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Effects of Moving Core Velocity and Viscous Dissipation on Fully Developed Laminar Heat Transfer in Concentric Annuli

by

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Fully developed laminar heat transfer of a Newtonian fluid in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation of the flowing fluid. The effects of the relative velocity of a moving core and viscous dissipation on the temperature distributions and Nusselt numbers at the tube walls have been discussed.

1. Introduction

Problems involving fluid flow and heat transfer with an axially moving core of solid body or fluid in an annular geometry can be found in many manufacturing processes, such as extrusion, drawing and hot rolling, etc. In such processes, a hot cylindrical rod continuously exchanges heat with the surrounding environment. For such cases, the fluid involved may be Newtonian or non-Newtonian and the flow situations encountered can be either laminar or turbulent.

Another example which involves viscous dissipation effect is seen in microchannel cooling using liquid coolant\(^1\). The increasing scales of circuit integration of electronic components accompanied by reducing feature size of integrated circuit (IC) chips have increased the problems associated with cooling. Laminar regime viscous dissipation problems are widely involved in microchannel heat transfer such as efficient cooling techniques for IC chips.

In the previous report\(^1\), exact solutions of the momentum and energy equations were obtained for fully developed laminar flow of Newtonian fluid flowing for an annular geometry. There the effects of viscous dissipation on heat transfer was omitted.

In this study the energy equation with the viscous dissipation term was exactly solved for the boundary conditions of constant wall heat flux at one tube wall with the other insulated.

Nusselt numbers at the inner and outer tubes were presented for the wide ranges of parameters: the radius ratio, the relative core velocity and Brinkman number.

Nomenclature

- \(a\) radius ratio \(= R_i/R_o\)
- \(\eta\) dimensionless axial coordinate \(= z/(2(R_o-R_i)Pe)\)
- \(\theta\) dimensionless temperature
- \(\mu\) viscosity
- \(\nu\) kinematic viscosity \(= \mu/\rho\)
- \(\rho\) density
- \(B\) \(= (a^2 - 1) / \ln a\)
- \(B^*\) \(= uB\)
- \(E\) \(= (a^2 - (B/2))(a^2 - 1)\)
- \(M\) \(= 1 + a^2 - B\)

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2. Analysis

The physical model for the analysis is shown in Fig. 1. The outer tube is stationary and the core tube is axially moving at a constant velocity. The assumptions used in the analysis are:

1. The flow is incompressible and steady-laminar, and fully developed, hydrodynamically and thermally.
2. The fluid is Newtonian and physical properties are constant.
3. The body forces and axial heat conduction are neglected.

2.1 Fluid Flow

The momentum equation together with the assumptions described above is

\[
\frac{1}{r} \frac{d}{dr} \left( ru \right) = \frac{1}{\mu} \frac{dP}{dz}
\]

The boundary conditions are:

\[
\begin{align*}
  u &= U & \text{at } r &= R_i \\
  u &= 0 & \text{at } r &= R_o
\end{align*}
\]

Solving Eq.(1) with Eq.(2), \( u \) is obtained as

\[
u = \frac{2um}{M} \left( 1 - EU^* \right) \left( 1 - r^2 + B^* ln r^* \right)
\]

where \( u_m \) is the average velocity, given as

\[
u_m = \frac{1}{\pi (R_o^2 - R_i^2)} \int_{R_i}^{R_o} u \cdot 2 \pi rdr
\]

\[
u_m = \frac{R_o^2}{4\mu} \left( - \frac{dP}{dz} \right) M + EU.
\]

The dimensionless fluid velocity, \( u^* \), is written as

\[
\frac{dP}{dz} = \frac{8R_i}{R_o(1 + \alpha)} \left( 1 + \frac{R_o}{R_j} B r_j \cdot V_B \right)
\]

where

\[
\eta = \frac{z}{2(R_o - R_j) Pe}
\]

\[
Br_j = \frac{\mu u_m^2}{(1 - \alpha) R_o q_j} \quad \text{(Brinkman number)}
\]

\[
V_B = (1 - \alpha) \left[ \int_{a}^{1} r^* \left( \frac{du^*}{dr^*} \right)^2 dr^* \right]
\]

\[
= 4(1 - \alpha)(1 - \alpha^2) \left[ \frac{(1 - EU^*)^2}{M} \right] \times (1 + \alpha^2 - 2B^* + B^* \omega)
\]
The energy equation and the boundary conditions may be expressed in dimensionless form as
\[
\frac{1}{r^* \, d^*} \, \left( r^* \, \frac{d}{dr^*} \right) \left( \frac{d}{dr^*} \right) = \left[ \frac{2S}{(1+a) \,(1-a)^2} \right] u^* + B rij \cdot V \tag{17}
\]

Case A:
\[
\begin{align*}
\frac{d\hat{\theta}}{dr^*} &= \frac{1}{a-1} \quad \text{at} \quad r^* = a \\
\frac{d\hat{\theta}}{dr^*} &= 0 \quad \text{at} \quad r^* = 1
\end{align*}
\]

Case B:
\[
\begin{align*}
\frac{d\hat{\theta}}{dr^*} &= 0 \quad \text{at} \quad r^* = a \\
\frac{d\hat{\theta}}{dr^*} &= \frac{1}{1-a} \quad \text{at} \quad r^* = 1
\end{align*}
\]

where \( \alpha \) is for Case A and \( \alpha = 1 \) for Case B.

