Laminar heat transfer in concentric annuli with viscous dissipation and fluid axial heat conduction
(Part II: Thermal boundary condition of the second kind)

by

Ganbat DAVAA*, Toru SHIGECHI**, Satoru MOMOKI** and Odgerel JAMBAL*

The present paper which is an extension of the previous study[(1)] on the combined effects of viscous dissipation, fluid axial heat conduction, relative velocity of the core and radius ratio on thermally developing laminar flow heat transfer, deals with the thermal boundary condition of constant heat flux. The solution is based on coordinate transformation of the elliptic energy equation. The present numerical solutions were compared with the relevant data[(2)-(7)] by the previous researchers and the authors, and the agreement was very well.

1. Introduction

The numerical study of the thermally developing heat transfer in concentric annuli has been presented in the previous report[(1)] for the boundary condition of constant wall temperature and this is an extension of the previous work to the boundary condition of constant heat flux at the tube walls.

The aim of this paper is to clarify the combined effects of viscous dissipation, fluid axial heat conduction, relative velocity of the core and radius ratio on the thermally developing heat transfer between the annuli and laminar flow of non-Newtonian fluids.

Heat transfer in annuli with axially moving cores subject to the boundary condition of constant heat flux at the walls has been considered by the authors and the results are presented in the previous reports[(2)-(4)]. The effects of viscous dissipation and relative velocity of the moving core on the fully developed laminar heat transfer had been studied analytically by solving the energy equation exactly for Newtonian fluids in[(2)]. Fully developed laminar heat transfer of non-Newtonian fluids was studied in[(3)] by investigating the effects of viscous dissipation and relative velocity of the moving core. In the previously presented work[(4)] thermal entrance region heat transfer of non-Newtonian fluids in concentric annuli with moving cores was investigated by neglecting fluid axial heat conduction.

In this study the energy equation including the viscous dissipation term and the fluid axial heat conduction term has been solved numerically with the finite difference method by applying the fully developed velocity profiles of non-Newtonian fluids presented in the previous report[(8)].

Received on October 25, 2002
* Graduate School of Science and Technology
** Department of Mechanical Systems Engineering

Nomenclature

Br	Brinkman number
c_p	specific heat at constant pressure, [J/(kg· K)]
D_h	hydraulic diameter $\equiv 2(R_o - R_i)$, [m]
E	constant of the axial transformation
h	heat transfer coefficient, [W/(m^{2}K)]
k	thermal conductivity, [W/(m· K)]
m	consistency index [N $\cdot$ s^n/m^2]
n	flow index
Nu	Nusselt number
Pe	Peclet number
Pr_M	modified Prandtl number
r	radial coordinate, [m]
r^*	dimensionless radial coordinate $\equiv r/D_h$
R	radius, [m]
Re_M	modified Reynolds number
T	temperature, [K]
u	velocity of the fluid, [m/s]
u_m	average velocity of the fluid, [m/s]
u^*	dimensionless velocity $\equiv u/u_m$
U	axial velocity of the moving core, [m/s]
U^*	dimensionless relative velocity of the moving core $\equiv U/u_m$
z	axial coordinate, [m]
z^*	dimensionless axial coordinate $\equiv z/(PeD_h)$
z_t	transformed axial coordinate

Greek Symbols

α	radius ratio $\equiv R_i/R_o$
β	dimensionless shear rate parameter
η_a	apparent viscosity, [kg/(m· s)]
η_a^*	dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0	viscosity at zero shear rate, [kg/(m· s)]
η^*	reference viscosity, [kg/(m· s)]
ρ	density, [kg/m^3]
θ	dimensionless temperature
ξ	transformed dimensionless radial coordinate $\equiv [2(1-\alpha)r^* - \alpha]/(1-\alpha)$

Subscripts

b	bulk
e	entrance
fd	fully developed
i	inner tube
o	outer tube

2. Analysis

The physical model for the analysis is shown in Fig.1. The core tube moves axially at a constant velocity, U. The assumptions and conditions used in the analysis are:

- The flow is incompressible, steady-laminar, and fully developed hydrodynamically.
- The fluid is non-Newtonian and the shear stress may be described by the modified power-law model(9), and physical properties are constant except viscosity.
- The body forces are neglected.
- The entering fluid temperature, T_e, is uniform at upstream infinity ($z \to -\infty$) and the outer tube wall is insulated. Constant heat flux at the inner moving core for $0 \leq z$, whereas for $z < 0$ the inner tube is insulated.

The energy equation together with the assumptions above is written as

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\eta_a\left(\frac{du}{dr}\right)^2=\rho c_p u\frac{\partial T}{\partial z} \quad (1)$$

in $R_i \leq r \leq R_o$, and $-\infty \leq z \leq \infty$

$$\begin{cases} -k\frac{\partial T}{\partial r} = q_i & \text{at} \quad r = R_i \quad 0 \leq z \\ k\frac{\partial T}{\partial r} = 0 & \text{at} \quad r = R_o \quad 0 \leq z \\ k\frac{\partial T}{\partial r} = 0 & \text{at} \quad r = R_i \quad z < 0 \\ k\frac{\partial T}{\partial r} = 0 & \text{at} \quad r = R_o \quad z < 0 \\ T = T_e & \text{at} \quad R_i \leq r \leq R_o \quad z \to -\infty \\ T = T_{fd} & \text{at} \quad R_i \leq r \leq R_o \quad z \to +\infty \end{cases} \quad (2)$$

