr>
<tr>
<td>$D_{tp}$</td>
<td>1.1</td>
<td>1.1 ~ 1.7</td>
<td>Fractal dimension for nonlinear streamline of fluid flow in a pore/capillary</td>
</tr>
<tr>
<td>$e_{max}$</td>
<td>10 mm</td>
<td>10 ~ 90 mm</td>
<td>Maximum fracture aperture</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>2 mm</td>
<td>1 ~ 7 mm</td>
<td>Maximum pore/capillary diameter</td>
</tr>
</tbody>
</table>

The orientation of fracture plane (i.e., $\alpha$ and $\theta$) could change the flow length and flow area, and would in turn change the permeability of a fracture network. The variations of $K^+$ with varying $\alpha$ and $\theta$ are shown in Fig. 8-6. When $\alpha$ increases from 0° to 80°, $K^+$ diminishes approximately one order of magnitude, whereas when $\theta$ increases from 0° to 80°, $K^+$ increases approximately one order of magnitude. These show that the larger $\alpha$, the larger flow area and the greater conductivity, while the larger $\theta$, the longer flow path and the weaker conductivity. According to Eq. (8-26), $K^+$ is linearly proportional to $\cos \alpha / (\cos \theta)^{0.5}$. In 2-D fracture network models, the fracture width, $W$, and its orientation angle, $\alpha$, were not considered [Long et al. 1982; Zhao et al. 2011; Lei et al. 2014, 2015], which may produce great deviations of estimated permeability from that of the 3-D models since a wide spectrum of values of $\alpha$ and $\theta$ could exist in natural rock masses.
Figs. 8-7 ~ 8-9 show the relationships between $K^+$ and $D_{tf}$, $D_e$, and $e_{max}$, respectively. With the increasing $D_{tf}$ from 1.1 to 1.9, $K^+$ increases approximately three orders of magnitude, indicating that the rougher the fracture surface represented by $D_{tf}$, the weaker the permeability of rock fracture networks, $K_f$, resulting in the larger dimensionless permeability, $K^+$. With the increment of $D_e$ from 1.1 to 1.9, $K^+$ decreases within one order of magnitude, indicating that the denser the fracture networks represented by $D_e$, the better connectivity and the greater $K_f$, and consequently the smaller $K^+$. As shown in Eq. (8-26), the variations of $e_{max}$ and $\lambda_{max}$ would robustly change the magnitude of $K^+$, because the exponents on $e_{max}$ and $\lambda_{max}$ are $2+D_{tf}$ and $3+D_{tp}$, respectively. Fig. 8-9 shows the relationship between $K^+$ and $e_{max}$, where $K^+$ decreases significantly when $e_{max}$ increases in a range of [10, 90] mm. By fitting these data, three expressions for $K^+$ can be empirically obtained as:

$$K^+ = \alpha_{tf} D_{tf}^{7.49}$$  \hspace{1cm} (8-31a)

$$K^+ = \alpha_e D_e^{-1.92}$$  \hspace{1cm} (8-31b)

$$K^+ = \alpha_\omega e_{max}^{-3.10}$$  \hspace{1cm} (8-31c)

where, $\alpha_{tf}$ is the proportionality coefficient of the $K^+ \sim D_{tf}$ relationship, $\alpha_e$ is the proportionality coefficient of the $K^+ \sim D_e$ relationship, and $\alpha_\omega$ is the proportionality coefficient of the $K^+ \sim e_{max}$ relationship.
Fig. 8-7 Variations of $K^+$ with the varying $D_{hf}$ at different $D_{tp}$.

Fig. 8-8 Variations of $K^+$ with the varying $D_{e}$ at different $D_{tp}$.

