Laminar heat transfer in the thermal entrance region of concentric annuli with moving heated cores  
(Part I: The cases with the first and second kinds of thermal boundary condition)  
by  
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Consideration is given to the effects of viscous dissipation on the developing heat transfer between a fully developed laminar non-Newtonian fluid flow and a concentric annular geometry with a moving heated core. In this report, the results with the first and second kinds of thermal boundary condition are presented. Applying the shear stress described by the modified power-law model, the energy equation including the viscous dissipation term is solved numerically. The effects of radius ratio, flow index, relative core velocity, dimensionless shear rate parameter and Brinkman number on temperature distribution and Nusselt number are discussed.

1. Introduction  
The problems of fully developed heat transfer to non-Newtonian fluids in a concentric annulus with an axially moving core have been studied numerically for the thermal boundary conditions of constant heat flux at either tube(1)(2). In this paper, the entrance-region heat transfer between a fully developed laminar fluid flow and a concentric annular geometry with a moving heated core is studied numerically. Applying the fully developed velocity profile reported for the modified power-law model in the previous report(3), the energy equation including the viscous dissipation term is solved numerically using the finite difference method for the thermal boundary conditions of first kind and second kind. The effects of radius ratio, relative velocity of the core, flow index and dimensionless shear rate parameter and Brinkman number on developing temperature distribution and Nusselt number are discussed.

Nomenclature  
\( c_p \) specific heat at constant pressure  
\( D_h \) hydraulic diameter = \( 2(R_o - R_i) \)  
\( k \) thermal conductivity  
\( m \) consistency index  
\( n \) flow index  
\( Pe \) Peclet number  
\( r \) radial coordinate  
\( r^* \) dimensionless radial coordinate = \( r/D_h \)  
\( R \) radius  
\( q \) wall heat flux  
\( T \) temperature  
\( u_m \) average velocity of the fluid  
\( u^* \) dimensionless velocity \( \equiv u/u_m \)  
\( U^* \) dimensionless relative velocity of the moving core \( \equiv U/u_m \)  
\( z \) axial coordinate  
\( z^* \) dimensionless axial coordinate = \( z/(PeD_h) \)  

Greek Symbols  
\( \alpha \) radius ratio \( \equiv R_i/R_o \)  
\( \beta \) dimensionless shear rate parameter  
\( \eta_0 \) viscosity at zero shear rate  
\( \eta^* \) reference viscosity  
\( \rho \) density  
\( \xi \) transformed dimensionless radial coordinate = \( [2(1-\alpha)r^* - \alpha]/(1-\alpha) \)  

Subscripts  
\( b \) bulk  
\( e \) inlet  
\( i \) inner tube  
\( ii \) at the inner wall with the inner heated  
\( o \) outer tube  
\( oi \) at the outer wall with the inner heated  

2. Analysis  
The physical model for the analysis is shown in Fig.1. The core tube moves axially at a constant velocity, \( U \). The assumptions used in the analysis are:
1. The flow is incompressible, steady-laminar, and fully developed hydrodynamically.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model, and the physical properties are constant except viscosity.
3. The body forces and axial heat conduction are neglected.

Heat transfer

The energy equation together with the assumptions above is written as

\[ \frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \tau \left( \frac{du}{dr} \right) = \rho c_p u \frac{\partial T_b}{\partial z}. \] (1)

The velocity, \( u \), and its gradient, \( \frac{du}{dr} \), have been evaluated and reported in the previous report. The thermal boundary conditions:

(1) The first kind (constant wall temperature at the moving core and the temperature of the outer tube is kept equal to the uniform entering fluid temperature):

\[
\begin{align*}
T^{(1)} &= T_1 \quad \text{at} \quad r = R_1 \\
T^{(1)} &= T_e \quad \text{at} \quad r = R_o
\end{align*}
\] (2)

(2) The second kind (constant heat flux at the moving core with the outer tube insulated):

\[
\begin{align*}
-k \frac{\partial T^{(2)}}{\partial r} &= q_i \quad \text{at} \quad r = R_1 \\
\frac{\partial T^{(2)}}{\partial r} &= 0 \quad \text{at} \quad r = R_o
\end{align*}
\] (3)

