Effects of viscous dissipation and fluid axial heat conduction on entrance-region heat transfer in parallel plates (Part II: The thermal boundary condition of the second kind)

by

Odgerel JAMBAL*, Toru SHIGECHI**, Satoru MOMOKI** and Ganbat DAVAA*

In the present paper, the extension of the work on the combined effects of viscous dissipation, fluid axial heat conduction and moving boundary to cover the thermal boundary condition of constant wall heat flux is considered. The laminar forced convection in parallel plates is investigated under the assumptions of hydrodynamically fully developed non-Newtonian fluid flow, the uniform temperature at upstream infinity and a step change in wall heat flux at $z = 0$. The temperature distribution of the fluid for $-\infty < z < \infty$ is determined and the effects of relative velocity of the moving plate, Brinkman number and Peclet number on developing temperature distribution and Nusselt number are discussed.

1. Introduction

The analysis of thermally developing heat transfer in parallel plates has been considered for the boundary conditions of constant heat flux and this is an extension of the previous report(1). This paper is concerned with the laminar heat transfer of non-Newtonian fluids in parallel plates of infinite extend. The aim of this paper is to clarify the combined effects of viscous dissipation of the flowing fluid, relative velocity of the moving plate and fluid axial heat conduction on the developing heat transfer.

Nomenclature

Br	modified Brinkman number
c_p	specific heat at constant pressure, [J/ (kg·K)]
D_h	hydraulic diameter, $D_\mathrm{h} = 2L$ [m]
E	constant of the axial transformation
f	friction factor
h	heat transfer coefficient [W/(K m^2)]
k	thermal conductivity, [W / (m·K)]
L	distance between the parallel plates, [m]
m	fluid consistency [N s^n / m^2]
n	flow index
Nu	Nusselt number
Pe	Peclet number
Re_M	modified Reynolds number
Pr_M	modified Prandtl number
T	temperature, [K]
u	fully developed velocity profile, [m/s]
u_m	average velocity of the fluid $u_\mathrm{m} \equiv \frac{1}{L}\int_0^L u dy$ [m/s]
u^*	dimensionless velocity $\equiv u/u_\mathrm{m}$
U	axial velocity of the moving plate, [m/s]
U^*	dimensionless relative velocity of the moving plate $\equiv U/u_\mathrm{m}$
y	coordinate normal to the fixed plate, [m]
y^*	dimensionless coordinate
z	axial coordinate, [m]
z^*	dimensionless axial coordinate
z_t	transformed axial coordinate

Greek Symbols

β	dimensionless shear rate parameter
η_a	apparent viscosity, [kg/(m·s)]
η_a^*	dimensionless apparent viscosity $\equiv \eta_a/\eta$
η_0	viscosity at zero shear rate, [kg/(m·s)]
η	reference viscosity, [kg/(m·s)]
ρ	density, [kg/m^3]
τ	shear stress, [N/m^2]
θ	dimensionless temperature

Subscripts

II	the second kind of boundary condition
b	bulk
e	entrance or inlet
fd	fully developed
lw	lower plate
uw	upper plate

2. Analysis

The physical model for the analysis is shown in Fig.1. The lower plate moves axially at a constant velocity, U. The assumptions and conditions used in the analysis are:

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* Graduate School of Science and Technology
** Department of Mechanical Systems Engineering

- The flow is incompressible, steady-laminar, and fully developed hydrodynamically.
- The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[2], and physical properties are constant except viscosity.
- The body forces are neglected.
- The entering fluid temperature, T_e, is uniform at upstream infinity ($z \to -\infty$) and the upper wall is insulated. Constant heat flux at the lower wall for $0 \leq z$, whereas for $0 < z$ the lower wall is insulated.

The energy equation together with the assumptions above is written as

$$\rho c_p u \frac{\partial T}{\partial z} = k \left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \eta_a \left[\frac{du}{dy} \right]^2 \quad (1)$$

$$\text{in} \quad 0 \leq y \leq L, \text{ and } -\infty \leq z \leq \infty$$

$$\begin{cases} k\frac{\partial T}{\partial y}=0 & \text{at} \quad y=0, \quad 0 \leq z \\ k\frac{\partial T}{\partial y}=q_{\mathrm{lw}} & \text{at} \quad y=L, \quad 0 \leq z \\ k\frac{\partial T}{\partial y}=0 & \text{at} \quad y=0, \quad z < 0 \\ k\frac{\partial T}{\partial y}=0 & \text{at} \quad y=L, \quad z < 0 \\ \lim_{z \to -\infty} & T=T_e, \quad 0 < y < L \\ \lim_{z \to +\infty} & T=T_{fd}(y,z), \quad 0 < y < L. \end{cases} \quad (2)$$