Fig. 8-9 Variations of $K^+$ with the varying $e_{max}$ at different $\lambda_{max}$. 
As shown in Fig. 8-7, when increasing \( D_{tp} \) from 1.1 to 1.7, \( \alpha_{tf} \) decreases around two orders of magnitude, indicating that the stronger tortuosity of capillary/pore represented by \( D_{tp} \) would further reduce the permeability of porous matrix, \( K_p \), and \( K^+ \). As shown in Figs. 8-8 and 8-9, \( \alpha_e \) increases around 1.29 times when \( D_p \) varies from 1.1 to 1.7, and \( \alpha_{eo} \) varies more than three orders of magnitude when \( \lambda_{max} \) varies in a range of [1, 7] mm. By fitting these data, another three power-law functions were obtained as:

\[
\alpha_{tf} = 1.86E - 08D_{tp}^{8.10} \quad (8-32a)
\]

\[
\alpha_e = 3.16E - 08D_{p}^{0.58} \quad (8-32b)
\]

\[
\alpha_{eo} = 1.39E - 03\lambda_{max}^{4.10} \quad (8-32c)
\]

Substituting these equations into Eq. (8-31) leads to the empirical expressions for \( K^+ \) as follows:

\[
K^+ = 1.86E - 08D_{tp}^{8.10}D_{tf}^{7.49} \quad (8-33a)
\]

\[
K^+ = 3.16E - 08D_{p}^{0.58}D_{e}^{1.92} \quad (8-33b)
\]

\[
K^+ = 1.39E - 03\lambda_{max}^{4.10}e_{max}^{3.10} \quad (8-33c)
\]

In Eq. (8-33a), the exponent of \( D_{tf} \) (7.49) is slightly smaller than that of \( D_{tp} \) (8.10); therefore, \( K^+ \) is less sensitive to \( D_{tf} \) than that to \( D_{tp} \). Eq. (8-33b) implies that \( K^+ \) is more sensitive to \( D_{e} \) with an exponent of 1.92 than that to \( D_{p} \) with an exponent of 0.58. Eq. (8-33c) shows that \( \lambda_{max} \) has a more significant influence on \( K^+ \) than that of \( e_{max} \), because the exponent of \( \lambda_{max} \) (4.10) is larger than that of \( e_{max} \) (3.10).

8.5.3 Effect of aperture distribution

The present Chapter derived an expression for fractal aperture distribution (Eq. (8-11)), and an expression for estimating the dimensionless permeability (Eq. (8-26)). However, a great number of previous studies have confirmed that the lognormal aperture distribution extensively exists in nature [Dverstorp and Andersson 1989; Cacas et al. 1990; De Dreuzy et al. 2001b, 2002; Li and Zhang 2010]. To gain an insight into the effect of the type of aperture distribution on the dimensionless permeability, models in which the apertures follow lognormal distribution were established, and the
calculated dimensionless permeability was compared with those of the models with fractal aperture distribution.

In the lognormal distribution model, the mean aperture was obtained by averaging the apertures generated by using Eq. (8-11), with a variance of 0.1%. Other parameters involved in model construction have identical values to those for the models of fractal aperture distribution as shown in Table 8-1. Since all fractures were assumed cutting through the model as shown in Fig. 8-1, the total flow rate of fractures, \( Q_f \), could be calculated by summing the flow rate of each fracture according to Eq. (8-13). Then substituting the calculated \( Q_f \) into Eq. (8-26), the dimensionless permeability of model with apertures following lognormal distribution could be estimated. When the aperture was lognormally distributed, its value depended significantly on the variations of random number; therefore, we randomly generated ten sets of apertures for each \( D_e \), and calculated the average \( K^+ \) to obtain the best fitted curve. Liu et al. [2015b] characterized fluid flow in fracture networks with the value of \( D_f \) in the range of [1.3, 1.6], below which the connectivity of fracture networks is too poor to effectively conduct fluid flow, and above which the rock masses are so fragmented that rarely exist in nature. Chelidze and Guguen [1990] used the box-counting method to calculate the \( D_f \) of fracture networks described by Nolen-Hoeksema and Gordon [1987], and found that for the studied 2-D fracture network, \( D_f = 1.6 \). Barton and Zoback [1992] studied the fractal properties of fracture networks with fracture length spanning ten orders of magnitude, and concluded that \( D_f = 1.3 \sim 1.7 \). Therefore, in the present Chapter, we investigated the variation of \( K^+ \) with \( D_e (= D_f) \) ranging from 1.3 to 1.7, to cover the general patterns of aperture distribution of fracture networks.