The parameter \( V \) is related to viscous dissipation, defined as
\[
V = \frac{\nu \, \alpha B^*}{(1+a)^2} \tag{18}
\]

Nusselt numbers, \( Nu_{ij} \), on the inner and outer walls are calculated as
\[
Nu_{ij} = \left[ \sum \frac{\alpha}{(1+a)^2} \right] \tag{19}
\]

The dimensionless bulk temperature, \( \theta_b \), is defined as
\[
\theta_b = T_b / \left[ \sum \frac{\alpha}{(1+a)^2} \right] \tag{20}
\]

The temperature difference is obtained as
\[
\theta_i - \theta_b = \left( \frac{2}{1-a^2} \right) \int_a^1 \, r^*(\theta_i - \theta) \, dr^* \tag{21}
\]

The temperature difference is obtained as
\[
\theta_i - \theta_b = \left( \frac{2}{1-a^2} \right) \int_a^1 \, r^*(\theta_i - \theta) \, dr^* \tag{22}
\]

Case A: In this case, Eq.(13) with \( j = i \) is written as
\[
\frac{d\theta_i}{d\eta} = \frac{8 \, a}{(1+a)} \left( \frac{1}{1-a} \right) \frac{a}{Br_i} \cdot \frac{V_i}{k} \tag{23}
\]

The temperature difference is obtained as
\[
\theta_i - \theta_b = \left( \frac{2}{1-a^2} \right) \int_a^1 \, r^*(\theta_i - \theta) \, dr^* \tag{24}
\]

The temperature difference is obtained as
\[
\theta_i - \theta_b = \left( \frac{2}{1-a^2} \right) \int_a^1 \, r^*(\theta_i - \theta) \, dr^* \tag{25}
\]

The temperature difference is obtained as
\[
\theta_i - \theta_b = \left( \frac{2}{1-a^2} \right) \int_a^1 \, r^*(\theta_i - \theta) \, dr^* \tag{26}
\]

The dimensionless inner wall temperature and the coefficients \( c_1, c_2, c_3, c_4, c_5 \) and \( c_6 \) are
\[
c_1 = \frac{a}{(1+B^*)} \left[ \frac{(1-a^2)}{(1+a)^2} \right] + 4Br_i
\]

\[
c_2 = \frac{1}{4} \left[ \left( \frac{a}{1+a} \right) \left( \frac{1-a^2}{1-a^2} \right) \right]
\]

\[
c_3 = \left[ \frac{a B^*}{(1+a)^2} \right]
\]

\[
c_4 = \frac{1}{4} \left[ \left( \frac{a}{1+a} \right) \left( \frac{1-a^2}{1-a^2} \right) \right]
\]

\[
c_5 = \left[ \frac{a B^*}{(1+a)^2} \right]
\]

\[
c_6 = \frac{1}{4} \left[ \left( \frac{a}{1+a} \right) \left( \frac{1-a^2}{1-a^2} \right) \right]
\]
\[ g_{41} = \left( \frac{1}{2} - \frac{2\omega}{3} \right) c_2 + \left[ \frac{2 - 3\omega}{3B} \right] c_3 + \left[ \frac{(1 - 2\omega)}{B^2} \right] c_4 \]

**Case B:** Equation (13) with \( j = 0 \) is written as

\[ \frac{d\theta_o}{d\eta} = \frac{8}{(1 + \alpha)} (1 + B_{\alpha} \cdot V_B) \]

The temperature difference is obtained as

\[ \theta_o - \theta = \left[ \frac{1 - EU^*}{M} \right] \left[ d_1 \alpha^2 + d_2 \alpha^4 + d_3 \alpha^2 \ln \alpha^* + d_4 (\ln \alpha^*)^2 + b_1 \ln \alpha^* + b_2 \right] \]

where \( \theta_o \) is the dimensionless outer wall temperature and the coefficients \( d_1, d_2, d_3, d_4, b_1 \) and \( b_2 \) are