From the solutions of Eq.(1) with Eq.(2), some thermal quantities are obtained as

- bulk temperature, T_b:

$$T_b \equiv \frac{\int_0^L uT\,dy}{\int_0^L u\,dy} \quad (3)$$

- Nusselt number, Nu:

$$Nu \equiv \frac{hD_h}{k} \quad (4)$$

- heat transfer coefficient, h:

$$h = \frac{q_{\mathrm{lw}}}{T_{\mathrm{lw}} - T_b} \quad (5)$$

The following dimensionless quantities are introduced

$$y^* = \frac{y}{D_h} \qquad z^* = \frac{z}{PeD_h} \quad (6)$$

$$u^* = \frac{u}{u_m} \qquad \beta = \frac{\eta_0}{m} \left(\frac{u_{\mathrm{m}}}{D_{\mathrm{h}}} \right)^{1-n} \quad (7)$$

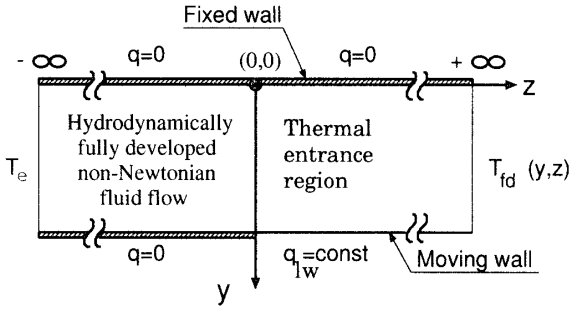

Fig.1 Geometrical configuration

$$\theta = \frac{T - T_e}{q_{\mathrm{lw}} D_{\mathrm{h}}/k} \quad (8)$$

$$Br_{II} = \eta \frac{u_m^2}{q_{\mathrm{lw}} D_{\mathrm{h}}} \quad (9)$$

Where

$$Pe = Re_{\mathrm{M}} \cdot Pr_{\mathrm{M}} = \frac{u_m D_h}{\alpha} \quad (10)$$

$$Re_{\mathrm{M}} \equiv \frac{\rho u_{\mathrm{m}} D_{\mathrm{h}}}{\eta} \qquad Pr_{\mathrm{M}} \equiv \frac{c_p \eta}{k} \quad (11)$$

With the substitution of the above quantities into the dimensional formulation, the dimensionless energy equation and boundary conditions are obtained as

$$u^* \frac{\partial \theta}{\partial z^*} = \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial z^{*2}} + Br_{II} \eta_a^* \left(\frac{du^*}{dy^*} \right)^2 \quad (12)$$

$$\text{in} \quad 0 \leq y^* \leq \tfrac{1}{2} \text{ and } -\infty \leq z^* \leq \infty$$

$$\begin{cases} \frac{\partial \theta}{\partial y^*}=0 & \text{at} \quad y^*=0, \quad 0 \leq z^* \\ \frac{\partial \theta}{\partial y^*}=1 & \text{at} \quad y^*=1/2, \quad 0 \leq z^* \\ \frac{\partial \theta}{\partial y^*}=0 & \text{at} \quad y^*=0, \quad z^* < 0 \\ \frac{\partial \theta}{\partial y^*}=0 & \text{at} \quad y^*=1/2, \quad z^* < 0 \\ \lim_{z^* \to -\infty} & \theta=0, \quad 0 < y^* < 1/2 \\ \lim_{z^* \to +\infty} & \theta=\theta_{fd}(y^*, z^*), \quad 0 < y^* < 1/2. \end{cases} \quad (13)$$

For infinitely large values of the axial distance ($z^* \to \infty$), thermally fully developed region is reached. In the thermally fully developed region, the temperature solution is a linear function of the axial coordinate. Therefore the solution is sought in the form:

$$\theta_{fd} = Cz^* + \psi(y^*) \quad (14)$$

Substitution of Eq.(14) into Eq.(12) yields

$$\frac{d^2\psi}{dy^{*2}} = Cu^* - V \tag{15}$$

$$\begin{cases} \frac{d\psi}{dy^*}=0 & \text{at} \quad y^*=0, \\ \frac{d\psi}{dy^*}=1 & \text{at} \quad y^*=1/2 \end{cases} \tag{16}$$

where

$$V = Br_{II}\eta_a^* \left(\frac{du^*}{dy^*}\right)^2 \tag{17}$$

On the other hand, in the thermally developed region

$$\rho c_p u \frac{dT_b}{dz} = k\frac{\partial^2 T_{fd}}{\partial y^2} + \eta_a \left(\frac{du}{dy}\right)^2 \tag{18}$$