Fig. 8-10 shows the comparison of \( K^+ \) between the calculations of models based on the fractal aperture distribution and the lognormal aperture distribution. For both distributions, with the increment of \( D_e \), \( K^+ \) decreases following power-law functions, and the correlation coefficients of the best fitted curves, \( R^2 \), are larger than 0.93. The exponent on \( D_e \) (1.92) for the models with fractal aperture distribution is smaller than that (6.92) with lognormal aperture distribution, indicating that \( K^+ \) is more sensitive to \( D_e \) when the aperture is lognormally distributed. The difference of the magnitude of \( K^+ \) calculated by the two kinds of distributions is generally within one order of magnitude. They have especially close values when \( D_e \) is smaller than 1.5. The main reason might be that in the fracture networks with fractal aperture distribution, the aperture is truncated within a range of [0.01, 10] mm, while in the fracture networks with lognormal aperture distribution, a number of large aperture values typically exist
especially at large $D_e$ that vary significantly with different random numbers and subsequently change the permeability of fracture networks [Baghbanan and Jing 2007]. The large aperture values would give rise to the permeability of fracture networks and subsequently reduce the value of $K^+$. Nevertheless, as a first order estimation, the calculated $K^+$ of the fracture networks based on the proposed fractal aperture distribution has close magnitude with those of models with widely accepted lognormal aperture distribution, especially when $D_e$ is smaller than 1.5.

![Fig. 8-10 Comparison of calculated $K^+$ between the models based on the fractal aperture distribution and the lognormal aperture distribution.]

**8.4 Conclusions**

In the present Chapter, a multiple fractal model for estimating the permeability of dual-porosity media embedded with randomly distributed fractures was established and the validity of proposed fractal aperture distribution was verified by comparing with the in-situ measurement results reported in literature. Analytical expressions for the fractal aperture distribution ($e_i$), the total flow rate ($Q$), the total equivalent permeability ($K$), and the dimensionless permeability ($K^+$) were proposed, where the dimensionless permeability is defined as the ratio of the permeability of porous media ($K_m$) to that of fracture networks ($K_f$). The proposed multiple fractal model involves a group of geometric parameters that do not contain any empirical constants. The relationships between the dimensionless permeability with the structural parameters (i.e., $\alpha$, $\theta$, $D_{lf}$, $D_{tp}$, $D_{tp}$, $D_{tf}$, $D_{tp}$, $D_{tp}$).
\(D_e, D_p, e_{\text{max}}, \lambda_{\text{max}}\) of dual-porosity media have been systematically investigated. The results showed that the dimensionless permeability is explicitly correlated with these structural parameters, following power-law functions. The dimensionless permeability is more sensitive to the fractal dimension for size distribution of fracture aperture \((D_e)\) than to that for size distribution of pore/capillary diameter \((D_p)\). The maximum pore/capillary diameter \((\lambda_{\text{max}})\) has a greater impact on the dimensionless permeability than that of the maximum fracture aperture \((e_{\text{max}})\). The predicted dimensionless permeability of the models based on the proposed fractal aperture distribution has close values with those based on the widely accepted lognormal aperture distribution, especially when the fractal dimension for the size distribution of fracture aperture is less than 1.5.

In practices, the parameters associated with pore/capillary could be estimated based on samplings and measurement of intact rocks, and the parameters for fractures could be estimated based on field mapping on outcrops and/or boring holes. Their fractal dimensions could be calculated using typical fractal estimation methods such as the box-counting method. Then the permeability of fractured rock masses could be easily assessed using the proposed method as a first order estimation.