The inlet condition is:

\[ z = 0 : \quad T^{(k)} = T_i \quad \text{for} \quad R_i \leq r \leq R_o \quad (k = 1 \text{ or } 2) \] (4)

\( \tau \) in Eq.(1) is the shear stress defined as

\[ \tau \equiv \eta_a \frac{du}{dr} \] (5)

where \( \eta_a \) is the apparent viscosity defined by

\[ \eta_a = \frac{\eta_0}{1 + \frac{\eta_0}{m} \left( \frac{du}{dr} \right)^{1-n}} \quad \text{for} \quad n < 1, \] (6)

\[ \eta_a = \eta_0 \left( 1 + \frac{m}{\eta_0} \left( \frac{du}{dr} \right)^{n-1} \right) \quad \text{for} \quad n > 1. \] (7)

Dimensionless apparent viscosity is

\[ \eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1 + \beta}{1 + \beta \left( \frac{\eta^*}{\eta_0} \right)^{1-n}} \quad \text{for} \quad n < 1, \] (8)

Fig.1 Schematic of a concentric annulus with an axially moving core

Heat transfer coefficients are defined as:

\begin{align*}
h^{(1)}_{ii} &\equiv \frac{-k \frac{\partial T^{(1)}}{\partial r}}{T^{(1)}_i - T_b} \\
h^{(1)}_{io} &\equiv \frac{k \frac{\partial T^{(1)}}{\partial r}}{T^{(1)}_i - T_b}
\end{align*}

for

\begin{align*}
h^{(2)}_{ii} &\equiv \frac{q_i}{T^{(2)}_i - T_b} \\
h^{(2)}_{io} &\equiv \frac{q_i}{T^{(2)}_i - T_b}
\end{align*}
Thus, Nusselt numbers are calculated as:

\[
N_{hi}^{(1)} = \frac{D_h}{T_1^{(1)} - T_b^{(1)}} \left[ -\frac{\partial T^{(1)}}{\partial r} \right]_{R_i} \tag{20}
\]

\[
N_{ho}^{(1)} = \frac{D_h}{T_o^{(1)} - T_b^{(1)}} \left[ -\frac{\partial T^{(1)}}{\partial r} \right]_{R_o} \tag{21}
\]

\[
N_{li}^{(2)} = \frac{D_h}{T_1^{(2)} - T_b^{(2)}} \left[ \frac{q_i}{k} \right] \tag{22}
\]

Introducing a dimensionless temperature, \( \theta \), defined as

\[
\theta^{(1)} \equiv \frac{T^{(1)} - T_e}{T_1^{(1)} - T_e} \tag{23}
\]

\[
\theta^{(2)} \equiv \frac{k \left[ T^{(2)} - T_e \right]}{q_i D_h} \tag{24}
\]

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

\[
\frac{1}{r^* \partial r^*} \left( r^* \frac{\partial \theta}{\partial r^*} \right) + Br \cdot \eta_a \left( \frac{du^*}{dr^*} \right)^2 = u^* \frac{\partial \theta}{\partial z^*} \tag{25}
\]

(1) The boundary condition of the first kind:

\[
\begin{align*}
\theta^{(1)} &= 1 \quad \text{at} \quad r^* = \frac{2(1-\alpha)}{2(1-\alpha)} \\
\theta^{(1)} &= 0 \quad \text{at} \quad r^* = \frac{1}{2(1-\alpha)}
\end{align*} \tag{26}
\]

(2) The boundary condition of the second kind:

\[
\begin{align*}
\frac{d\theta^{(2)}}{dr^*} &= -1 \quad \text{at} \quad r^* = \frac{2(1-\alpha)}{2(1-\alpha)} \\
\frac{d\theta^{(2)}}{dr^*} &= 0 \quad \text{at} \quad r^* = \frac{1}{2(1-\alpha)}
\end{align*} \tag{27}
\]