$$\begin{cases} \frac{\partial T_{fd}}{\partial y}=0 & \text{at} \quad y=0 \\ k\frac{\partial T_{fd}}{\partial y}=q_{\mathrm{lw}} & \text{at} \quad y=L \end{cases} \tag{19}$$

dT_b/dz in Eq.(18) is evaluated, from an energy balance, as

$$\frac{dT_b}{dz} = \frac{q_{\mathrm{lw}}}{\rho c_p u_m L}\left[1 + \frac{\int_0^L \eta_a \left(\frac{du}{dy}\right)^2 dy}{q_{\mathrm{lw}}}\right] \tag{20}$$

Substitution of the above balance into Eq.(18) gives,

$$k\frac{\partial^2 T_{fd}}{\partial y^2} + \eta_a \left(\frac{du}{dy}\right)^2 = \frac{2q_{\mathrm{lw}}u}{u_m D_h}\left[1 + \frac{\int_0^L \eta_a \left(\frac{du}{dy}\right)^2 dy}{q_{\mathrm{lw}}}\right] \tag{21}$$

By introducing the relevant dimensionless quantities the above equation becomes

$$\frac{\partial^2\theta_{fd}}{\partial y^{*2}} = 2u^*\left(1 + \int_0^{0.5} V dy^*\right) - V \tag{22}$$

According to Eq.(14)

$$\frac{\partial^2\theta_{fd}}{\partial y^{*2}} = \frac{d^2\psi}{dy^{*2}} \tag{23}$$

Thus

$$\frac{d^2\psi}{dy^{*2}} = 2u^*\left(1 + \int_0^{0.5} V dy^*\right) - V \tag{24}$$

The coefficient C was obtained from Eqs.(15) and (24) as

$$C = 2\left(1 + \int_0^{0.5} V dy^*\right) \tag{25}$$

$\psi(y^*)$ was calculated from Eq.(15) with Eq.(16) by the finite difference method. $\theta_{fd}(y^*, z^*)$ was obtained by calculating Eq.(14) and used as a boundary condition to solve the problem.

In order to convert the upstream and downstream infinities, the dimensionless axial coordinate z^* is transformed according to the relation employed by Verhoff and Fisher[3] as follows:

$$z^* = E\tan\pi z_t \quad or \quad z_t = \frac{1}{\pi}\arctan\frac{z^*}{E} \tag{26}$$

By introducing the transformed coordinate z_t, the energy equation Eq.(12) and the boundary conditions Eq.(13) become

$$A\frac{\partial\theta}{\partial z_t} = \frac{\partial^2\theta}{\partial y^{*2}} + B\frac{\partial^2\theta}{\partial z_t^2} + \eta_a^* Br_{II}\left(\frac{du^*}{dy^*}\right)^2 \tag{27}$$

$$\text{in} \quad 0 \le y^* \le \tfrac{1}{2} \text{ and } -0.5 \le z_t \le 0.5$$

where

$$A = \frac{\cos^2\pi z_t}{\pi E}\left(u^* + \frac{1}{Pe^2}\frac{\sin 2\pi z_t}{E}\right) \tag{28}$$

$$B = \frac{1}{Pe^2}\left(\frac{\cos^2\pi z_t}{\pi E}\right)^2 \tag{29}$$

$$\begin{cases} \frac{\partial\theta}{\partial y^*}=0 & \text{at} & y^*=0, & 0 \le z_t \\ \frac{\partial\theta}{\partial y^*}=1 & \text{at} & y^*=1/2, & 0 \le z_t \\ \frac{\partial\theta}{\partial y^*}=0 & \text{at} & y^*=0, & z_t < 0 \\ \frac{\partial\theta}{\partial y^*}=0 & \text{at} & y^*=1/2, & z_t < 0 \\ \lim_{z_t\to -0.5} & \theta=0, & 0 < y^* < 1/2 & \\ \lim_{z_t\to +0.5} & \theta=\theta_{fd}, & 0 < y^* < 1/2. & \end{cases} \tag{30}$$

Nu at the lower wall is

$$Nu = \frac{1}{(\theta_{\mathrm{lw}} - \theta_b)} \tag{31}$$

3. Results and discussion

In the previous report[1], the analysis of the thermally developing laminar forced convection in parallel plates with one plate moving had been performed by taking into account viscous dissipation of flowing non-Newtonian fluids and fluid axial heat conduction. In this paper the work presented previously[1] for constant wall temperature