\begin{align*}
    d_1 &= \frac{(1 - B^*)}{(1 + \alpha)(1 - \alpha^2)} + 4B_{\alpha} \\
    &\times \left[ \frac{(1 + \alpha^2) + (w - 2 + (w - 1)(2\alpha^2 - B^*)B^*}{|1 + (2\alpha - 1)\alpha^2 - B^*|^2} \right] \\
    d_2 &= \frac{-1}{4}\left[ \frac{1}{(1 + \alpha)(1 - \alpha^2)} \right] \\
    &- \frac{B_{\alpha}}{1 + (2\alpha - 1)\alpha^2 - B^*} \left[ 2 + 2\alpha a^2 + (w - 3)B^* \right] \\
    d_3 &= \frac{B^*}{(1 + \alpha)(1 - \alpha^2)} \\
    &+ 4B_{\alpha} \left[ \frac{1 + \alpha^2 + (w - 2)B^*}{|1 + (2\alpha - 1)\alpha^2 - B^*|^2} \right] \\
    d_4 &= -2B_{\alpha} \left[ \frac{B^*}{1 + (2\alpha - 1)\alpha^2 - B^*} \right] \\
    b_1 &= -[(2d_1 + d_3)\alpha^2 + 4d_2\alpha^4 + 2d_3\alpha^2 \ln \alpha + 2d_4 \ln \alpha] \\
    b_2 &= -(d_1 + d_2) \\
\end{align*}

NuSSelt number is determined as

\[ Nu_{\infty}^{(2)} = 2\left[ 1 + (2\omega - 1)\alpha^2 - B^* \right]^{\frac{1}{2}} / G_{\infty} \]

where

\[ G_{\infty} = g_{51} + g_{52} \alpha^2 + g_{53} \alpha^4 + g_{54} \alpha^6 \]

\[ g_{51} = -\left( \frac{1}{3} - \frac{B^*}{4} \right) d_1 - \left( \frac{1}{6} - \frac{B^*}{9} \right) d_2 + \left( \frac{5}{36} - \frac{B^*}{8} \right) d_3 - \left( \frac{7}{8} - \frac{3B^*}{2} \right) d_4 \\
    + \left( \frac{3}{4} - B^* \right) b_1 - (1 - B^*)b_2 \]

\[ g_{52} = -\left( \frac{1}{3} - \frac{B^*}{4} \right) d_1 - \left( \frac{1}{6} - \frac{B^*}{9} \right) d_2 + \left( \frac{5}{36} - \frac{B^*}{8} \right) d_3 - \left( \frac{7}{8} - \frac{3B^*}{2} \right) d_4 \\
    + \left( \frac{3}{4} - B^* \right) b_1 - (1 - B^*)b_2 \]

3. Result and Discussion

3.1 Effects of moving core velocity

The effects of the relative velocity of the moving core \( U^* \) on the velocity profiles across the annulus for the cases of \( \alpha = 0.2, 0.5 \) and 0.8 are shown in Fig.2. It is seen clearly that the profiles of the fluid velocity are deformed by the moving core with a relative velocity \( U^* \). For \( U^* < 0 \), the velocity profile is parabolic having a larger maximum value with increasing values of \( \alpha \).

The velocity gradient and parameter \( V \) govern the heat transfer with viscous dissipation through Eq.(16). The behaviors of velocity gradient and \( V \) are shown in terms of \( (1 - \alpha)^2 (du^*/dr^*)^2 \) and \( (1 - \alpha)^2 V \), respectively, in Figs.3 and 4, for \( \alpha = 0.2, 0.5 \) and 0.8. The absolute values of velocity gradient and \( V \) become larger near the moving wall (\( \xi = 0 \)) with a decrease in \( U^* \).

Figure 5 shows the magnitude of parameter \( V_n \). It is seen that \( V_n \) increases with a decrease in \( U^* \).

In Fig.6, the magnitude of \( d\theta_o / d\eta \) is shown for Cases A and B respectively. It is seen from these figures that \( d\theta_o / d\eta \) changes sharply with \( \alpha \) for \( U^* = -2 \), and depends weakly.
on $\alpha$ for $U^*=0$ and $U^*=2$. $d\theta_b/d\eta$ increases with an increase in $Br_j$.

In Figs.7 and 8, the effect of $U^*$ on the dimensionless temperature differences ($\theta_i - \theta_b$, $\theta_i - \theta$) and ($\theta - \theta_b$), are shown respectively for Cases A and B. ($\theta_i - \theta$) decreases with an increase in $U^*$ near the fixed wall, but ($\theta_i - \theta_b$) increases with an increase in $U^*$ near the moving wall. ($\theta - \theta_b$) decreases with an increase in $U^*$ near the moving wall for Cases A and B.

In Fig.9 Nusselt numbers, $Nu_j$, for Cases A and B are shown. From these figures it is seen that $Nu_j$ changes clearly with an increase in $U^*$ for Case A and depends weakly on $U^*$ for Case B.

3.2 Effects of viscous dissipation

In both Cases of A and B, $d\theta_b/d\eta$ increases with an increase in Brinkman number, $Br_j$ for heating process ($Br_j>0$) and decreases with an increase in $Br_j$ for cooling process ($Br_j<0$) (see Fig.6).