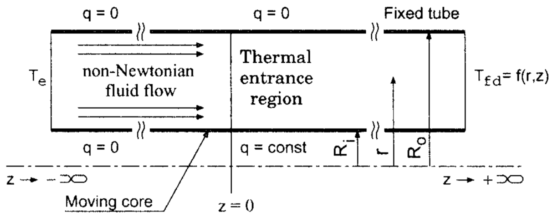

Fig.1 Schematic of a concentric annulus with an axially moving core

Bulk temperature and Nusselt number are defined as

$$T_b \equiv \frac{2}{u_m\left(R_o^2 - R_i^2\right)}\int_{R_i}^{R_o} uTrdr \quad (3)$$

$$Nu_i \equiv \frac{h_i D_h}{k} \quad (4)$$

Heat transfer coefficient is:

$$h_i = \frac{q_i}{T_i - T_b} \quad (5)$$

The following dimensionless variables are introduced

$$r^* = \frac{r}{D_h} \qquad z^* = \frac{z}{PeD_h} \quad (6)$$

$$u^* = \frac{u}{u_m} \qquad \beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n} \quad (7)$$

$$\alpha = \frac{R_i}{R_o} \qquad \theta = \frac{k(T-T_e)}{q_i D_h} \quad (8)$$

where

$$Pe = Re_M \cdot Pr_M \quad (9)$$

$$Re_M \equiv \frac{\rho u_m D_h}{\eta^*} \qquad Pr_M \equiv \frac{c_p \eta^*}{k} \quad (10)$$

$$Br = \frac{\eta^* u_m^2}{q_i D_h} \quad (11)$$

With the substitution of the above quantities into the dimensional formulation, the dimensionless energy equation and boundary conditions are obtained as

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta}{\partial r^*}\right)+\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial z^{*2}}+Br\cdot\eta_a^*\left(\frac{du^*}{dr^*}\right)^2 = u^*\frac{\partial\theta}{\partial z^*} \quad (12)$$

in $\frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)}$ and $-\infty \leq z^* \leq \infty$

$$\begin{cases} \frac{d\theta}{dr^*} = -1 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} & 0 \leq z^* \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)} & 0 \leq z^* \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} & z^* < 0 \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)} & z^* < 0 \\ \theta = 0 & \text{at} \quad \frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)} & z^* = -\infty \\ \theta = \theta_{fd} & \text{at} \quad \frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)} & z^* = +\infty \end{cases} \tag{13}$$

In order to convert the upstream and downstream infinities, the dimensionless axial coordinate z^* is transformed according to the relation employed by Verhoff and Fisher[10] as follows:

$$z^* = E\tan(\pi z_t) \quad \text{or} \quad z_t = \frac{1}{\pi}\arctan\frac{z^*}{E} \tag{14}$$

By introducing the transformed coordinate, z_t, the energy equation and the boundary conditions become

$$\frac{\partial^2\theta}{\partial r^{*2}} + B\frac{\partial^2\theta}{\partial z_t^2} + \frac{1}{r^*}\frac{\partial\theta}{\partial r^*} + Br\cdot\eta_a^*\left(\frac{du^*}{dr^*}\right)^2 = A\frac{\partial\theta}{\partial z_t} \tag{15}$$

in $\frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)}$ and $-\infty \leq z^* \leq \infty$

where

$$A = \frac{\cos^2(\pi z_t)}{\pi E}\left[u^* + \frac{1}{Pe^2}\frac{\sin(2\pi z_t)}{E}\right] \tag{16}$$

$$B = \frac{1}{Pe^2}\left[\frac{\cos^2(\pi z_t)}{\pi E}\right]^2 \tag{17}$$

$$\begin{cases} \frac{d\theta}{dr^*} = -1 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)}, & 0 \leq z_t \leq 0.5 \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)}, & 0 \leq z_t \leq 0.5 \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)}, & -0.5 \leq z_t < 0 \\ \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)}, & -0.5 \leq z_t < 0 \\ \theta = 0 & \text{at} \quad \frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)} & z^* = -0.5 \\ \theta = \theta_{fd} & \text{at} \quad \frac{\alpha}{2(1-\alpha)} \leq r^* \leq \frac{1}{2(1-\alpha)} & z^* = 0.5 \end{cases} \tag{18}$$

For infinitely large values of the axial distance ($z^* \to \infty$), thermally fully developed region is reached. Since the definition of dimensionless temperature is different in this uniform heat flux boundary condition case, the fully developed temperature profile is calculated differently from that in the previous report[1]. It can be derived as follows. To seek the expression for θ_{fd}, for example, a solution of the form:

$$\theta_{fd} = Cz^* + \psi(r^*) \tag{19}$$

is assumed on the fact that in the thermally fully developed region, the temperature solution is a linear function of z^*. Substitution of Eq.(19) into Eqs.(12), (13) yields

$$\frac{d^2\psi}{dr^{*2}} + \frac{1}{r^*}\frac{d\psi}{dr^*} = Cu^* - V \tag{20}$$

$$\begin{cases} \frac{d\psi}{dr^*} = -1 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} \\ \frac{d\psi}{dr^*} = 0 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)} \end{cases} \tag{21}$$