The inlet condition is:

\[
z^* = 0 : \quad \theta^{(k)} = 0
\]

for \( \frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)} \) (k = 1 or 2) \tag{28}

Brinkman number is defined as follows

\[
Br^{(1)} \equiv \frac{\eta_a u_m^2}{k \left[ T_1^{(1)} - T_e \right]} \tag{29}
\]

\[
Br^{(2)} \equiv \frac{\eta_a u_m^2}{D_h q_i} \tag{30}
\]

Nusselt number at the tube walls:

\[
N_{hi}^{(1)} = -\left. \frac{1}{1 - \theta_b^{(1)}} \frac{\partial \theta^{(1)}}{\partial r^*} \right|_{z^* = 0} \tag{31}
\]

\[
N_{ho}^{(1)} = \left. \frac{1}{\theta_b^{(1)} - \theta_b^{(1)}} \frac{\partial \theta^{(1)}}{\partial r^*} \right|_{z^* = 0} \tag{32}
\]

\[
N_{li}^{(2)} = \frac{1}{\theta_b^{(2)} - \theta_b^{(2)}} \tag{33}
\]

where the dimensionless bulk temperature, \( \theta_b \), is defined as

\[
\theta_b^{(k)} = \frac{8(1 - \alpha)}{1 + \alpha} \int_{z^* = 0}^{z^* = 1} u^* \theta^{(k)} r^* dr^* \tag{34}
\]

3. Results and discussion

The calculation has been carried out by using the finite difference method. The range of parameters considered are:

- The radius ratio: \( 0.2 \leq \alpha \leq 1.0 \)
- The relative velocity: \( 0 \leq U^* \leq 1.0 \)
- The flow index: \( 0.5 \leq n \leq 1.5 \)
- The dimensionless shear rate parameter: \( 10^{-5} \leq \beta \leq 10^5 \)
- Brinkman number: \( 0.0, 0.01, 0.05 \) and \( 0.1 \).

The mesh sizes used in the numerical calculation are shown below.

a. Axial direction (\( z^* \)):

\[
0 < z^* \leq 10^{-3} : 10^{-3} < z^* \leq 1
\]

b. Radial direction (\( r^* \)):

\[
r^* = \frac{1}{100}.
\]

The development of the non-dimensional temperature profiles in the thermal entrance region of a concentric annulus with a heated core for the two kinds of the boundary conditions (\( k = 1 \) and 2) are presented in Fig.2a and Fig.2b, respectively, for the same condition ( \( \alpha = 0.5, n = 0.5 \), power law fluid (\( \beta = 10^5 \)) and \( U^* = 1.0 \) ). The figures illustrate clearly how the temperature profiles develop for the two different boundary conditions.

The effects of the relative velocity, \( U^* \), on the development of temperature profiles are demonstrated in Figs.2b and 2c for the second kind of boundary condition (\( k = 2 \)). It is seen that the fluid temperature increase is less rapid for larger values of the core velocity.

The effects of viscous dissipation on the development of temperature profiles for \( \alpha = 0.5, n = 0.5, \beta = 10^5 \) and \( U^* = 1.0 \) for the second kind of boundary condition (\( k = 2 \)) are shown in Fig.2b and 2d.

The effect of the moving core velocity on Nusselt number at the tube walls are shown in Figs.3a to 3c at given values of \( Br = 0.0 \) and \( \alpha = 0.5 \) for three different fluids (\( n < 1.0 \) pseudoplastic, \( n = 1.0 \) Newtonian and \( n > 1.0 \) dilatant).

The calculation results of the particular case of Newtonian fluids (\( n = 1.0 \)) with neglected viscous dissipation (\( Br = 0.0 \)) are compared with the predictions by Shah and London(6) for the stationary...
core \( (U^* = 0) \) and by Shigechi and Araki\(^6\) for
the moving core \( (U^* = 1.0) \), respectively. Even at
small values of \( z^* \), it can be seen in Figs.3a, 3b
and 3c that the agreement is excellent. The effect
of the relative velocity of the moving core tube
is always to increase the values of Nusselt num­
bers, \( Nu_{ii} \) and to decrease the values of Nusselt
numbers, \( Nu_{oi} \), for the given conditions of \( \alpha \) and
\( Br \).