This Chapter presents a multiple fractal model for permeability of fluid flow in dual-porosity media. The two-phase flow and multi-phase flow through dual-porosity media are also important issues that will be explored in our next works.

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9 Summaries

The nonlinear flow and fractal properties of rock fracture networks are discussed in this doctoral thesis. The main conclusions are summarized in this chapter.

In Chapter 2, the equivalent permeability of fracture networks is strongly correlated to the geometric properties of fractured rock masses, e.g., length, aperture, orientation, and connectivity of fractures. It is more difficult to reach a REV size when the aperture of the fracture is linked with the fracture length, comparing with that with an identical mean aperture. Fractal dimension is a useful tool to represent the geometric properties and can be utilized to predict the value of equivalent permeability. The stress applied on the boundary of the DFNs would significantly affect the equivalent permeability tensor and the REV size. In the dual-porosity models, the fracture network is usually more permeable than the rock matrix. The permeability tensor is utilized to represent the anisotropy of the fractured rock masses, and is affected significantly by the scale effects of the DFN models. Since the 2-D fracture network is just a cut plane of the 3-D model, the equivalent permeability of the 2-D DFN models is usually underestimated by a factor of 3 to up to 3 orders of magnitude.

In Chapter 3, the derivations of the Navier-Stokes equations and its simplification forms, such as Stokes equations, Reynolds equations, and cubic law were presented. According to the geometries of fracture surface and the flow rate, the associated governing equations should be selected.

In Chapter 4, the mechanical apertures of experimental models can be measured using the visualization techniques with a CCD camera. The numerical simulations examined the accuracy of the measured mechanical apertures, and the flow tests verified the reliability of the FVM code by solving the Navier-Stokes equations to quantify the hydraulic properties of fluid flow at fracture intersections. The flow tests show that the nonlinearity of fluid flow through crossed fracture models is potentially caused by fracture intersection, aperture variation, hydraulic gradient, inflow/outflow tanks, and fracture surface roughness. The numerical simulations based on the large scale models show that with the increment of the hydraulic gradient, the ratio of the flow rate to the hydraulic gradient, \( Q/J \), decreases and the relative deviation, \( \delta \), increases, due to the gradually increasing inertial effects. When taking account of the fracture surface roughness with JRC = 0 ~ 20, the values of \( Q/J \) and \( \delta \) would reduce by 0 ~ 26.55% and increase by 0 ~ 4.24%, respectively. The values of \( Q/J \) vary significantly when the radius of the truncating circles, \( R_t \), is less than 50 mm, and gradually become stable for
$R_r > 200$ mm. In the linear flow regimes, i.e., $J < 10^{-3}$, $Q/J$ decreases with the increment of $R_r$, yet in the nonlinear flow regimes, i.e., $J > 10^{-2}$, $Q/J$ increases with the increment of $R_r$. The influences of intersecting angles of the two crossed fractures can be negligible on $Q/J$ and $\delta$ in spite of the value of $J$. Their influences on the normalized flow rate, $R_a$, and the ratio of the hydraulic aperture to the mechanical aperture, $e/E$, can also be neglected when $J < 10^{-3}$, however, the values of $R_a$ and $e/E$ are quite different for different intersecting angles when $J > 10^{-2}$ and their influences have to be considered. By analyzing the relations of $Q \sim J$, $R_a \sim J$, and $e/E \sim J$ based on the crossed fracture models, it can be found that they all have linear relationships where the inertial effects can be negligible, weak nonlinear relationships where the inertial effects are weak, and strong nonlinear relationships where the inertial effects are strong, corresponding to $J < 10^{-3}$, $10^{-3} < J < 10^{-2}$, and $J > 10^{-2}$, respectively. By fitting the results of the numerical simulations based on 30 cases, an empirical expression of calculating $e/E$ was proposed, which was a function of $J$ and $E/R_r$. The predictions of the proposed empirical expression agree well with the numerical simulation results with $J = 10^{-5} \sim 10^0$ and $E/R_r = 0.002 \sim 0.2$ for different combinations of inlet and outlets, indicating that this expression is sufficiently reliable to characterize hydraulic properties of fluid flow at fracture intersections. Using the proposed expression, $R_r$ was extended to 10000 mm to quantify the scale effects and the results depicted that for $E/R_r > 10^{-2}$, $e/E$ varies significantly and the scale effect has to be considered, while when $E/R_r < 10^{-3}$, the scale effect is less significant and can be neglected. It can be inferred that one of the conservative conditions to apply the local cubic law at fracture intersections is: $J < 10^{-3}$, and $E/R_r < 10^{-3}$, and $\text{JRC} = 0$. However, further works are needed to verify whether this criteria is also applicable for fluid flow in DFNs that contain hundreds/thousands of fracture intersections and segments.