where

$$V = Br\cdot\eta_a^*\left(\frac{du^*}{dr^*}\right)^2 \tag{22}$$

On the other hand, in the thermally developed region

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{fd}}{\partial r}\right) + \eta_a\left(\frac{du}{dr}\right)^2 = \rho c_p u\frac{dT_b}{dz} \tag{23}$$

$$\begin{cases} -k\frac{\partial T_{fd}}{\partial r} = q_i & \text{at} \quad r = R_i \\ k\frac{\partial T_{fd}}{\partial r} = 0 & \text{at} \quad r = R_o \end{cases} \tag{24}$$

dT_b/dz in Eq.(23) is evaluated, from an energy balance, as

$$\frac{dT_b}{dz} = \frac{2\,R_i q_i}{\rho c_p u_m\left(R_o^2 - R_i^2\right)}\left[1 + \frac{\int_{R_i}^{R_o} r\,\eta_a\left(\frac{du}{dr}\right)^2 dr}{R_i q_i}\right] \tag{25}$$

Substitution of the above balance into Eq.(23) gives,

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{fd}}{\partial r}\right) + \eta_a\left(\frac{du}{dr}\right)^2 = \frac{2uR_i q_i}{u_m\left(R_o^2 - R_i^2\right)}\left[1 + \frac{\int_{R_i}^{R_o} r\,\eta_a\left(\frac{du}{dr}\right)^2 dr}{R_i q_i}\right] \tag{26}$$

By introducing the relevant dimensionless quantities the above equation becomes

$$\frac{\partial^2\theta_{fd}}{dr^{*2}} + \frac{1}{r^*}\frac{\partial\theta_{fd}}{dr^*} = \frac{4u^*}{(1+\alpha)}\left\{\alpha + 2(1-\alpha)\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} V\cdot r^* dr^*\right\} - V \tag{27}$$

According to Eq.(19)

$$\frac{\partial^2 \theta_{fd}}{\partial r^{*2}} = \frac{d^2\psi}{dr^{*2}} \quad \text{and} \quad \frac{\partial \theta_{fd}}{\partial r^*} = \frac{d\psi}{dr^*} \tag{28}$$

Thus

$$\frac{d^2\psi}{dr^{*2}} + \frac{1}{r^*}\frac{d\psi}{dr^*}$$
$$= \frac{4u^*}{(1+\alpha)}\left\{\alpha + 2(1-\alpha)\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} V\cdot r^* dr^*\right\} - V \tag{29}$$

The coefficient C was found by comparing Eqs.(20) and (29) as

$$C = \frac{4}{(1+\alpha)}\left\{\alpha + 2(1-\alpha)\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} V\cdot r^* dr^*\right\} \tag{30}$$

$\psi(r^*)$ was calculated from Eq.(20) with Eq.(21) by the finite difference method. θ_{fd} was calculated from Eq.(19) and used as a boundary condition to solve the problem.
Nusselt number at the heated core is

$$Nu_{\mathrm{i}} = \frac{1}{(\theta_{\mathrm{i}} - \theta_{\mathrm{b}})} \tag{31}$$

3. Results and discussion

The work presented previously for laminar heat transfer to non-Newtonian fluids in concentric annuli under conditions of constant wall temperature has been extended to the case of constant wall heat flux. The results are discussed in terms of temperature profiles and Nusselt numbers for the different values of relative velocity of the core, Brinkman number, Peclet number and radius ratio. The calculation procedure to compute the heat transfer problem in this study has already been described in the previous report(1).

The results are presented graphically in dimensionless form and the effects of moving boundary, viscous dissipation, fluid axial heat conduction, rheological properties and radius ratio are demonstrated.

The range of parameters considered and the calculation technique are the same as in the counterpart reported in the previous report(1). In order to verify the accuracy of the present calculation, the results are compared for the special cases and shown in Fig.2. Temperature of a Newtonian fluid at the outer and inner tubes and Nu at the heated core are shown for $Br = 0.0$ and for various Pe number in Fig.2 for $U^* = 0.0$ and 1.0, respectively. It is shown the Nusselt number and the temperature at the walls corresponding to $Pe \to \infty$ agree excellently with those obtained by Shah and London(5), and Shigechi and Araki(6), who analyzed the corresponding problem by assuming negligible axial fluid heat conduction and viscous dissipation. From these figures demonstrating the effect of fluid axial heat conduction on Nu for a Newtonian fluid, it is seen the effect of fluid axial heat conduction is considerable for small values of Pe and in the fully developed region there is no fluid axial heat conduction.

Figure 3 illustrates the variation of local fluid temperature profiles for the cases of $U^* = 0.0$ and 1.0, respectively. These results, which are shown for $Br = 0.0$, 0.05, 0.1 and, for $Pe = 10$ and $Pe \to \infty$ show the combined effects of viscous dissipation and fluid axial heat conduction on the developing temperature profiles with regard to relative velocity, U^*. In the adiabatic tube wall region ($z^* < 0$) the fluid temperature is seen to be sufficiently large for large Br or small Pe. In fact, it can be seen that the fluid temperature increases significantly before the fluid reaches the heated wall because of the heat generated by viscous dissipation and the heat conducted upstream into the adiabatic wall region. This is main reason why Nu remains almost constant throughout the thermal entrance region for small Pe and large Br.

That is not the case for $Pe \to \infty$ and $Br = 0.0$. The fluid temperature does not increase in $z^* < 0$. As soon as z^* becomes positive, the fluid temperature profile undergoes rather rapid change because of the wall heat flux, causing a relatively abrupt decrease of Nu at the wall as it is seen in Nu curves.