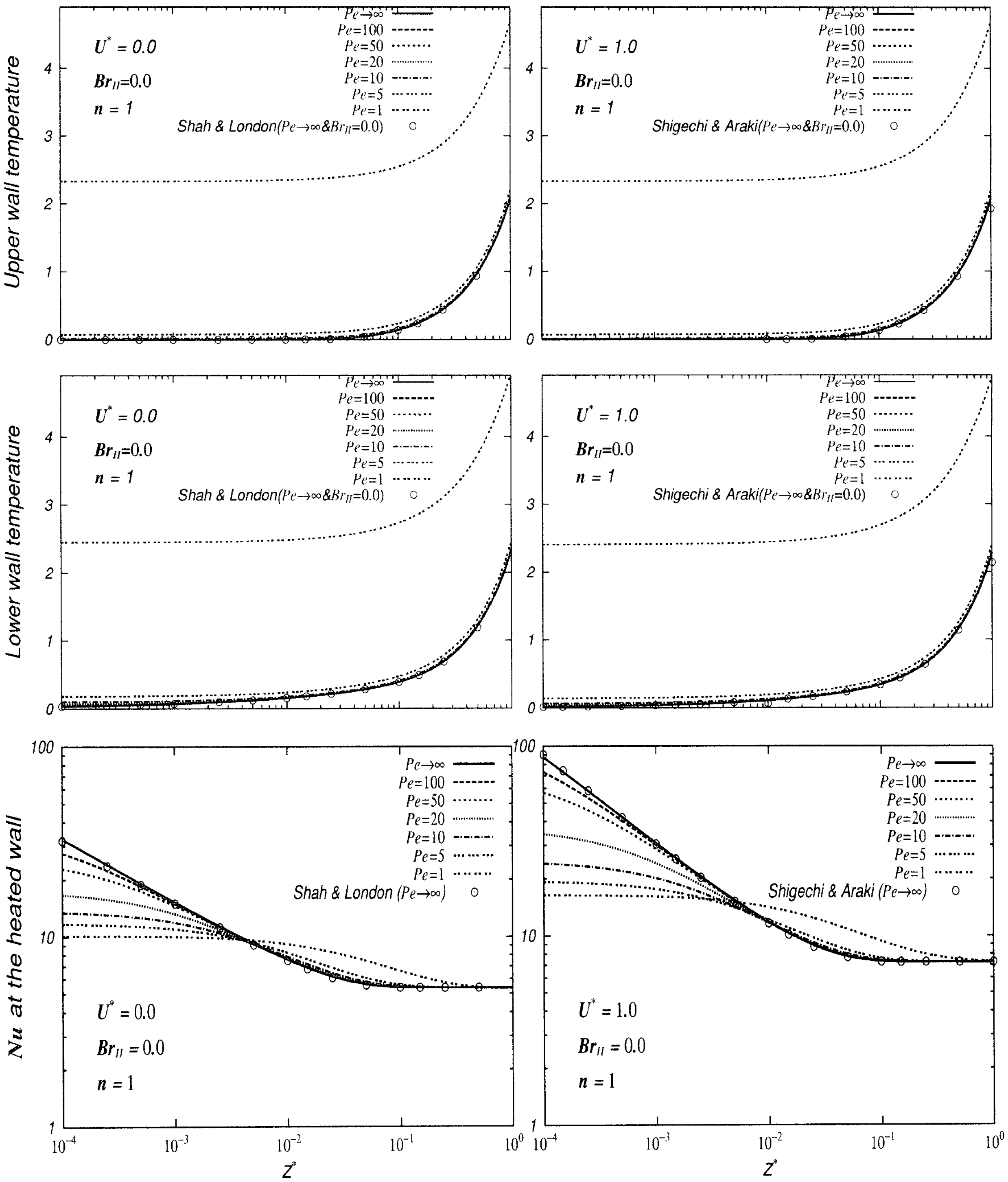

Fig.2 Wall temperatures and Nu for various Pe ($U^* = 0$ and $U^* = 1$)

has been extended to the case of constant wall heat flux. The results are discussed in terms of temperature profiles and Nu numbers for different relative velocity of the lower wall and fluid properties. The results are presented graphically in dimensionless form and the effects of relative velocity of the moving plate, viscous dissipation and fluid axial heat conduction are demonstrated.