In Chapter 5, the nonlinearity of fluid flow in DFNs originates from the inertial effects enhanced by surface roughness and intersections of fractures at high hydraulic gradients, which are positively correlated with the Reynold numbers. Fluid flow in DFNs can be quantified by Forchheimer’s law, in which $A Q$ and $B Q^2$ represent the linear and nonlinear components, respectively. At sufficiently low hydraulic gradients or Reynold numbers, the nonlinear term $B Q^2$ drops out and Forchheimer’s law reduces to the cubic law. Transition from the linear regime to the nonlinear regime would be found at a lower hydraulic gradient in a DFN with rougher fracture surfaces, greater mechanical apertures, and a larger number of intersections. The critical hydraulic gradient is most sensitive to the mechanical aperture, followed by the number of
intersections and surface roughness. The reasons are that (1) the Reynolds number in each fracture in a DFN changes proportionally with the cubic change of the mechanical aperture, and (2) the intersection and surface roughness that are the sources of the frictional loss and inertial efforts will only have significant influences at large Reynolds numbers. Predicted values by the empirical equations fit well with simulation results of DFNs with correlated fracture length - aperture distributions, after introducing a parameter $\lambda$ to account for the reduction of aperture induced by aperture variations. When the imposed hydraulic gradient on a DFN is below the predicted critical hydraulic gradient, application of the cubic law in conjunction with necessary modifications considering aperture reductions induced by surface roughness would give reasonable solutions. When the hydraulic gradient is larger than the critical value, one may apply Forchheimer’s law to the studied models using predicted values of $A$ and $B$ based on the proposed equations, which will profoundly improve the reliability of obtained pressure – flow rate relationships, compared with those made by linear predictions.

In Chapter 6, the correlation of fracture number and fracture length based on the proposed fractal length distribution in this study agrees well with reported values from the literature, which confirmed the reliability of the proposed length distribution approach. Comparisons of the fractal dimension $D_f$ between the values calculated using the box-counting method and the theoretical values agree with each other, which verified the validity of the fractal DFN models that were developed using the proposed fractal length distribution. When $D_f$ is small (e.g., less than 1.5), fluid flow mainly occurs in a few long fractures that are subparallel to the flow direction and particularly in the fractures that intersect the inlet and outlet boundaries of the models. When $D_f$ exceeds a certain value (e.g., 1.5), the flow rate distribution becomes more homogeneous, and shorter non-persistent fractures dominate the preferential flow paths. The equivalent permeabilities of models generated using different random numbers vary significantly with changes of $D_f$ when $D_f$ is small (e.g., less than 1.5), and they become more stable when $D_f$ is relatively large (e.g., greater than 1.6). This behavior is consistent with the observations of the flow paths, which show that the models become more homogeneous at larger values of $D_f$. Therefore, a mathematical expression between the equivalent permeability $K$ and the fractal dimension $D_f$ (e.g., the exponential relationship presented in this study) can be expected to be applicable for models with large values of $D_f$. For models with small values of $D_f$, other parameters, such as connectivity, should be taken into account to improve the accuracy of the predictions. Compared with the parallel plate model, the maximum deviation of the
calculated flow volume that considers the effect of tortuosity \((D_T)\) can be as high as 19.51\% when \(D_T = 1.018\), which corresponds to a JRC value of 20. These results show that both the geometric characteristics of the fracture distributions and the geometric characteristics (surface roughness) of single rock fractures (the source of tortuosity) have significant influence on the hydraulic behavior of fracture networks.