It is also seen if the channel is infinitely long, there may be generated a significant amount of heat due to viscous shear heating ($Br > 0$) if the walls are insulated. Thus the value of the fluid temperature increases sufficiently.

From the developing temperature profiles it is seen for the second kind of boundary condition, the wall-to-fluid temperature difference is small, whereas the effect of viscous dissipation on heat transfer is more significant.

Figure 4 presents the calculation results for Nu, defined with respect to the temperature difference of bulk to wall temperature for a pseudoplastic fluid whose $n = 0.5$, $\beta = 1$ and $Br = 0.0$, 0.01, 0.05, 0.1. These curves clearly exhibit the trend of

Nu in the thermal entrance region by comparison them with the relative velocity U^*. The circles in Fig.4 correspond to the calculation results for the fully developed Nusselt number values presented in the previous report[3].

It can be observed that, from the Nu curves (Figs.2 and 4) for $Br = 0.0$, Nu tends to decrease near $z^* = 0.0$ as Pe reduces below 100. This tendency is exactly identical to the results by Hsu[7] on heat transfer with neglected viscous dissipation in a concentric annuli. The Nusselt curves for various Pe numbers, however, cross each other and reverse their orders of magnitude before reaching the fully developed values. It can also be noted that, for a fixed value of z^*, Nu at the moving core is larger than the corresponding Nu at the

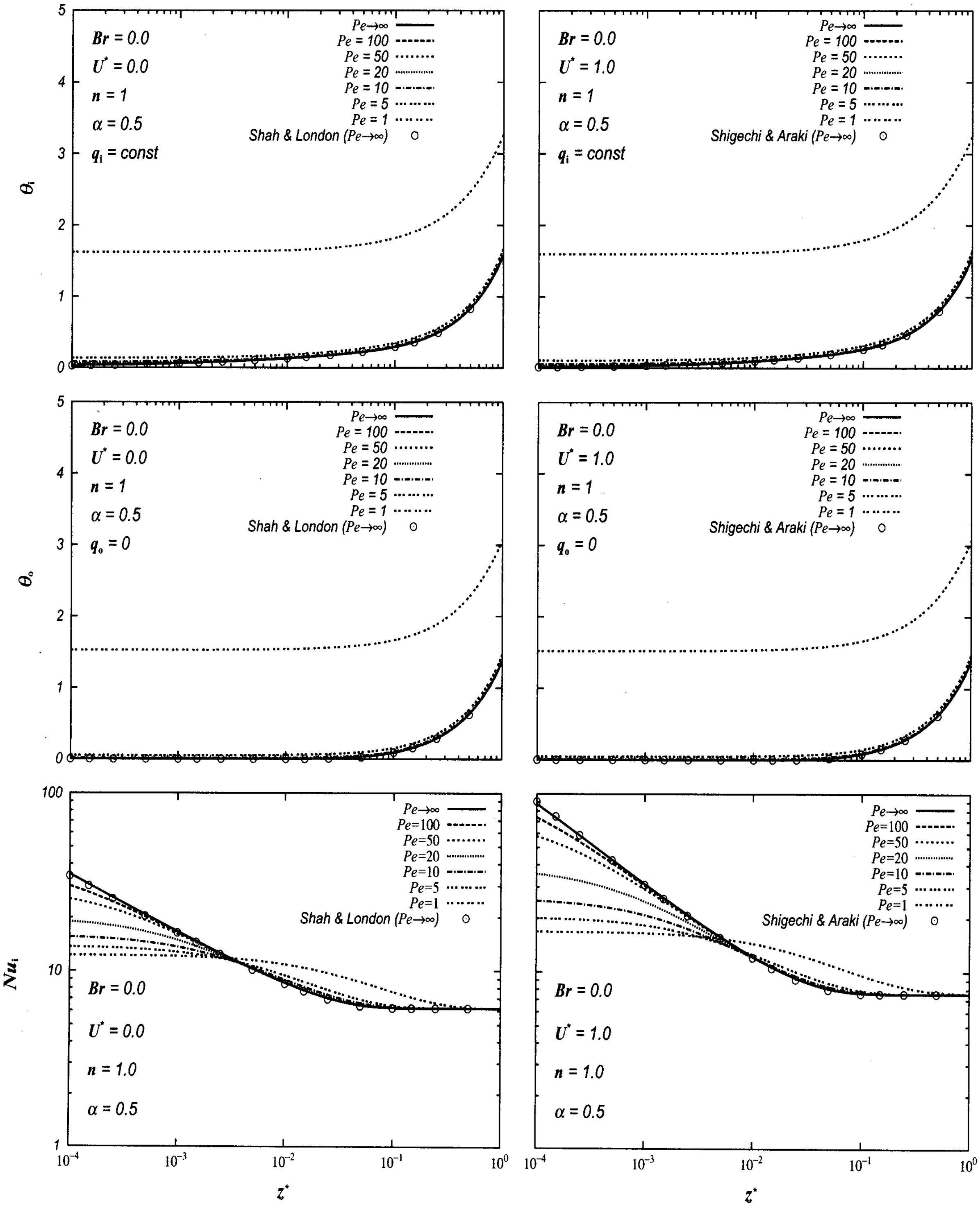

Fig.2 θ_i, θ_o and Nu_i for $n = 1$ ($U^* = 0$ and $U^* = 1$)