The range of parameters considered and the calculation technique are the same as in the counterpart reported in the previous report[1]. In order to verify the accuracy of the present calculation, the results are compared for the special cases and shown in Fig.2. Temperature of a Newtonian fluid at the upper and lower walls and Nu at the heated lower wall are shown for $Br = 0.0$ and for various Pe number in Fig. 2 for the relative velocity of U^* 0.0 and 1.0. It is shown the Nusselt number and the temperature at the walls corresponding to $Pe \to \infty$ agree excellently with those obtained by Shah and London[4], and Shigechi and Araki[5], who analyzed the corresponding problem by assuming negligible fluid axial heat conduction and viscous dissipation. From the Nu curves demonstrating the effect of fluid axial heat conduction for a Newtonian fluid, it is seen the effect of fluid axial heat conduction is considerable for small values of Pe and in the fully developed region there is no fluid axial heat conduction effect.

Figure 3 illustrates the variation of local fluid temperature profiles for the cases of U^* 0.0 and 1.0. These results, which are for $Br = 0.0$; 0.05; 0.1 and, for $Pe = 10$ and $Pe \to \infty$ show the combined effects of viscous dissipation and fluid axial heat conduction on the developing temperature profiles with regard to U^*. The solid and dashed lines stand for the results for negligible ($Pe \to \infty$) and considerable ($Pe = 10$) fluid axial heat conduction cases, respectively. In the insulated wall region ($z^* < 0$) the fluid temperature is seen to be sufficiently large for large Br and small Pe. In fact, it can be seen that the fluid temperature increases significantly before the fluid reaches the heated wall because of the heat generated by viscous dissipation and the heat conducted upstream.

That is not the case for $Pe \to \infty$ and $Br = 0.0$. The fluid temperature does not increase in $z^* < 0$. As soon as z^* becomes positive, the fluid temperature profile undergoes rather rapid change because of the wall heat flux, causing a relatively abrupt decrease of Nu at the wall as it is seen in Nu curves.

It is also seen if the channel is infinitely long, there may be generated a significant amount of heat due to viscous shear heating ($Br > 0$) when the walls are insulated. Thus the value of the fluid temperature increases sufficiently.

From the developing temperature profiles it is seen for the second kind of boundary condition, the wall-to-fluid temperature difference is small, whereas the effect of viscous dissipation on heat transfer is more significant.

Figures 4-6 present the calculation results on the Nu number, defined with respect to the temperature difference of bulk to wall temperature for a pseudoplastic fluid whose $n = 0.5$ and $\beta =$ 1. These curves clearly exhibit the trend of Nu in the thermal entrance region by comparison them with the relative velocity U^*.

It can be observed that, from the Nu curves (Figs.2 , 4-6), Nu tends to decrease near z^*=0.0 as Pe reduces below 100. This tendency is exactly identical to that found in a paper[6] by previous researchers who studied heat transfer with neglected viscous dissipation in a fixed concentric annuli. The Nusselt curves for various Pe numbers, however, cross each other and reverse their orders of magnitude before reaching the fully developed values. It can also be noted that, for a fixed value of z^*, Nu at the moving wall is larger than the corresponding Nu at the stationary wall. It is seen that Nu remains almost constant throughout the thermal entrance region if Pe is small. This trend also had been observed for the thermal boundary condition of the first kind.

In Fig. 7 the combined effect of viscous dissipation and fluid axial heat conduction is shown by comparing them with the relative velocity of the lower plate. From Fig.7 it is seen that the effect of Br on Nusselt number is different depending on the relative velocity, U^*. In the fully developed region, viscous dissipation has a definite effect for both cases of a fixed walls ($U^* = 0.0$) and a moving wall ($U^* = 1.0$). Nu decreases with an increase in Br for a fixed wall ($U^* = 0.0$) and vice versa for a moving wall ($U^* = 1.0$). These behaviors of Nu due to viscous dissipation are comprehended as follows.

The value of Nu is calculated using the dimensionless temperature difference ($\theta_{\mathrm{lw}} - \theta_{\mathrm{b}}$) as is shown by Eq.(31). Both of θ_{lw} and θ_{b} increases in the flow direction of z^*. In terms of the increments of θ_{lw} and θ_{b} due to viscous dissipation, the increment of θ_{lw} is larger than that of θ_{b} for U^* = 0.0 and vice versa for $U^* = 1.0$.

The effect of fluid axial heat conduction ($Pe =$ 10) accounts for the change in the curve shape in the thermal entrance region.

4. Conclusions

The problem of thermally developing heat transfer of Couette-Poiseuille laminar flow including viscous dissipation of the flowing fluid and fluid axial heat conduction with the boundary condition of constant heat flux is analyzed by considering an infinite axial domain.