The viscous dissipation effects on Nusselt num­
ber are shown in Figs.4 to 6 for three different flu­
dids. With an increase in Brinkman number, \( Nu_{ii} \)
decreases for \( U^* = 0.0 \) and \( Nu_{oi} \) increases for \( U^* = 0.0 \) and \( U^* = 1.0 \).

For the first kind of boundary condition, \( Br \) has
a strong effect on \( Nu_{ii} \) at the thermally fully de­
veloped region while the core is fixed. But for the
case of the moving core the effect of the Brinkman
number is very small at both the thermal entrance
region and the fully developed region. It is also
seen that for dilatant fluids \( Br \) on \( Nu_{ii} \) is stronger
than for Newtonian and pseudoplastic fluids.

For the second kind of boundary condition \( Br \)
affects strongly on \( Nu_{ii} \) in both the thermally de­
veloping and developed regions when the core is
fixed. \( Nu_{ii} \) decreases with an increase in \( Br \). For
the moving core in the thermal entrance region
the effect of the \( Br \) on \( Nu_{ii} \) is almost negligible
but in the fully developed region \( Nu_{ii} \) increases
with an increase in \( Br \).

\( \begin{align*}
\alpha &= 0.5 \\
n &= 0.5 \\
U^* &= 0.0 \\
Br &= 0.05 \\
\beta &= 10^3
\end{align*} \)

\( \begin{align*}
\alpha &= 0.5 \\
n &= 0.5 \\
U^* &= 0.8 \\
Br &= 0.05 \\
\beta &= 10^3
\end{align*} \)

\( \begin{align*}
\alpha &= 0.5 \\
n &= 0.5 \\
U^* &= 1.0 \\
Br &= 0.05 \\
\beta &= 10^3
\end{align*} \)

\fig{2.a}{Development of temperature profiles
(1st kind)}

\fig{2.b}{Development of temperature profiles
(2nd kind)}

\fig{2.c}{Development of temperature profiles
for different \( U^* \) (2nd kind)}

\fig{2.d}{Effect of \( Br \) on the development
of temperature profiles (2nd kind)}
4. Conclusions

The heat transfer between a fully developed laminar fluid flow and concentric annular geometry with a moving heated core of fluid or solid body is studied with the two different boundary conditions.

It may be concluded that the viscous dissipation effect on heat transfer is stronger for dilatant fluids. $Br$ affects strongly on Nusselt number at the unheated fixed tube.

In this report the heat transfer results for the first and second kinds of thermal boundary conditions are only discussed. The counterpart for the third and fourth kinds of boundary conditions will be reported in the next report.

References

3. Ganbat Davaa, Toru Shigechi and Satoru Momoki, “Fluid flow for modified power law


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Fig. 4.b Nusselt numbers, $N_{ui}$, for $n = 1.0$, $U^* = 0.0$ and 1.0 (1st kind)

Fig. 4.c Nusselt numbers, $N_{ui}$, for $n = 1.5$, $U^* = 0.0$ and 1.0 (1st kind)
Fig. 5.a Nusselt numbers, \( N_{\text{ub}} \), for \( n = 0.5, U^* = 0.0 \) and 1.0 (1st kind)

Fig. 5.b Nusselt numbers, \( N_{\text{ub}} \), for \( n = 1.0, U^* = 0.0 \) and 1.0 (1st kind)
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Fig. 5.c Nusselt numbers, $N_{u_{01}}$, for $n = 1.5, U^* = 0.0$ and 1.0 (1st kind)

Fig. 6.a Nusselt numbers, $N_{u_{02}}$, for $n = 0.5, U^* = 0.0$ and 1.0 (2nd kind)
Fig. 6.b Nusselt numbers, $N_u_{ii}$, for $n = 1.0$, $U^* = 0.0$ and $1.0$ (2nd kind)

Fig. 6.c Nusselt numbers, $N_u_{ij}$, for $n = 1.5$, $U^* = 0.0$ and $1.0$ (2nd kind)