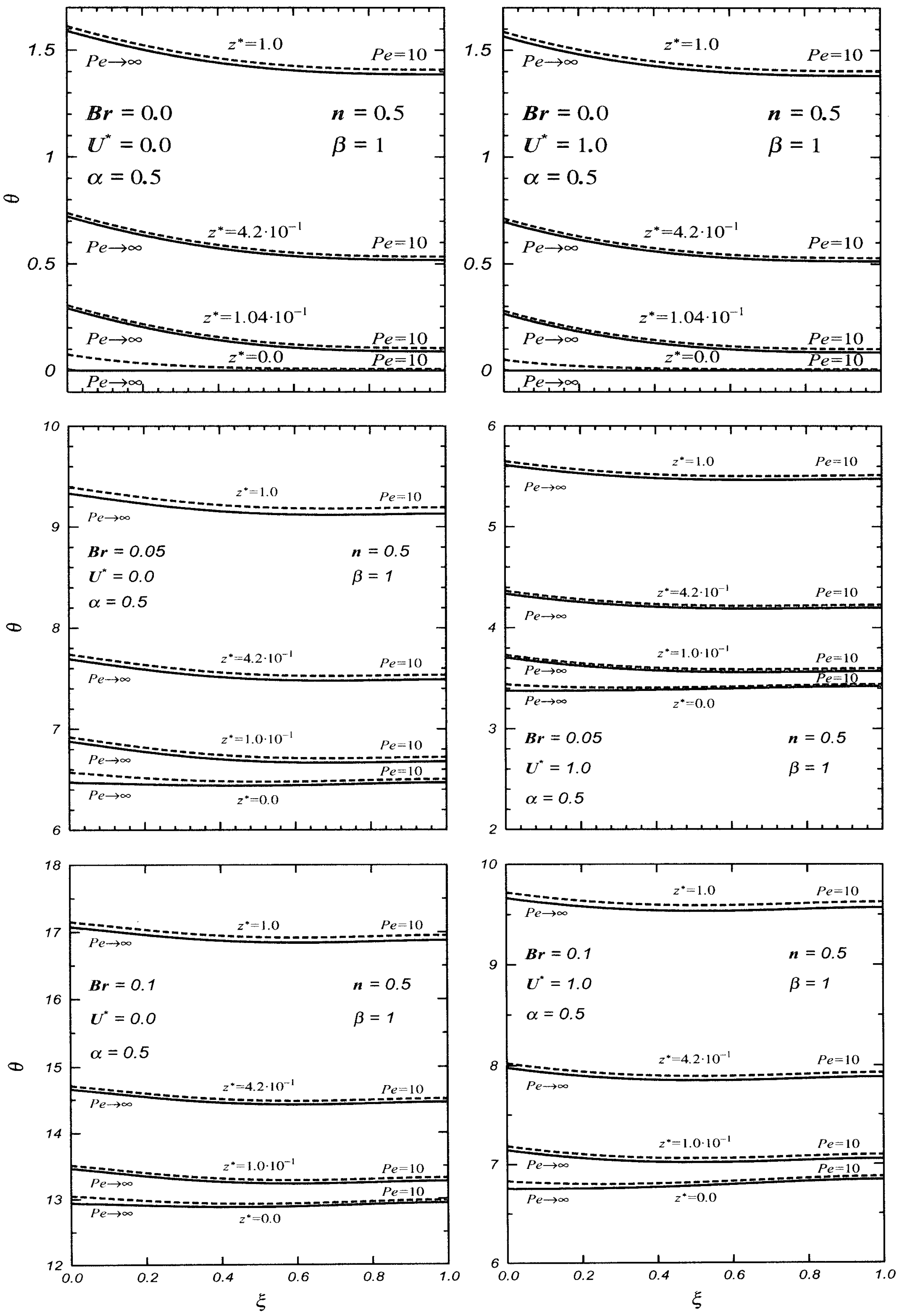

Fig.3 Temperature profiles for $n = 0.5$, $\beta = 1$ ($U^* = 0$ and $U^* = 1$)