The sample results to demonstrate the combined effects of relative velocity of the moving

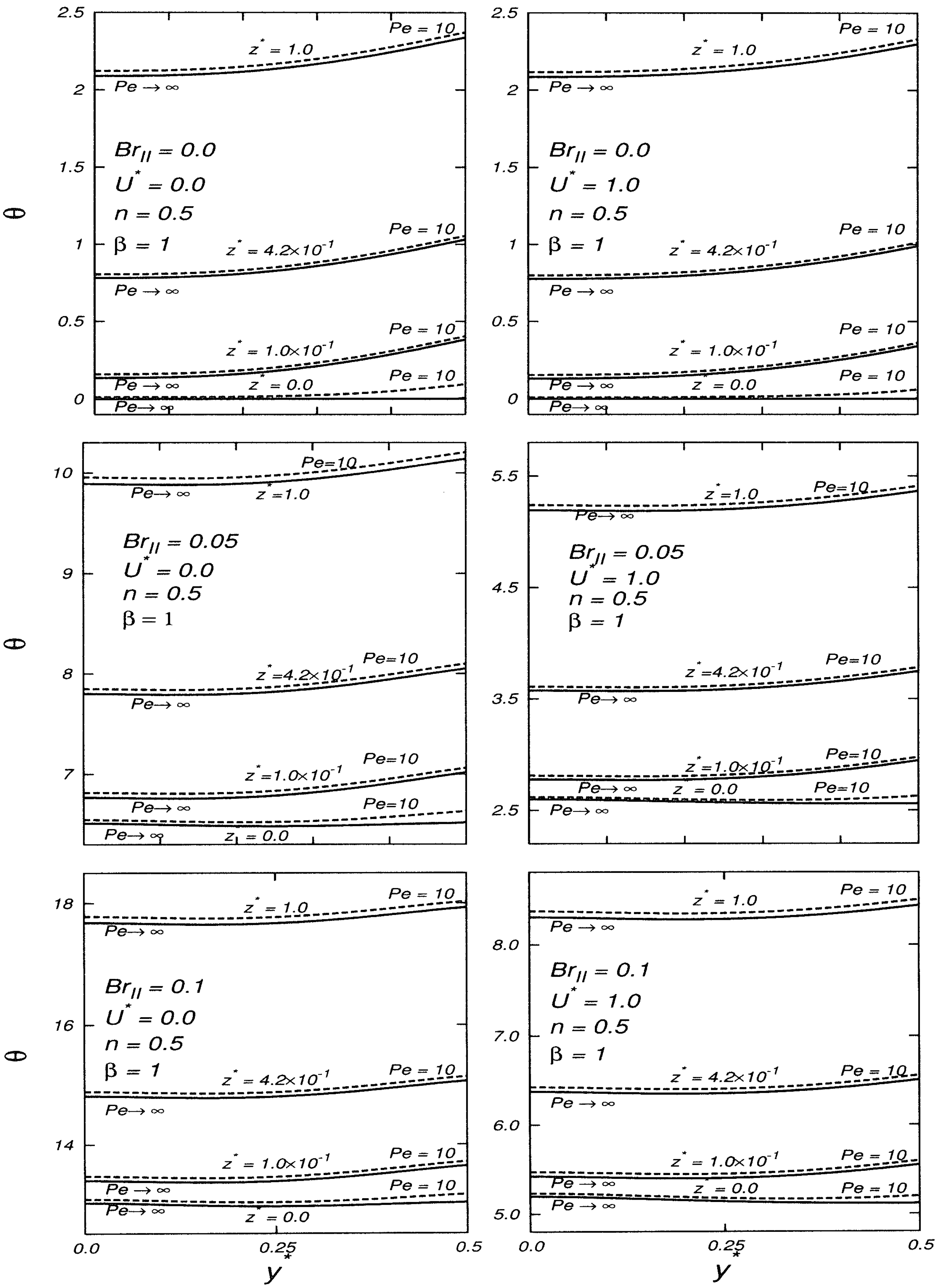

Fig.3 Temperature profiles for $U^* = 0$ and $U^* = 1$