In Chapter 7, an expression for generating fracture lengths was theoretically derived corresponding to different situations where the fracture density can be changed by inputting the required fractal dimension \(D_f\). Based on the fractal length distribution model, a governing equation for fluid flow in fractures that considers the effects of tortuosity and takes into account the out-of-plane geometry of fractures was proposed. Finally, the REV size of DFNs and the effect of random number on equivalent permeability were estimated. The results showed that \(D_f\) has a linear relation with the power law exponent \(a\), revealing that the fractures in the proposed fractal model also follow power law distributions that are well-accepted distributions in previous studies. The value of \(a\) ranges [1.17, 3.39], which is consistent with the value of similar models reported in literature. The flow rate in the proposed governing equation for fluid flow in single fractures is proportional to \(e^{6-D_T}\), where \(D_T\) ranges \([1, 2]\). This model fits better with several datasets of in-situ measurements than the cubic law in which the flow rate is proportional to \(e^3\). By taking into account the out-of-plane geometry of fractures, the proposed governing equation incorporated the 3-D geometry of opening-mode fractures into a 2-D framework to facilitate efficient solutions for the fluid flow in DFNs. The REV decreases with increasing \(D_f\), because the flow paths become more homogeneous as increasing number of fractures in a DFN. The random number utilized to generate the fracture length has larger impacts on the calculated equivalent permeability than those for generating the orientation and center point of fractures.

In Chapter 8, a multiple fractal model for estimating the permeability of dual-porosity media embedded with randomly distributed fractures was established and the validity of proposed fractal aperture distribution was verified by comparing with the in-situ measurement results reported in literature. Analytical expressions for the fractal aperture distribution \((e_i)\), the total flow rate \((Q)\), the total equivalent permeability \((K)\), and the dimensionless permeability \((K^+)\) were proposed, where the dimensionless permeability is defined as the ratio of the permeability of porous media \((K_m)\) to that of fracture networks \((K_f)\). The proposed multiple fractal model involves a group of geometric parameters that do not contain any empirical constants. The relationships between the dimensionless permeability with the structural parameters (i.e., \(\alpha, \theta, D_{sf}, D_{tp}\), \(K_m\)),
$D_e$, $D_p$, $e_{\text{max}}$, $\lambda_{\text{max}}$) of dual-porosity media have been systematically investigated. The results showed that the dimensionless permeability is explicitly correlated with these structural parameters, following power-law functions. The dimensionless permeability is more sensitive to the fractal dimension for size distribution of fracture aperture ($D_e$) than to that for size distribution of pore/capillary diameter ($D_p$). The maximum pore/capillary diameter ($\lambda_{\text{max}}$) has a greater impact on the dimensionless permeability than that of the maximum fracture aperture ($e_{\text{max}}$). The predicted dimensionless permeability of the models based on the proposed fractal aperture distribution has close values with those based on the widely accepted lognormal aperture distribution, especially when the fractal dimension for the size distribution of fracture aperture is less than 1.5. In practices, the parameters associated with pore/capillary could be estimated based on samplings and measurement of intact rocks, and the parameters for fractures could be estimated based on field mapping on outcrops and/or boring holes. Their fractal dimensions could be calculated using typical fractal estimation methods such as the box-counting method. Then the permeability of fractured rock masses could be easily assessed using the proposed method as a first order estimation.

The construction and maintenance of the underground projects, such as underground nuclear waste repositories, CO$_2$ sequestration and enhanced geothermal systems, are going on development, and the author expects that the researches during the doctoral course could contribute to the evolving discipline.