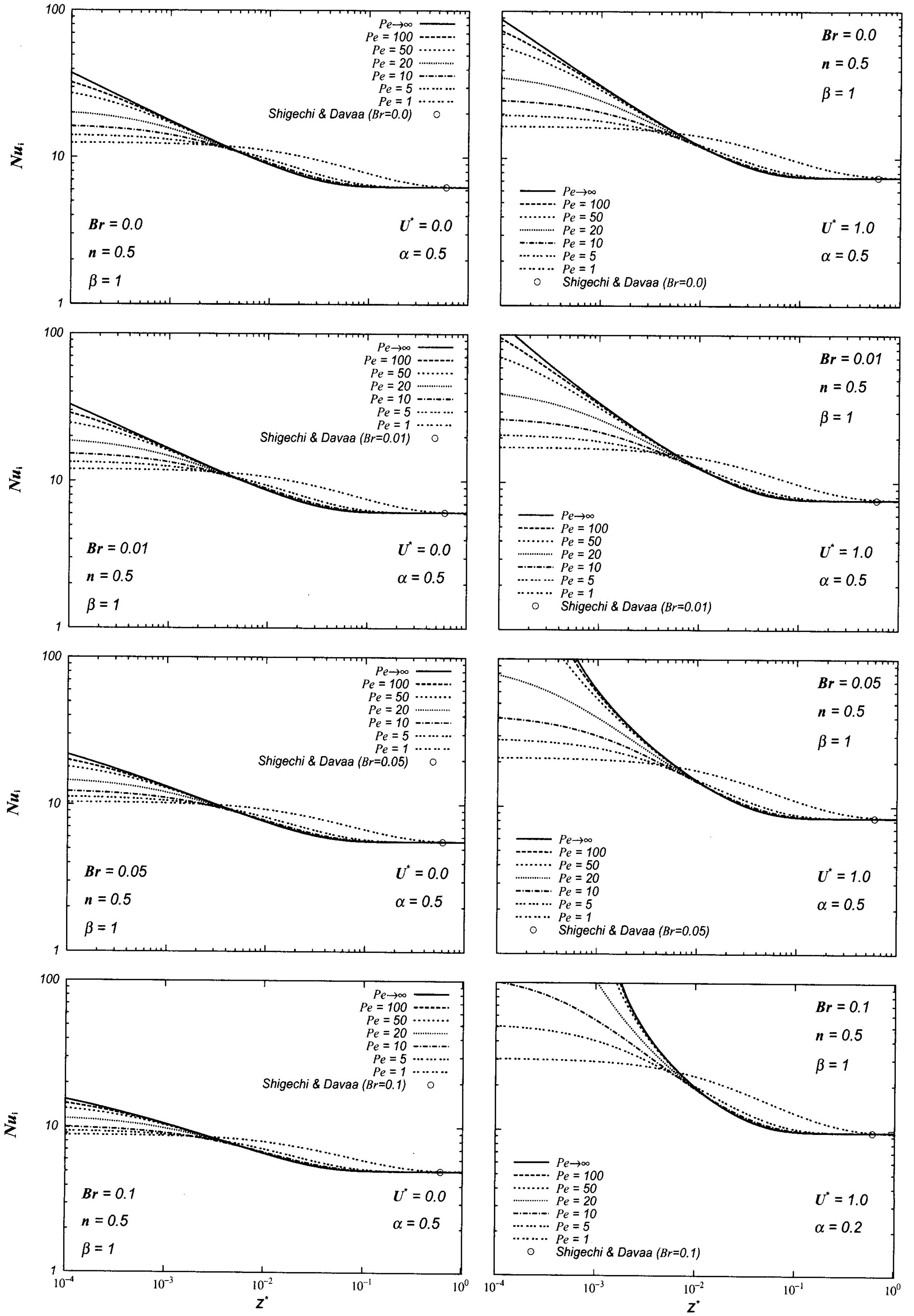

Fig.4 Nu_{i} variations for $n = 0.5$, $\beta = 1$ ($U^* = 0$ and $U^* = 1$)

stationary core. It is seen that Nu remains almost constant throughout the thermal entrance region if Pe is small.

Fig.5 presents the combined effects of viscous dissipation and fluid axial heat conduction on Nu at the tubes of the annuli (α = 0.2, 0.5 and 0.8) for a pseudoplastic fluid whose $n = 0.5$, $\beta = 1$ for $U^* = 0$ and $U^* = 1$, respectively. The present numerical solutions for the fully developed Nu agree very well with the previous study[(3)]. From these figures it is seen the effect of fluid axial heat conduction accounts for the change in the curve shape in the thermal entrance region. Br has a strong effect on Nu in both thermally developing and de-