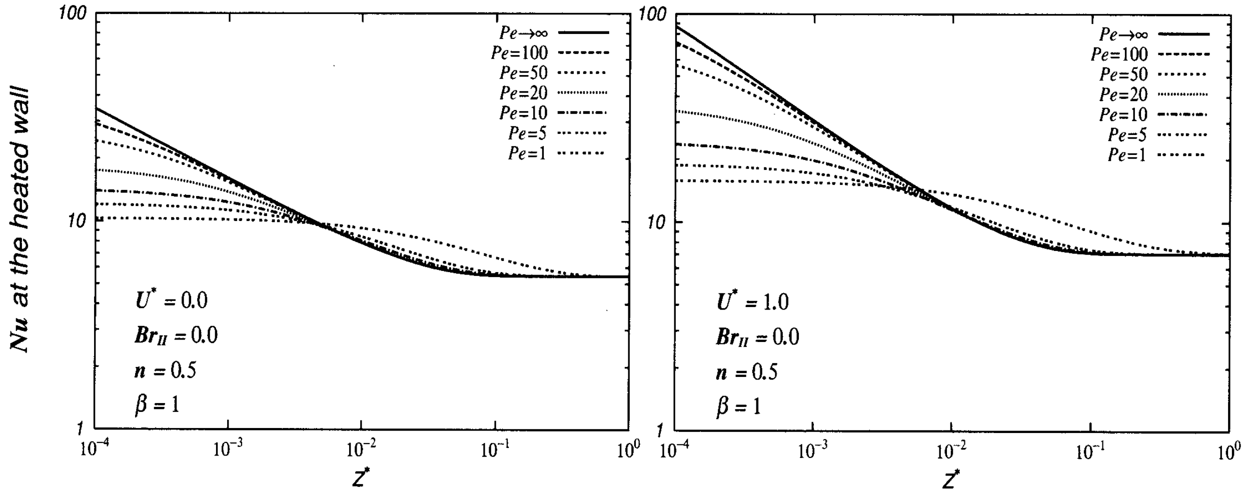

Fig.4 Nu for Br=0.0 ($U^* = 0$ and $U^* = 1$)

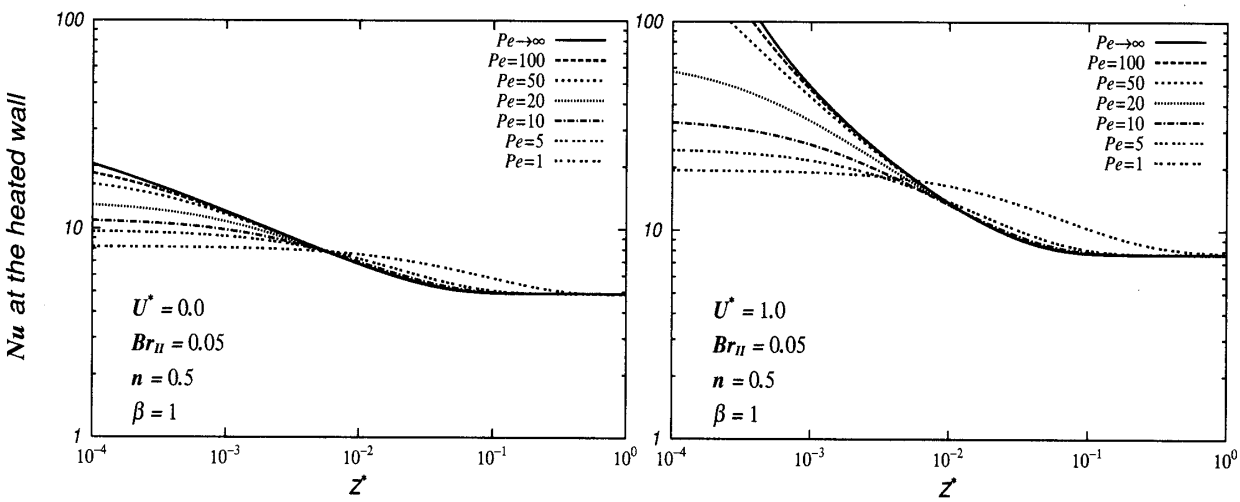

Fig.5 Nu for Br=0.05 ($U^* = 0$ and $U^* = 1$)