In Case A, $(\theta_i - \theta)$ increases with an increase in $Br_j$ for $U^*=-2$ and $U^*=0$, whereas it decreases with an increase in $Br_j$ for $U^*=2$ (see Fig.7). In Case B, $(\theta_i - \theta)$ decreases with an increase in $Br_j$ for $U^*=-2$, whereas it increases with an increase in $Br_j$ for $U^*=0$ and $U^*=2$ (see Fig.7).

In Cases A and B, for $Br_j>0$ and $U^*=0$ the bulk temperature difference $(\theta - \theta_b)$ increases with an increase in $Br_j$ near the walls (see Fig.8). But it decreases in the middle section. This is attributed to that near the walls the dimensionless velocity gradient is large (see Fig.3) and that the viscous dissipation effect is large. The heat axially transferred by convection is large in the middle section (see Fig.2). For $U^*=-2$, $(\theta - \theta_b)$ greatly increases with an increase in $Br_j$ near the moving wall, but increases a little near the fixed wall. In the middle section it decreases. The dimensionless velocity gradient is large near the moving wall and small near the fixed wall. For $U^*=2$, $(\theta - \theta_b)$ increases with an increase in $Br_j$ near the fixed wall and decreases near the moving wall. Near the moving wall the dimensionless velocity gradient is large, but small near the fixed wall (see Fig.3). The heat transferred by convection is larger near the moving wall (see Fig.2).

Nusselt numbers, $Nu_j$, increases with an increase in $Br_j$ for Case A and it decreases with an in $Br_j$ for Case B (see Fig.9).

3.3 Cooling process

The negative values for $Br_j$ correspond to the cooling process. In Figs.10 and 11, the effects of $U^*$ and viscous dissipation on the dimensionless temperatures ($\theta_i - \theta$), ($\theta_i - \theta_b$) and ($\theta - \theta_h$), are shown respectively for Cases A and B. In cooling processes the behavior of temperature differences tends to be contrary to that in heating process (see Figs.7 and 8).

The effects of viscous dissipation in cooling process are shown for Cases A and B in Fig.12. It is seen from these figures that Nusselt numbers depend on $Br_j$ and $U^*$. Especially, the behaviors of $Nu_j$ vary widely when the value of $Br_j$ is large.

4. Conclusions

The study showed that for equal conditions the following changes were observed in heating process.

a) With an increase in $U^*$,

- decrease in $d\theta_b/d\eta$ for Cases A and B
- decrease in $\theta_i - \theta$ for Case A
- increase in $\theta_i - \theta$ for Case B
- decrease in $\theta - \theta_h$ for Cases A and B
- increase in $Nu_j$ for Case A
- decrease in $Nu_j$ for Case B

b) With an increase of $Br_j$

- increase in $d\theta_b/d\eta$ for Cases A and B
- increase in $\theta_i - \theta$ ($U^* \leq 0$) and decrease in $\theta_i - \theta$ ($U^* > 0$) for Case A
- decrease in $\theta_i - \theta$ ($U^* < 0$) and increase in $\theta_i - \theta$ ($U^* \geq 0$) for Case B
- decrease in $\theta - \theta_h$ for Cases A and B
- increase in $Nu_j$ ($U^* < 0.4$) and decrease in $Nu_j$ ($U^* \geq 0.4$) for Case A
- decrease in $Nu_j$ for Case B

The effect of $Br_j$ on $d\theta_b/d\eta$, $\theta_i - \theta$, and $\theta - \theta_h$ is stronger for $U^*<0$. But it affects more strongly on $Nu_j$ (Case A) when $U^*>0$.

References


Fig. 2 Velocity profiles

Fig. 3 Square of velocity gradient

Fig. 4 Distribution of parameter $V$
Effects of Moving Core Velocity and Viscous Dissipation on Fully Developed Laminar Heat Transfer in Concentric Annuli

Fig. 5 Magnitude of parameter $V_b$

Case A

Case B

Fig. 6 $\frac{d\theta_n}{d\eta}$ vs. $\alpha$
Fig. 7 Dimensionless temperature difference ($\theta_j - \theta$) for Cases A and B

Fig. 8 Dimensionless temperature difference ($\theta - \theta_0$) for Cases A and B
Effects of Moving Core Velocity and Viscous Dissipation on Fully Developed Laminar Heat Transfer in Concentric Annuli

Fig. 9 Nusselt numbers for Cases A and B

Fig. 10 Dimensionless temperature difference ($\theta_l - \theta$) for Cases A and B for cooling process
Fig. 11 Dimensionless temperature difference ($\theta - \theta_n$) for Cases A and B for cooling process

Fig. 12 Nusselt numbers for Cases A and B for cooling process