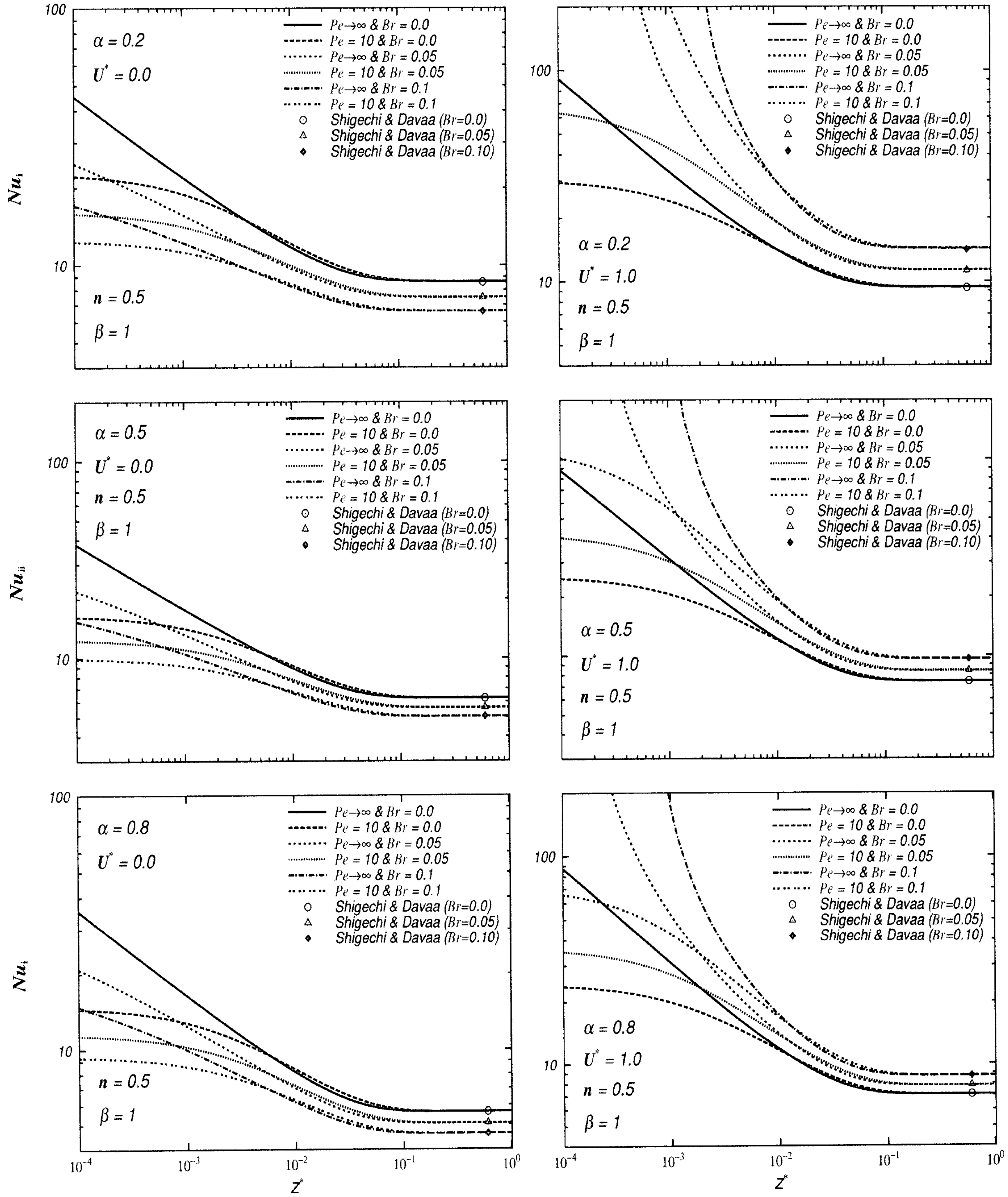

Fig.5 Nusselt number for $Pe = \infty$ and 10 ($U^* = 0$ and $U^* = 1$)

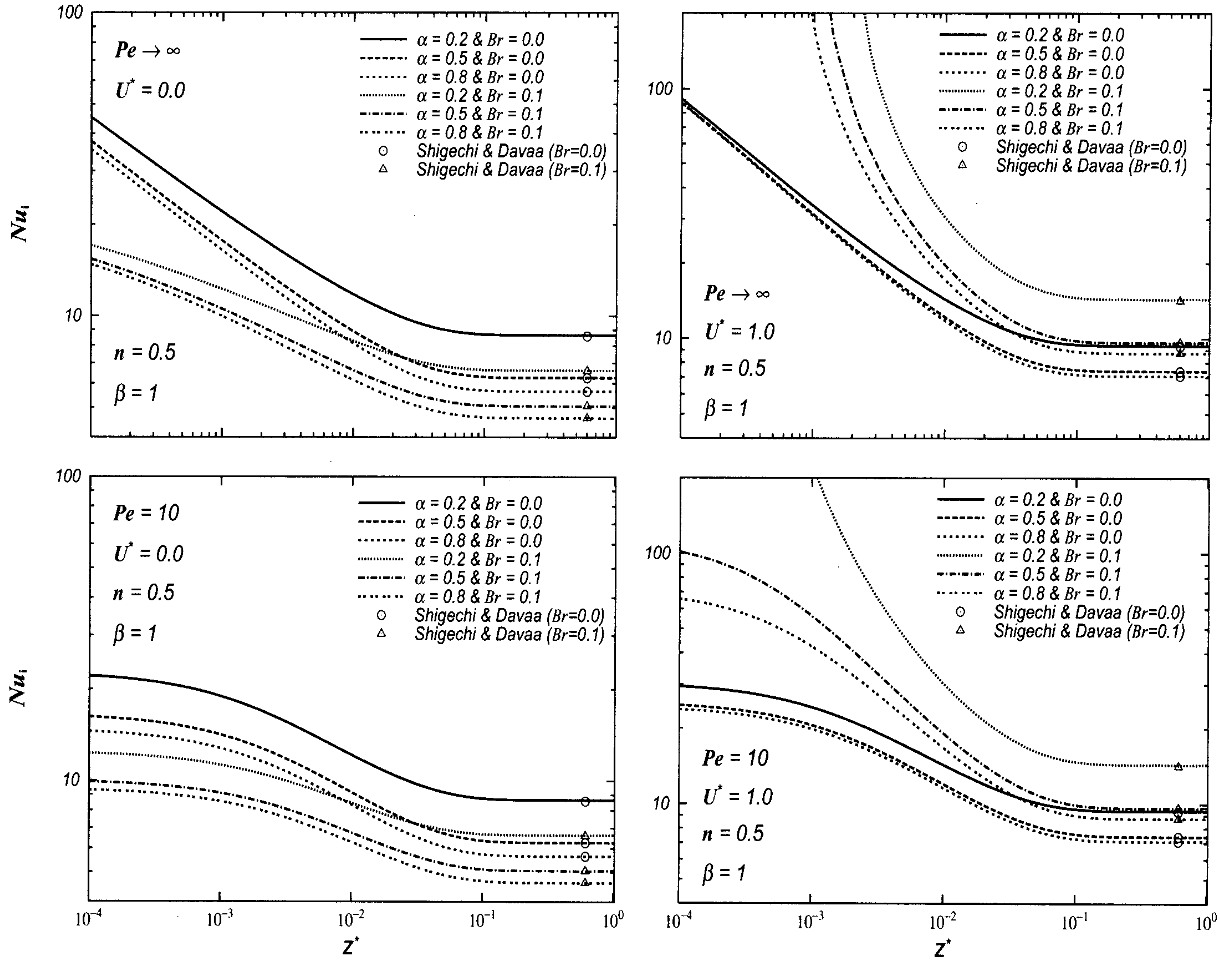

Fig.6 Nusselt number for α = 0.2, 0.5, 0.8 (U^* = 0 and U^* = 1)