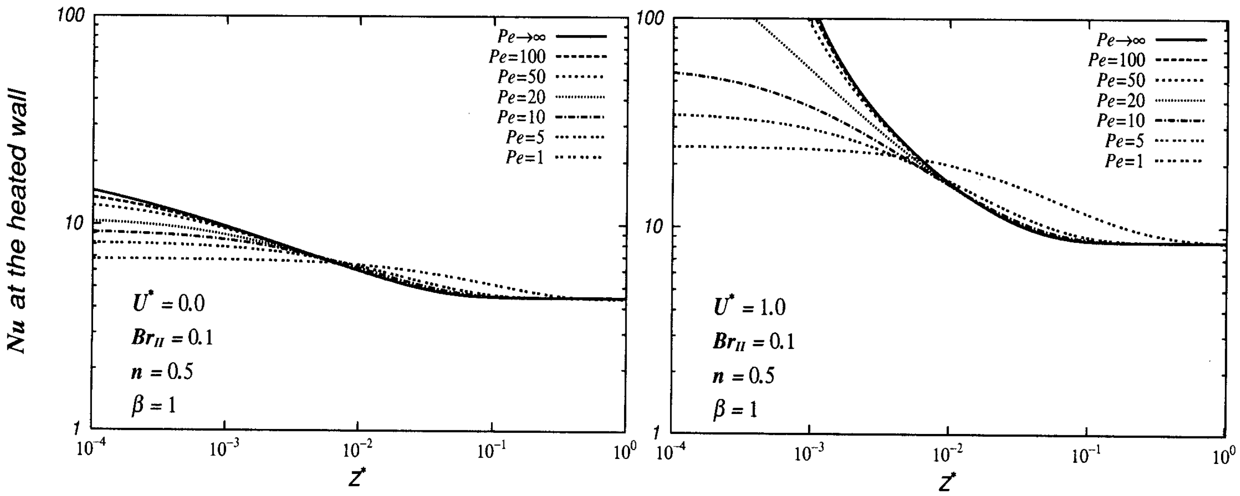

Fig.6 Nu for Br=0.1 ($U^* = 0$ and $U^* = 1$)

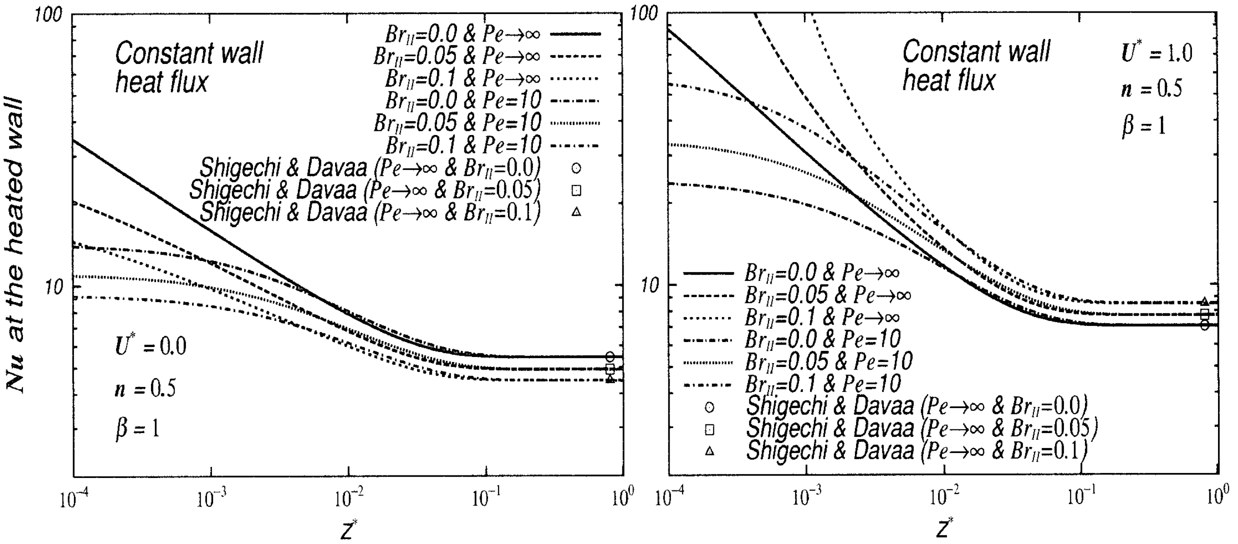

Fig.7 Nu for various Br and Pe
($U^* = 0$ and $U^* = 1$)

plate, viscous dissipation and fluid axial heat conduction are presented graphically in dimensionless form. The temperature solutions corresponding to the limiting cases $Pe \to \infty$, and $Br = 0.0$ show excellent agreement with those reported in(4) for $U^* = 0.0$ and in(5) for $U^* = 1.0$, who analyzed the developing heat transfer by neglecting viscous dissipation and fluid axial heat conduction.

For thermally developing flow viscous dissipation effect on Nu is different depending on U^*. From the developing temperature profiles it is seen for the second kind of boundary condition, the wall-to-fluid temperature difference is small, whereas the effect of viscous dissipation on heat transfer is more significant.

A comparison of developing temperature profiles for the different boundary conditions reveals that the increase in fluid temperature due to viscous heating for the second kind of boundary condition is much higher than for the first kind of boundary condition.

For constant heat flux boundary condition, including the effect of fluid axial heat conduction in the analyses results in lesser values of the Nu number near $z^* = 0$ (where the wall heat flux commences) compare to the case without axial heat conduction.

It may also be concluded that, the moving boundary may make less increase in the fluid temperature compared to the stationary wall boundary case and in turn it results higher values of Nusselt number at the wall as U^* increases.

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