veloped regions. For $U^* = 0$, Nusselt number at the core decreases with an increase in Brinkman number in the fully developed region. For $U^* = 1$ these are opposite.

In order to study the radius ratio effects, heat transfer results are shown in Fig.6 for $Pe \to \infty$ and $Pe = 10$, respectively. It seen the general behavior is quite similar for different values of Pe and the results show that the radius ratio $\alpha = 0.2$ is superior to $\alpha = 0.5$ and $\alpha = 0.8$ from the view point of heat transfer at the core under the same conditions.

4. Conclusions

The problem of laminar heat transfer in the thermal entrance region including viscous dissipation of the flowing fluid and fluid axial heat conduction with the boundary condition of constant heat flux is analyzed by considering an infinite axial domain.

The sample results to demonstrate the combined effects of moving boundary, viscous dissipation and axial fluid heat conduction are presented graphically in dimensionless form. The temperature solutions corresponding to the limiting cases $Pe \to \infty$, and $Br = 0.0$ show excellent agreement with those reported in[5] for $U^* = 0.0$ and in[6] for $U^* = 1.0$, who analyzed the entrance region heat transfer by neglecting viscous dissipation and axial fluid heat conduction for Newtonian fluids.

Viscous dissipation effect on Nu is different depending on U^* in both thermally developing and fully developed flows. From the developing temperature profiles it is seen for the second kind of boundary condition, the tube-to-fluid temperature difference is small, whereas the effect of viscous dissipation on heat transfer is more significant.

A comparison of developing temperature profiles for the different boundary conditions reveals that the increase in fluid temperature due to viscous heating for the second kind of boundary condition is much higher than for the first kind of boundary condition.

For constant heat flux boundary condition, when viscous dissipation is negligible, including the effect of fluid axial heat conduction in the analyses results in lesser values of the Nu near $z^* = 0$ (where the wall heat flux commences) compare to the case without axial heat conduction.

It may also be concluded that the moving boundary may make less increase in the fluid temperature compare to the stationary core boundary case and in turn it results higher values of Nu number at the wall as U^* increases.

References

1. Ganbat Davaa, Toru Shigechi, Satoru Momoki and Odgerel Jambal, "Laminar Heat Transfer in Concentric Annuli with Viscous Dissipation and Fluid Axial Heat Conduction" (Part I: Thermal boundary condition of the first kind), *Reports of the Faculty of Engineering, Nagasaki University*, vol.33, No.60, (2003).
2. Ganbat Davaa, Toru Shigechi and Satoru Momoki, "Effect of Moving Core Velocity and Viscous Dissipation on Fully Developed Laminar Heat Transfer in Concentric Annuli", *Reports of the Faculty of Engineering, Nagasaki University*, vol.31, No.56, 13-22, (2001).
3. Ganbat Davaa, Toru Shigechi and Satoru Momoki, "Heat Transfer for Modified Power Law Fluids in Concentric Annuli with Heated Moving Cores", *Reports of the Faculty of Engineering, Nagasaki University*, vol.32, No.58, 91-98, (2002).
4. Ganbat Davaa, Toru Shigechi and Satoru Momoki, "Laminar Heat Transfer in the Thermal Entrance Region of Concentric Annuli with Moving Heated Cores" (Part I: the first and second kinds of thermal boundary conditions), *Reports of the Faculty of Engineering, Nagasaki University*, vol.32, No.59, 41-50, (2002).
5. R.K.Shah and A.L.London, "Laminar Flow Forced Convection in Ducts", *Advances in Heat Transfer*, Supplement 1, Academic Press, (1970).
6. K.Araki, "Laminar Heat Transfer in Annuli", *Department of Mechanical Engineering, Nagasaki University*, Master Thesis, (1991) (in Japanese).
7. C.J.Hsu, "Theoretical Solutions for Low-Peclet-number Thermal-entry-region Heat Transfer in Laminar Flow Through Concentric Annuli", *Int.J. Heat Mass Transfer*, vol.13, No.12, 1907-1924, (1970).
8. Ganbat Davaa, Toru Shigechi and Satoru Momoki, "Fluid Flow for Modified Power Law Fluids in Concentric Annuli with Axially Moving Cores", *Reports of the Faculty of Engineering, Nagasaki University*, vol.32, No.58, 83-90, (2002).
9. S.Kakac, R.K.Shah and W.Aung (eds.), "Handbook of Single-Phase Convective Heat Transfer", *Wiley, New York*, 20.2-20.32, (1987).
10. F.H.Verhoff and D.P.Fisher, "A Numerical Solution of the Graetz Problem with Axial Conduction Included", *Journal of Heat Transfer, Trans. ASME*, vol.95, No.1, 132-134, (1973).